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Application of Fractional PID Controller to Single and Multi-Variable Non-Minimum Phase Systems

Pritesh Shah, Ravi Sekhar, Sudhir Agashe

Abstract: *A non-minimum phase system has the unique characteristic of undershoot or over and undershoot based on the number of zeros and the location of zeros in the systems. A fractional PID controller has the ability to capture more dynamics, as there are two more parameters to tune compared to a traditional PID controller. In this paper, a fractional PID controller is designed for single and multi-variable non-minimum phase systems. A simple optimization method for the tuning of a fractional PID controller has been applied. Six different single variable plants were simulated covering different cases of non-minimum phase systems. Simulation results showed that zero crossing is reduced to a greater extent by fractional PID controller as compared to traditional PID controller for single variable systems. This paper also provides experimental validations for design and tuning of a fractional PID controller for a multi-variable non-minimum phase quadruple-tank system. Fine tuned experimental results agree well with simulation results, thereby validating the applicability of the fractional PID controller for multi variable non-minimum phase systems.*

Index Terms: *Fractional PID Controller, Non Minimum Phase, Fractional Calculus, Quadruple Tank System, Multi-variable Control System.*

I. INTRODUCTION

If a transfer function has poles and/or zeros in the right-hand side of the s-plane, then this system is referred to as a non-minimum phase system [15]. Non-minimum phase systems exhibit the phenomena of undershoot or over- and undershoot based on the number of zeros and the location of zeros in the systems. Bicycle dynamics is a good example of a non-minimum phase system [4]. A design based on the fuzzy PID controller using a genetic algorithm for non-minimum phase system was discussed by Tzuu-Hseng et al. in 1997 [27, 51]. Astrom [5] in 1980 proposed direct methods for non-minimum phase systems. In his work, a pole-placement method and adaptive algorithm were discussed. A variable-structure approach was presented by Bartolini and many others in different papers [9, 8]. A PID controller is also used in the design of control systems having non-minimum phase behavior [24, 30, 52]. In most of these works, advanced control-system theory is used to control non-minimum phase systems [19, 38, 20]. The design of an advanced control system is very complex and time consuming. In case of a PID controller, work appears to be limited to simulation and covers only a few cases of non-minimum phase systems. So, a simple fractional PID (FPID) controller design approach is proposed to control non-minimum phase systems. Fractional calculus has been highly regarded by mathematicians since its inception. In the last few decades, the use of fractional calculus in science and engineering has progressed remarkably. Fractional PID controller is one of the applications of fractional calculus [33, 32, 17, 53]. In the current study, a fractional PID controller was used for reducing the effect of non-minimum phase system characteristics (zero crossing characteristics). A simple optimization approach was implemented to design fractional PID controller for non-minimum phase systems. Six different single variable non-minimum phase plants were simulated. Closed-loop responses of fractional PID controller were compared with those from a traditional PID controller. Multi variable plant simulation results were verified with a quadruple-tank system capable of being set up as a multi variable minimum or a non-minimum phase system. This paper is structured as follows. In the following section, fractional calculus and fractional PID controller are introduced. Different types of non-minimum phase systems are also described. Section 2 outlines methodology adopted in the current work, wherein firstly the six single variable simulation plants are described. Multi variable experimental set-up details are also provided. Tuning methodology of FPID controller is presented with a flow chart to show optimization approach used in this work. In Section 3, open loop / PID / FPID controller results of single / multi variable simulation plants and multi variable experimental model are compared and discussed. Tuned parameters and time domain specifications are also presented in this section. Conclusions and recommendations are provided in Section 4.

1.1 Non-Minimum Phase System

Minimum and non minimum phase systems can be described respectively by the following transfer functions,

$$G_1(s) = \frac{1+sT}{1+sT_1} \quad (1)$$

$$G_2(s) = \frac{1-sT}{1+sT_1} \quad (2)$$

Amplitude responses for both systems are same, but the phase responses are different.

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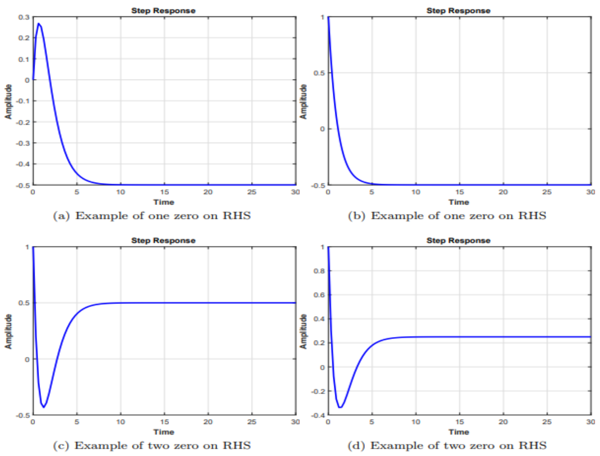
* Correspondence Author
Pritesh Shah*, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Pune, India
Ravi Sekhar, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Pune, India.
Sudhir Agashe, Department of Instrumentation and Control, College of Engineering, Pune, India.

In case of non-minimum phase systems, $G_2(s)$, phase contribution is more than that of minimum phase systems $G_1(s)$. In non minimum systems, mostly there is an undershoot or over and understood response. Zero crossing for this kind of response is better minimized using a fractional PID controller.
Typical examples of step responses for non-minimum phase systems are shown in Fig. 1,covering cases of one and two positive poles located on the right-hand side of the s-plane.



(a) Example of one zero on RHS (b) Example of one zero on RHS (c) Example of two zero on RHS (d) Example of two zero on RHS

Figure 1: Typical examples step responses of non minimum phase systems.

Non-minimum phase system behavior can be observed in many plants, e.g., DC motor with field regulation, hydraulic pumps, blast furnace [13], quadruple-tank system [25], conventional aircraft motion having pitch axis control surface aft of the center of mass [16], transportation lag in control system [15] etc. A system may become non-minimum phase system if its minimum phase continuous time system is transformed into an equivalent discrete model. A non-minimum phase system often occurs because of time delays that are not integral multiples of the sample interval [13]. For example, in case of level control of a volume of boiling water, when cold water is added to raise the level, the initial effect is bubble formation. This leads to an initial reduction in water level [26].
There are many issues related to control system design of non-minimum phase systems:

- System responds in the opposite direction of the steady state;
- Internal stability problem;
- Problem in phase response;
- Value of phase angle greater than 90 degrees;
- Limitation in control bandwidth, resulting in limited disturbance rejection; and
- Slower closed-loop response.

The following sub-section introduces fractional calculus to understand the basis of fractional PID controller used to control non minimum phase systems.

1.2 Fractional Calculus

Fractional calculus is three centuries older than classical calculus, but very less popular in the research field [14]. In the last few decades, many researchers have conducted investigations in different areas of science and engineering (control systems, speech signal processing, modeling, etc.) using fractional calculus [17].
There are numerous definitions of fractional differentiation-integral available in many books and papers on fractional calculus. From an engineering point of view, the Caputo definition is the most popular [12, 1, 32]. It's popularity is due to a straightforward connection between the type of initial conditions and the type of fractional derivative. As stated by Podlubny [41], this definition allows initial conditions such as $y''(0), y'(0)$ etc., unlike fractional condition such as $y^{0.4}(0)$. A derivative of constant is bounded in the case of the Caputo definition. This definition is given by:

$$D^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (3)$$

where,

$$(n-1) \leq \alpha \leq n,$$

n is an integer number and α is a real number and a and t are the limit of integration. Say, if α is 0.8, then n would be 1 as $0 \leq 0.8 \leq 1$. Two other definitions are given as follows. Riemann-Liouville fractional definition is defined by

$$D^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (4)$$

where,

$$(n-1) \leq \alpha \leq n$$

n is an integer number, α is a real number and J is the integral operator and a and t are the limits of integration.
Grunwald-Letnikov's fractional definition is defined by

$$D^{\alpha} = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{r=0}^{[\frac{t-a}{h}]} (-1)^r \binom{n}{r} f(t-rh) \quad (5)$$

where, $[\frac{t-a}{h}]$ is the integer part and a and t are limits of integration.

1.3 Fractional PID Controller

Fractional-order controller was introduced by Podlubny for fractional-order systems [41, 39, 40] in 1994. The beauty of this controller is that it is less sensitive to changes in system variables and tuning parameters of controller [28, 29]. Fractional PID controller has an iso-damping property and is more robust than other classical controllers [41]. A fractional PID controller has five parameters to tune as shown in Eq. (6). A block diagram of a fractional PID controller is shown in Fig. 2. It has the following structure [46, 7]

$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu}, (\lambda, \mu \geq 0) \quad (6)$$

where, $C(s)$ is the controller transfer function, $U(s)$ is the control signal, $E(s)$ is the error signal, K_P is the proportional constant gain, K_I is the integration constant gain, K_D is the derivative constant gain, λ is the order of integration and μ is the order of differentiation.

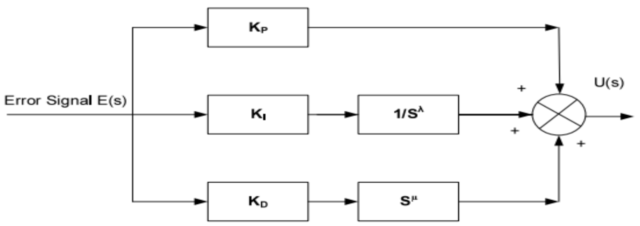


Figure 2: Block diagram of fractional PID controller.

Fractional-order system can be approximated by many methods cited in literature [18, 54, 31]. In this paper, approximation method is used to realize fractional PID controller. Oustaloup recursive approximation is the most popular method for approximate fractional order [11, 37, 36]. It is given by:

$$s^{v} \approx K\prod_{k=-N}^{N}\frac{1+s/\omega_k}{1+s/\omega'_k} \quad (7)$$

The approximation equation above can by synthesized using the following equations,

$$\omega_u = \sqrt{\omega_h\omega_b}$$

where ω_h, ω_b are the frequency bounds for approximation.

$$\omega'_0 = \alpha^{-0.5}\omega_u; \omega_0 = \alpha^{0.5}\omega_u;$$

$$\frac{\omega'_{k+1}}{\omega'_k} = \frac{\omega_{k+1}}{\omega_k} = \alpha\eta > 1$$

$$\frac{\omega'_{k+1}}{\omega_k} = \eta > 0; \frac{\omega_k}{\omega'_k} = \alpha > 0$$

$$N = \frac{\log(\omega_N/\omega_0)}{\log(\alpha\eta)}$$

The fractional PID controller is also implemented in real time applications using analog and digital approximation methods methods [22]. In most cases, the order of fractional PID controller is in the range of 0 to 2 [34, 2, 44]. More details on fractional PID controller can be found in [43, 42, 45].

II. METHODOLOGY

2.1 Single Variable Simulation Plants

In this section, a fractional PID controller is designed for non-minimum phase system. A simple optimization approach is described for the tuning of fractional PID controller. Table 1 shows summary of the single variable plants simulated in present work. First four plants are taken from Guy Beale [10]. In these examples, type 0 and type 1 with one and two zeros are located on the right-hand side (RHS) of the s-plane. Plant 3 and plant 4 are identical except for the sign of plant gain value. Plant 5 is referred from class notes of Roy Smith [47]. In plant 6, there is no real zero, but there are complex zeros on the RHS which show undershoot [23].

Table 1: Transfer functions used for simulation.

Plant	Transfer Function	Remarks
$P_1(s)$	$\frac{-0.5(s-0.1)}{s(s+1)^2}$	Type 1 and one positive zero at RHS
$P_2(s)$	$\frac{0.5(s-0.1)^2}{s(s+1)^2}$	Type 1 and two positive zero at RHS
$P_3(s)$	$\frac{-0.5(s-0.1)}{(s+1)^3}$	Type 0 and one positive zero at RHS
$P_4(s)$	$\frac{0.5(s-0.1)}{(s+1)^3}$	Type 0 and one positive zero at RHS
$P_5(s)$	$\frac{-s+3}{(s+1)(s+5)}$	One positive zero at RHS
$P_6(s)$	$\frac{(s^2-10s+27)}{(s+1)^3}$	Non real non minimum-phase zeros

All these plants are examples of non-minimum phase systems. Practically, one can find one or two zeros on the RHS of the s-plane in most of the non-minimum phase systems. Sometimes, this zero can be complex as shown in plant 6. Quadruple system is also an example of non-minimum phase system based on the location of zeros. In this system, there is only one zero on the RHS of the s-plane. Drum-boiler dynamics is another example of a non-minimum phase system. In this system, the complicated shrink-and-swell dynamics create a non-minimum phase behavior [3]. Even the inverted-pendulum linearized model has one pole and zero on the RHS for certain parameters of the system [23].

2.2 Experimental Set-up of Multi-variable System

A quadruple-tank system experiment set-up is devised to examine the proposed method of FPID controller tuning. This set-up can be configured as minimum and non-minimum phase systems. Moreover, this set-up has multiple inputs and multiple outputs (multi-variable) system with non linearity present in the system. The quadruple-tank experiment system has four tanks and two pumps. Control variables are levels of lower tanks ($h_1$and h_2). These variables are controlled by adjusting the speed of the pumps (υ_1 and υ_2). The flows of the two pumps (P_1 and P_2) are split up by three-way valves. Pump 1 feeds tanks 1 and 4, whereas pump 2 feeds tanks 2 and 3. The flow ratios of pump 1 and pump 2 are given respectively by γ_1 and γ_2.

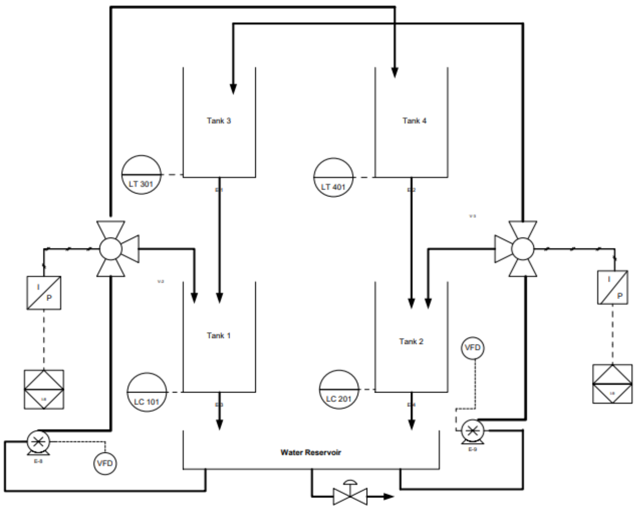


Figure 3: Schematic block diagram of the multi-variable experimental setup.

Figure 4: Multi-variable experimental set up for quadruple tank system.

Schematic block diagram of the experimental setup is shown in Fig. 3. Actual experimental set up is shown in Fig. 4. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are the settings of three-way valves prior to experiment. Changing γ_1 and γ_2 of the three-way valve determines whether the system phase is minimum or non-minimum. It is a non-minimum phase system if $0 < \gamma_1 + \gamma_2 < 1$ and minimum phase system if $1 < \gamma_1 + \gamma_2 < 2$ [25]. The positions of the three-way valves determine the location of multi-variable zero. Different constants for quadruple-tank system are shown in Table 2. During the experiment, γ_1=0.43 and γ_2 = 0.34 are selected such that the system behaves as a non-minimum phase system as explained above. In this set-up, pump flow is a manipulated variable which can be controlled by variable frequency drive (VFD).

Table 2: Constants for experimental set up.

Constant	Description	Value
A_i	Cross section area of tank i	196 cm^2
a_i	Cross section area of the outlet hole(for tank i)	0.64 cm^2
g	Acceleration due to gravity	981 cm/s^2
k_i	Pump flow constants	3.3 cm^3/sV

The quadruple-tank system was connected through Open Platform Communication (OPC) protocol. This protocol establishes communication of real-time plant data between control devices from different PLC manufacturers. Using OPC protocol, data can be read and written in milliseconds. In Simulink, the OPC client can be configured with local or remote host depending upon the location of the OPC server. For read and write operations, OPC read and write a block of Simulink was used with the appropriate tag as configured in the OPC server. The block diagram of hardware connection with Simulink is shown in Fig. 5.

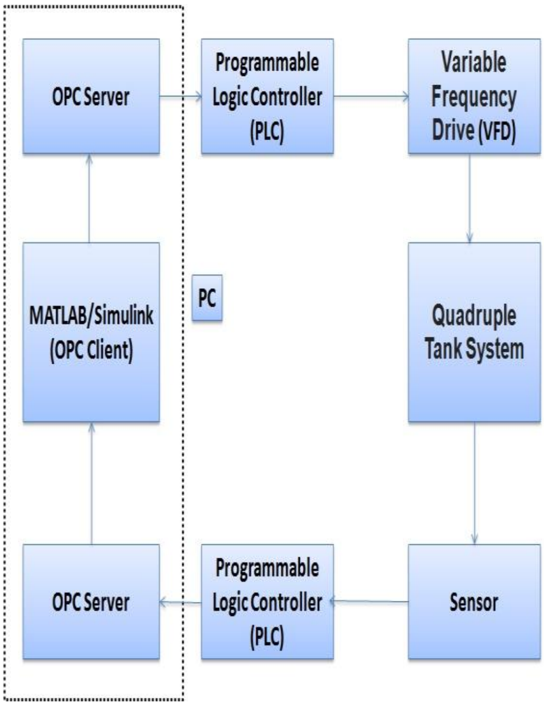


Figure 5: Experimental Set-up Interface with MATLAB/Simulink

The quadruple-tank system is governed by the following equations [25]:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}\upsilon_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}\upsilon_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}\upsilon_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}\upsilon_1 \quad (8)$$

where,

A_i is cross section area of tank i.
a_i is cross section area of the outlet hole (for tank i).
h_i is water level of tank i. (Range: 0 to 60 cm)
υ_i is the voltage applied to pump i.
k_i is the pump i flow constant.

Firstly, FPID controller is tuned using classical tuning method considering $\lambda = \mu = 1$. For tuning using the classical method, non-linear model (Eq. 8) is linearized at an operating point using a linearizing method (Taylor Series Expansion). This model has been derived by researchers as shown below [25, 21].

$$\dot{X} = \begin{pmatrix} -\frac{1}{T_1} & 0 & \frac{1}{T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{1}{T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{pmatrix} X + \begin{pmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{pmatrix} U$$
$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} X \quad (9)$$

where, x is the state of the system(h_1, h_2, h_3 and h_4), U is the system input (υ_1 and υ_2) and Y is the system output (h_1 and h_2).

In Eq. 9, areas of all tanks was the same. Constants' values for this equation are given in Table 2. Using the quadruple tank model, a mathematical model was developed in Simulink as shown in Fig. 6.

This model was tuned by

Simulink block of the PID controller.

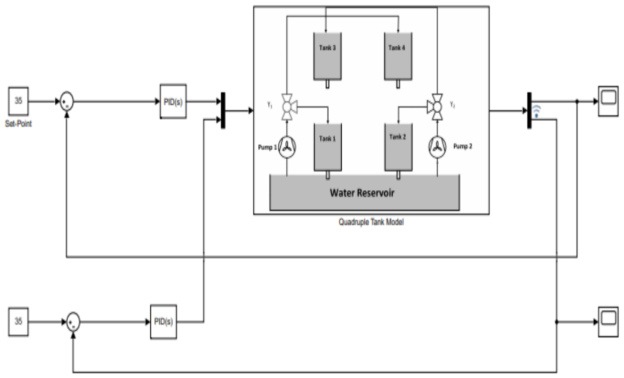


Figure 6: Simulation of multi-variable experimental model in Simulink.

2.3 Optimization Approach

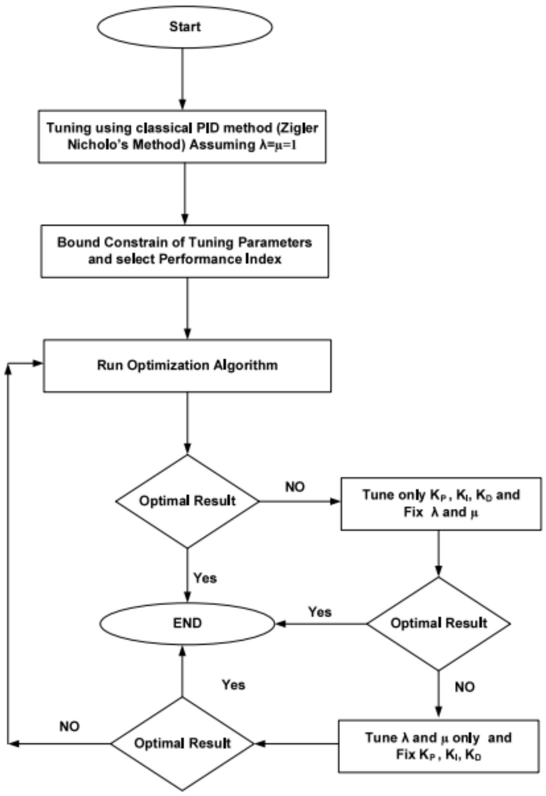


Figure 7: Optimization flowchart for tuning of a fractional PID controller.

In the optimization approach (Fig. 7), initially the system is tuned by a classical tuning method (Ziegler and Nichols) assuming $\lambda = \mu = 1$ [6]. For optimization of controller parameters, a simple Nelder Mead method is used [50, 48, 49, 35]. It is a heuristic search method that can converge on non-stationary points to optimize solutions. Following constrains are considered for the same:

$$0 \leq \lambda, \mu \leq 2$$

and

$$-10 \leq K_P, K_I, K_D \leq 10$$

An Integrated Square Error (ISE) performance index is used as the cost function, defined as:

$$ISE = \int_0^\infty e^2(t)dt \quad (10)$$

where, e(t) is the error signal.

After selecting constraints and performance index, optimization algorithm is applied on cost function. In the optimization algorithm, initial values of parameters are taken from results of the classical tuning method. If an optimal result is not achieved, then only gain parameters (K_P, K_I and K_D) are optimized keeping exponent parameters (λ and μ) constant. Once again, if optimal results are not achieved, then only exponent parameters are optimized, keeping gain parameters constant. The above steps are repeated until the desired results are obtained. Also, one may change constraints of fractional PID controller parameters based on the previous iteration results.

To compare with fractional PID controller results, a classical PID controller is also implemented in Simulink and tuned using Simulink auto-tune feature. A PID controller has the following form:

$$C(s) = P + I\frac{1}{s} + D\frac{N}{1 + N\frac{1}{s}} \quad (11)$$

where, P is the proportional gain, I is the integration gain, D is the derivative gain, and N is the filter coefficient. In this structure, a low pass filter removes high frequency noise from the system. If pure derivative action is required, then a higher value of N is required, generally around 1000 to 10000.

2.4 Fine tuning

In real time plants, fine tuning helps to get better response from the system by suitably varying controller parameters. For PID controller, there are general guidelines are available for fine tuning. Similarly, certain guidelines are also available for FPID controllers in [44]. By changing the orders of differentiation and integration, overshoot of the system can be minimized.The various simulation and experimental results are discussed in the next section.

III. RESULTS AND DISCUSSION

In this section, open loop and step responses of the six simulation plants are discussed. All of these simulated plants are single input single output systems.

3.1 Single Variable Simulation Plants

3.1.1 Single Variable Simulation Plants - Open Loop results

Fig. 8 shows open loop responses of all simulation plants. Table 3 shows zero crossing results of open loop system. Initial undershoots are observed for plants 1, 3, 5 due to presence of one zero in RHS of corresponding s-planes. This kind of phenomenon occurs if and only if the system has an odd number of positive zeros [23]. Longer initial undershoots may lead to inaccurate controller design. In case of plant 4, step response first overshot and then bent towards a negative value. For plant 2 and 6, two zero crossings are observed. First crossing is during initial overshoot and the second one is during second undershoot. This kind of phenomenon exhibits if and only if the system has an even number of positive zeros [23].

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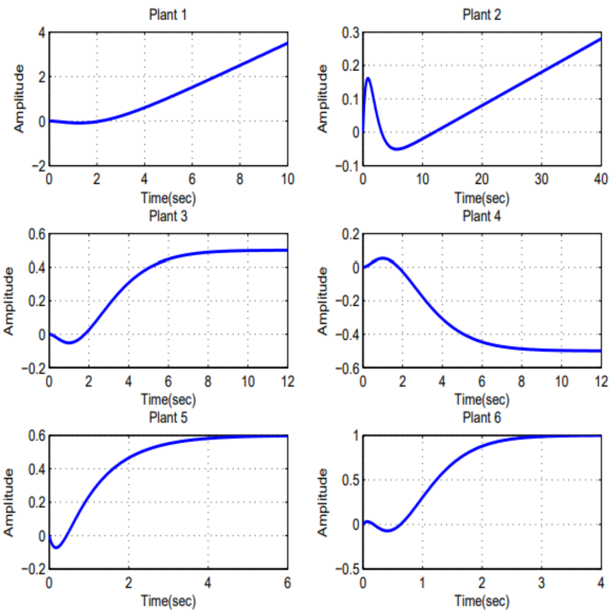


Figure 8: Step response of single variable non minimum phase systems without controller

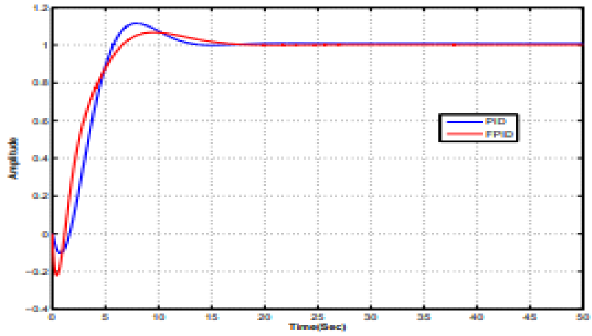
Table 3: Zero crossing for non minimum phase systems (seconds).

Plant	Open Loop
$P_1(s)$	2.1525
$P_2(s)$	21.9783
$P_3(s)$	1.7963
$P_4(s)$	1.7960(*)
$P_5(s)$	0.4427
$P_6(s)$	0.6337

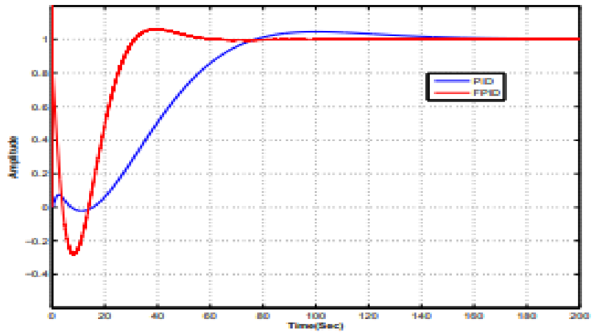
3.1.2 Single Variable Simulation Plants - PID vs FPID results (first overshoot)*

Tuned parameters for PID and FPID controllers are tabulated in Tables 4 and 5. Closed-loop system responses are shown in Fig. 11. Zero crossings for these controllers are tabulated in Table 6. Zero crossings of various simulation plants are plotted for comparisons in Fig. 12. Zero crossing improvements by FPID controller with respect to open loop and PID controller is shown in Fig. 13. The respective time domain specifications are shown in Table 7. Rise time, peak time and % peak overshoot indicate transient responses of the simulation plants considered in current study. Plants 1 and 3 show only one small undershoot in their PID and FPID responses. This is due to the presence of one zero in the RHS of s-plane (pole zero map) of the corresponding transfer functions. The effect of non-minimum phase was also observed to be reduced in FPID response as compared to PID response. For plant 2, there was overshoot and undershoot in both PID and FPID controllers responses because of two zeros in the RHS of s-plane. The FPID response for the second zero crossing was also reduced. In plant 4, closed-loop responses attained desired value very quickly, showing significant improvement over open loop results. Plant 5 closed loop responses also reach desired value fast. There is no overshoot in FPID controller response as observed in case of corresponding PID controller response. Initial undershoots were observed in closed loop responses of plants 4 and 5 due to one zero in RHS of respective s-planes.

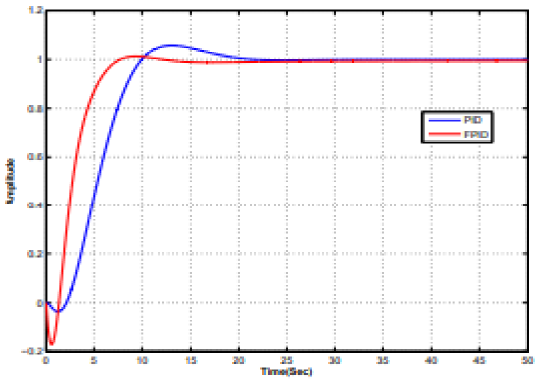
In case of plant 6, the FPID controller response is fast and exhibits higher under/overshoot amplitudes. The settling time of PID controller is observed to be lesser in comparison to FPID controller. This may be due to two conjugated complex zeroes on RHS of plant 6 s-plane.



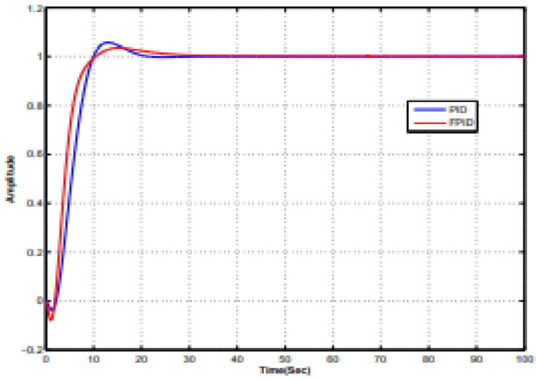
(a) Plant 1



(b) Plant 2



(c) Plant 3



(d) Plant 4

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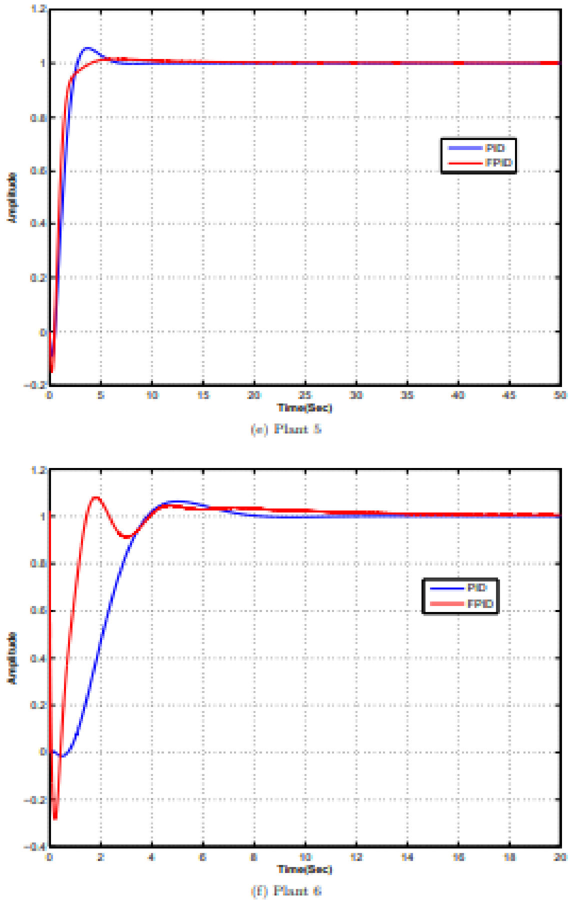

Figure 9: Closed loop responses of non minimum phase systems for unit step.

FPID zero crossing is reduced by 46 %, 37 %, 33% and 25 % for plants 1, 2, 6 and 3 respectively as compared to corresponding open loop system responses. Zero crossing for FPID controller has increased for plants 4 and 5. However, for the same plants (4 and 5) FPID zero crossing reduction is observed with respect to PID controller responses. FPID controller zero crossings decrease by 40 %, 37 %, 29 %, 15 %, 13 % and 5 % for plants 6, 3, 1, 2, 4 and 5 respectively as compared to PID controller responses. The average reduction of FPID zero crossing is 22 % as compared to open loop system and 23 % as compared to PID controllers. Overall, non-minimum phase systems effect was reduced as the fractional PID controller was able to capture more dynamics because of fractional order.

In comparison to PID controller, the fractional PID controller exhibits better response with respect to zero crossing and time domain specifications, such as rise time.

Table 4: Simulation results for PID controller.

Plant	PID Parameters			
	P	I	D	N
$P_1(s)$	0.62504	0.00169	0.58949	35.54286
$P_2(s)$	2.5140	0.00054	-18.6129	0.13506
$P_3(s)$	0.45099	0.37974	0	100
$P_4(s)$	-0.45099	-0.3797	0	100
$P_5(s)$	0.97832	1.31777	0	100
$P_6(s)$	0.15080	0.50278	0	100

Table 5: Simulation results for fractional PID controller.

Plant	Fractional PID Controller				
	K_P	K_I	λ	K_D	μ
$P_1(s)$	0.74678	4.0938e-08	0.12834	1.2749	1.2872
$P_2(s)$	5.4155	-0.36102	0.041858	10	1.0414
$P_3(s)$	1.4148	0.66235	0.97086	1.329	1.0063
$P_4(s)$	0.044327	-0.46071	1.0217	-1.2287	0.14235
$P_5(s)$	1.9301	1.3151	1.0609	-0.22508	0.098241
$P_6(s)$	1.5482	0.87392	1.1668	0.17681	1.4925

Table 6: Zero crossing for non minimum phase systems (seconds).

Plant	PID	Fractional PID
$P_1(s)$	1.6242	1.1598
$P_2(s)$	16.4	13.9
$P_3(s)$	2.1199	1.3445
$P_4(s)$	2.1184	1.8535
$P_5(s)$	0.4982	0.4750
$P_6(s)$	0.7110	0.4252

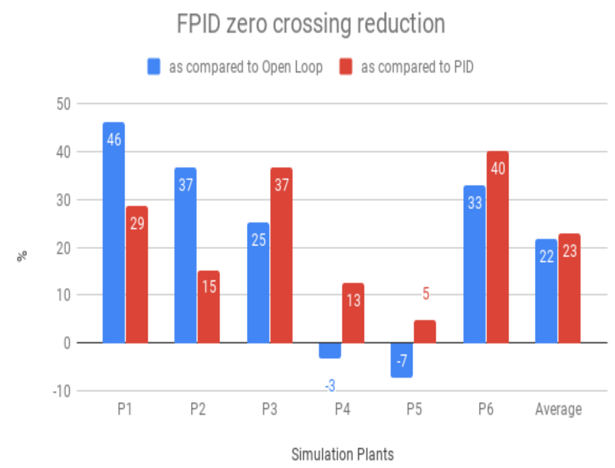

Figure 10: Zero crossing for single variable simulation plants

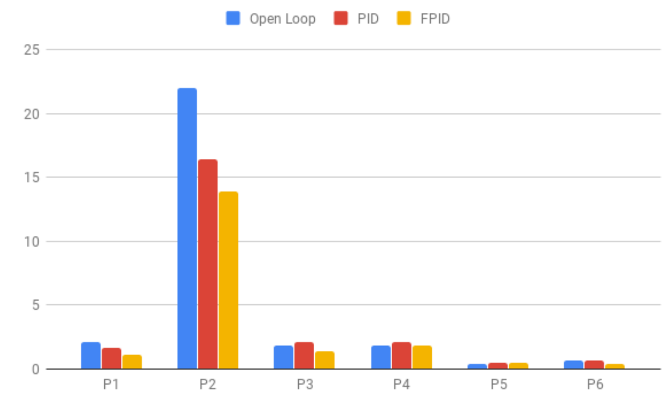

Figure 11: Zero Crossing Improvement for FPID with respect to open loop system and PID controller for single variable simulation plants

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Table 7: Time specification for simulation plants

Plants	Rise Time (sec)	Peak Time (sec)	Peak Overshoot (%)
Plant 1	6.5946	9.5545	6.7442
Plant 2	27.5311	36.99	5.81
Plant 3	7.7285	9.3137	1.26
Plant 4	10.4426	15.2346	3.48
Plant 5	4.0195	6.3950	1.56
Plant 6	1.4388	1.8068	8.08

3.2 Experimental Results of Multi-variable System

The mathematical model of quadruple tank system was simulated using closed loop control. This is an example of a multi input and multi output system. The resultant tuning parameters of PID and FPID controllers are shown respectively in Table 8 and Table 9. The PID and FPID controller responses for the same are shown respectively in Figs. 12 and 13. From these figures, the superiority of FPID responses over PID controller in terms of settling time, rise time etc. is clearly evident. Moreover, FPID controller is able to remove initial undershoots seen in PID responses.

Table 8: PID controller parameters for experimental model

Parameters	K_P	K_I	K_D	N
Level of Tank 1	-0.05822	-6.99 E -06	-107.7868	2.3510
Level of Tank 2	2.4279	0.0405	20.7294	0.0833

Table 9: Fractional PID controller parameters for experimental set up after optimization approach.

Parameters	K_P	K_I	λ	K_D	μ
Level of Tank 1	2.971	2.979	0.98	4.607	0.151
Level of Tank 2	2.966	4.962	0.987	4.88	0.108

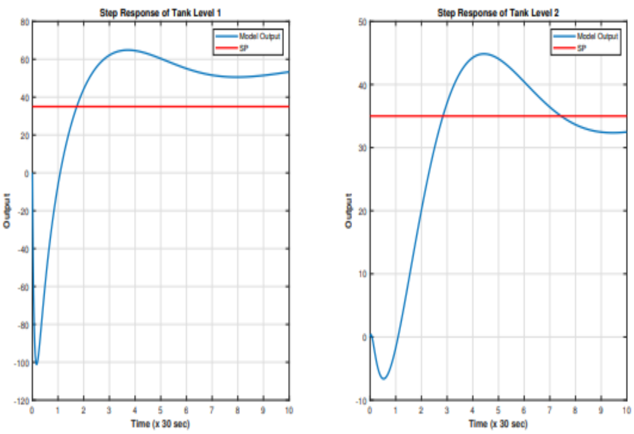

Figure 12: Step response of multi-variable mathematical model for PID controller

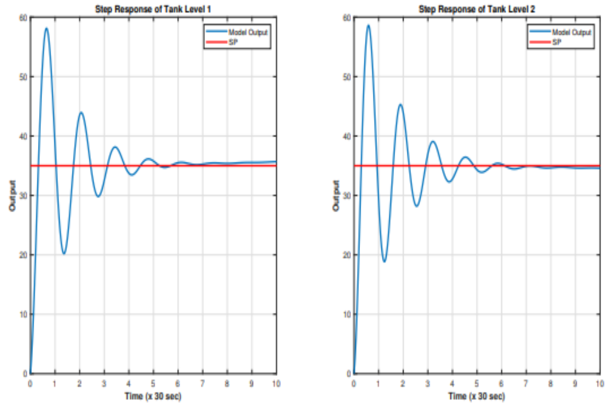

Figure 13: Step response of multi-variable mathematical model for FPID controller

To obtain real responses from the experimental set-up, FPID controller was set with the parameters of PID controller having $\lambda = 1$ and $\mu = 1$. The resultant response is shown in Fig. 14. This response shows that the output oscillates between wide limits of actual tank levels. In case of tank 1 level, it is not even reaching the desired set point, showing offset in the recorded response. For tank 2 level, output response is reaching the desired value with some overshoots. In the next step, FPID controller was set with optimised FPID controller parameters obtained via simulation of the quadruple tank mathematical model (Table 9). The resultant actual tank level responses are shown in Figs. 15 and 16. Thus, FPID controller gives much better response as compared to PID controller in case of real world systems also.

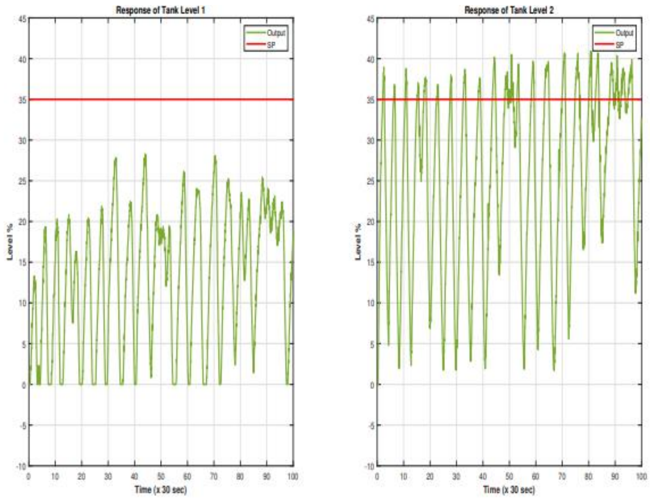

Figure 14: Step response of actual system using PID tuning method.

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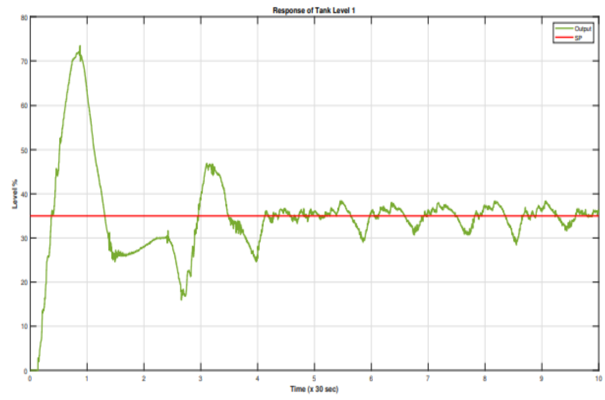

Figure 15: Step response of tank 1 after optimization method.

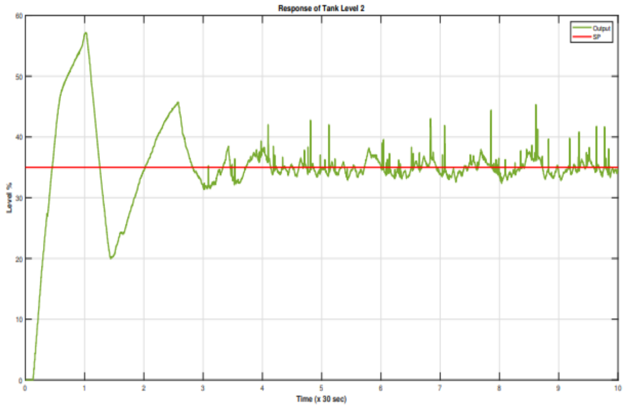

Figure 16: Step response of tank 2 after optimization method.

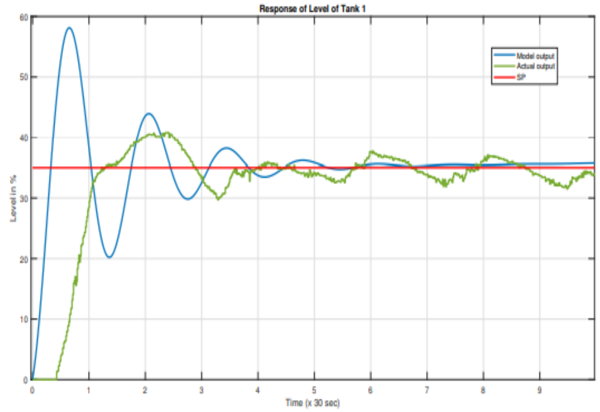

Figure 17: Comparison for Step response of tank 1 with model and actual plant after fine tuning.

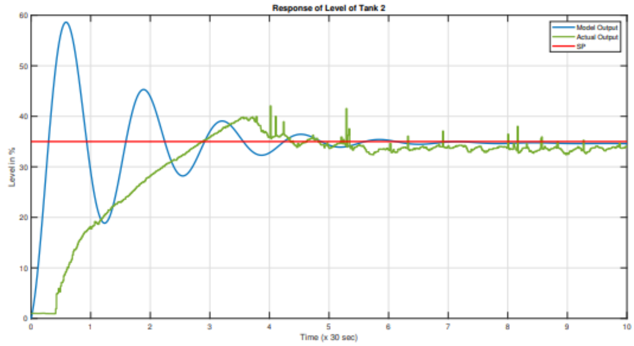

Figure 18: Comparison for Step response of tank 2 with model and actual plant after fine tuning.

As mentioned in methodology, further fine tuning was applied to closed loop control of real-time system under study. This was done to improve attainment of the desired set point. Experimental set up response was fine tuned by changing different FPID controller parameters, such as the orders of differentiation and integration [44]. By changing these orders, overshoot of the system was minimized. Even general guidelines for tuning of PID controller can be used for the fine tuning of the FPID controller. The fine tuned parameters of the fractional PID controller are shown in Table 10. Fine tuned responses of real time experimental set-up are compared to mathematical model FPID controller responses as shown in Figs. 17 and 18. These figures show that the simulation (mathematical model) and fine tuned experimental responses have good agreement. The various time domain specifications after fine tuning are tabulated in Table 11.

Table 10: Fractional PID controller parameters for experimental set up after fine tuning.

Parameters	K_P	K_I	λ	K_D	μ
Level of Tank 1	2	2	0.5	8	0.4
Level of Tank 2	2	4	0.5	2.5	0.4

Table 11: Performance specifications for experimental set up after fine tuning.

Output	Rise Time	Peak Time	Peak Overshoot
Level of tank 1	37 Sec	70 Sec	16.57 %
Level of tank 2	66 Sec	82 Sec	20 %

IV. CONCLUSION

The primary contribution of this paper is applying the FPID controller for non-minimum phase systems. FPID controller has been implemented for simulated and real time non-minimum phase systems using simple tuning approach. As many as six different single variable plants were simulated using closed loop controllers. Closed-loop responses of FPID controller were smoother than those by classical PID controller. The non-minimum phase system had an undershoot or over and undershoot effect. This effect (measured by zero crossing parameters) was reduced by FPID controller as compared to PID controller. The simple approach adopted to tuning of FPID controller yielded improved results. Also, implementation of the classical PID tuning method for optimization with initial conditions was shown to be effective and useful. Simulation results for multi-variable system were verified using an experimental setup of a quadruple-tank system.

A quadruple-tank system was linearized and the obtained model was used for the tuning of parameters. Communication was established using an OPC protocol which was implemented in Simulink. FPID controller showed characteristics of dealing with a multi-variable system without the use of a relative gain array (RGA) matrix. This RGA matrix is used for decoupling the multi-variable process. Zero crossing is present in experimental model for PID controller. Zero crossing is not observed in the FPID controller for experimental model and experimental set-up. So, the effect of non-minimum phase system is successfully reduced. Tuning of the experimental set up was obtained using the approach shown in the simulation. Later on, fine tuning of the FPID controller was also performed on the experimental set up. For optimization, initial parameters were obtained using the Simulink block of the PID. The close agreement among simulation and fine tuned experimental results show that it is possible to control a multi-variable system without decoupling theory. This also indicates that FPID controller is able to control the dynamics of a non-minimum phase system effectively. Based on current work it is recommended that for all kind of plants, FPID controller should be employed for better results as compared to PID controllers.

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AUTHORS PROFILE

Pritesh Shah is an Assistant Professor in Symbiosis Institute of Technology, Pune. He has completed PhD from college of Engineering, Pune in the area of fractional order controller. He has done work in the area of fractional calculus, system identification and process modeling. He is a life Member Instrument Society of India.

Ravi Sekhar is an Assistant Professor in Symbiosis Institute of Technology, Pune.

Sudhir Agashe is an experienced Professor with a demonstrated history of working in the Instrumentation and Control engineering industry. Skilled in Automation tools, Process Control, Engineering, C++, and Control Systems Design. Strong education professional with a Doctor of Philosophy (Ph.D.) focused in Process Automation from College of Engineering Pune. He is member of IEEE Instrumentation and Measurement Society, Life Member Instrument Society of India, Life Member Biomedical Engineering Society of India and Life Member Indian Society for Technical Education.