

Counter Machines & Distributed Automata

A Story about Exchanging Space and Time

Olivier Carton¹ Bruno Guillon² Fabian Reiter³

¹IRIF, Paris Diderot University

²University of Milan

³LSV, University of Paris-Saclay

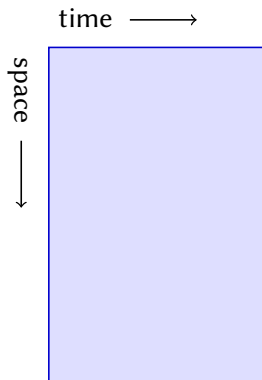
AUTOMATA'18, Ghent, 20 June 2018

A space-time duality

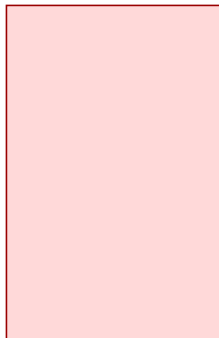
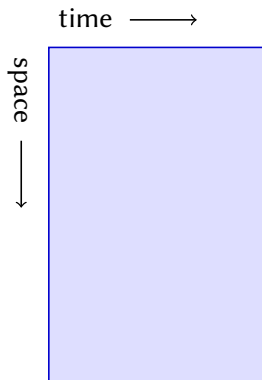


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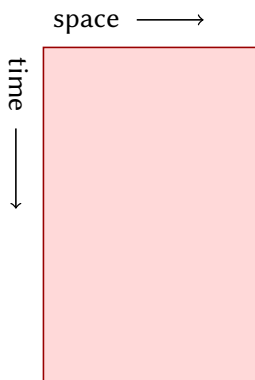
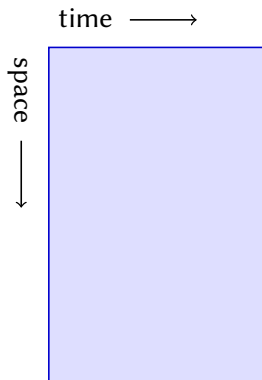
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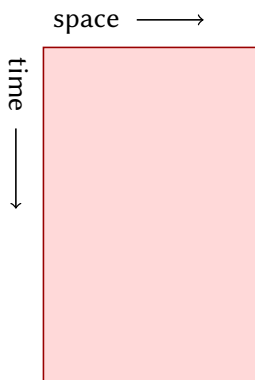
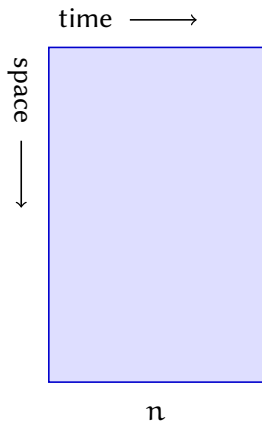
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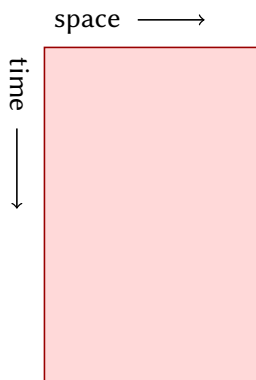
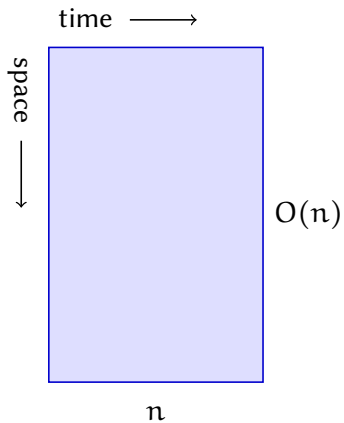
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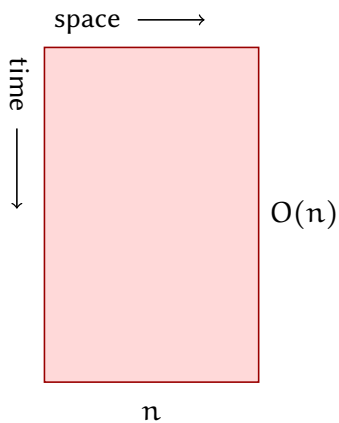
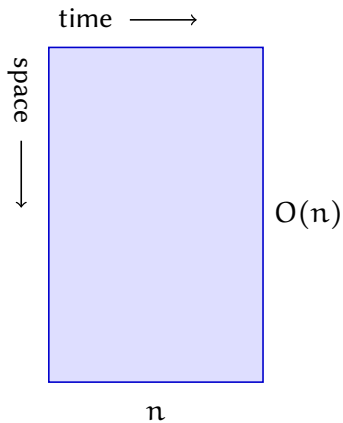
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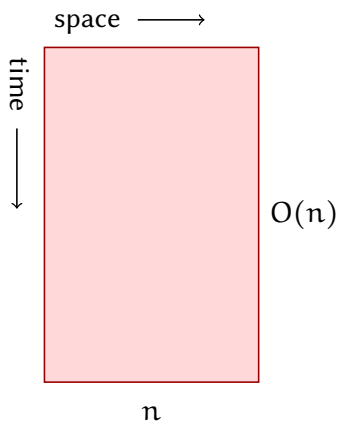
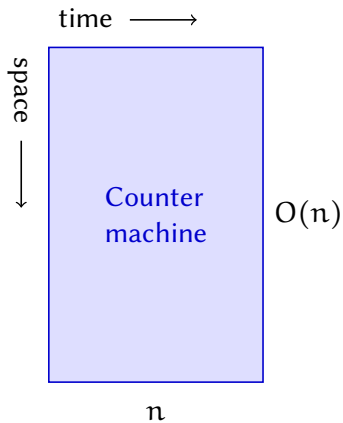
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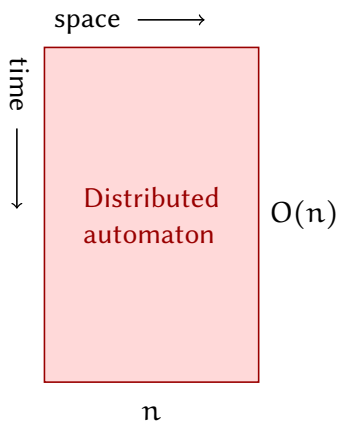
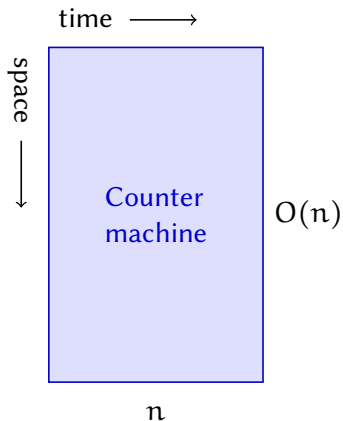
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Counter machines

Counter machines

a	b	a	c	a	c
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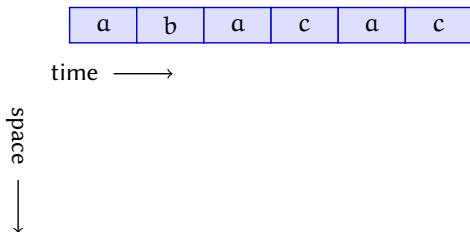
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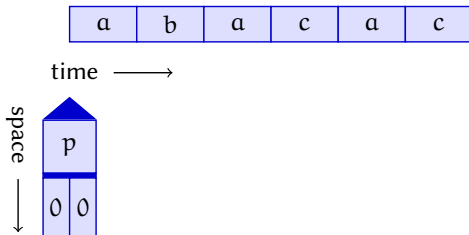
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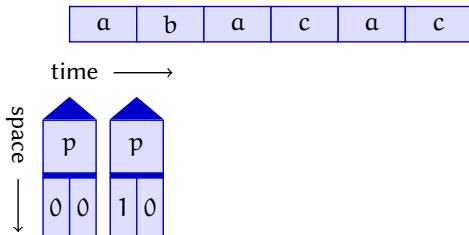
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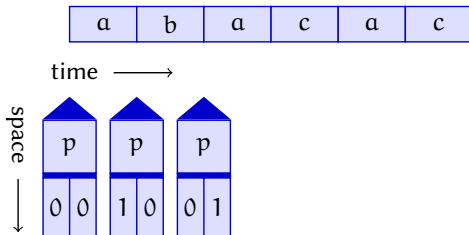
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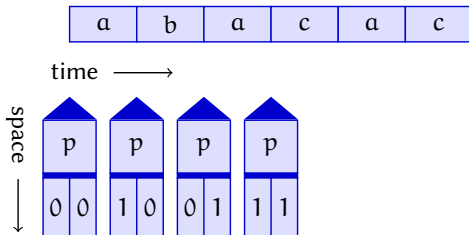
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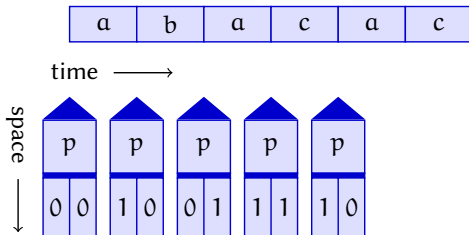
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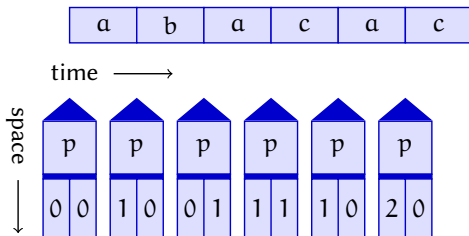
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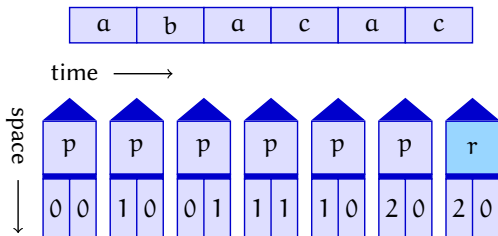
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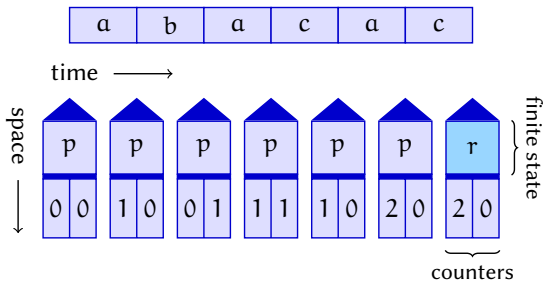
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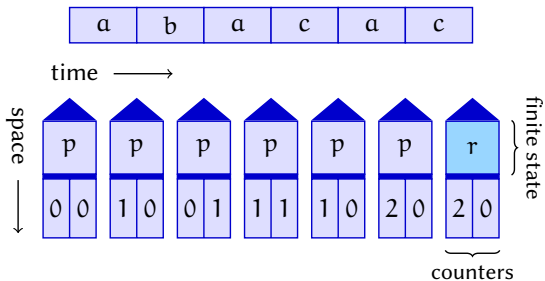
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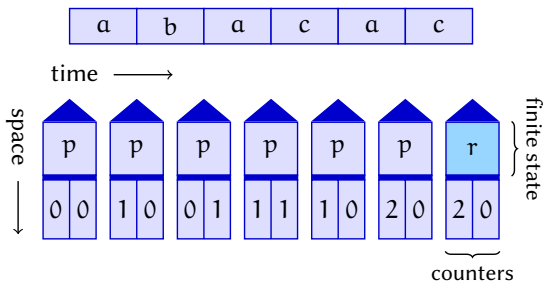
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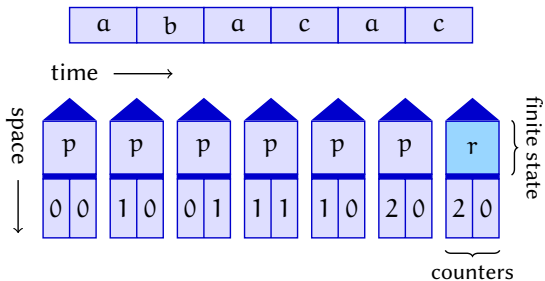
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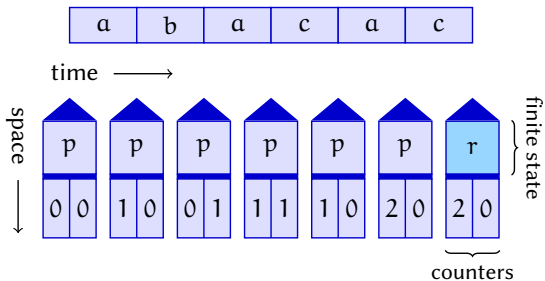
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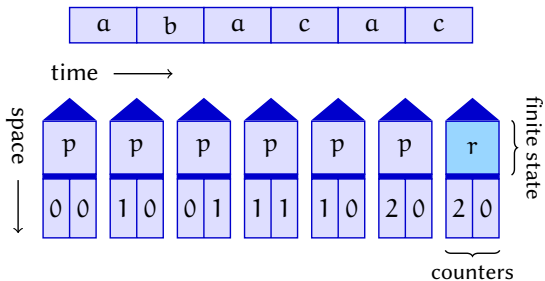
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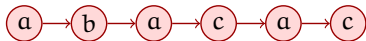
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Distributed automata

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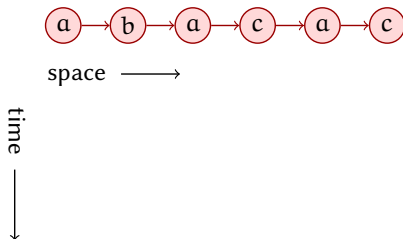
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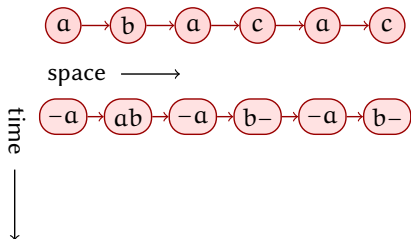
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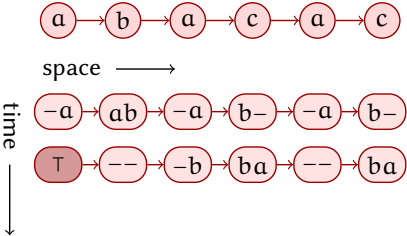
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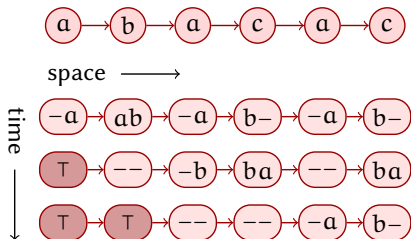
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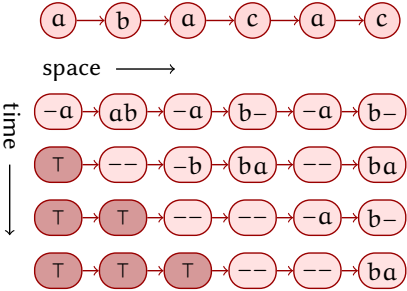
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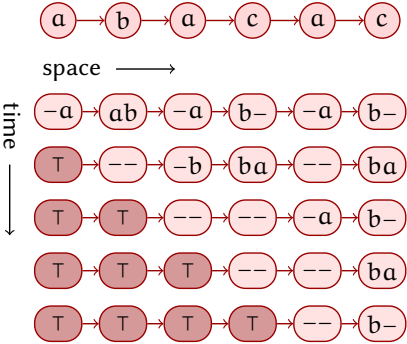
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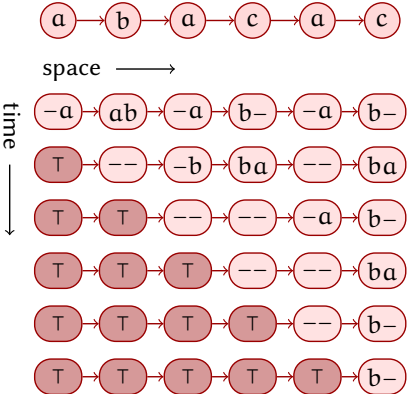
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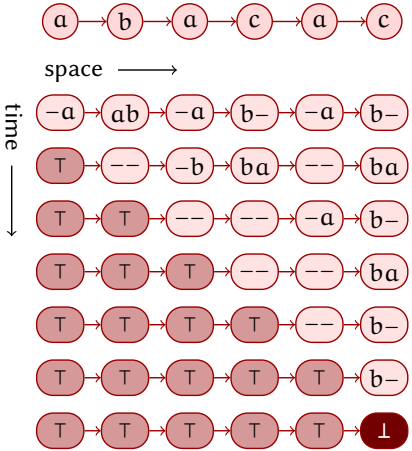
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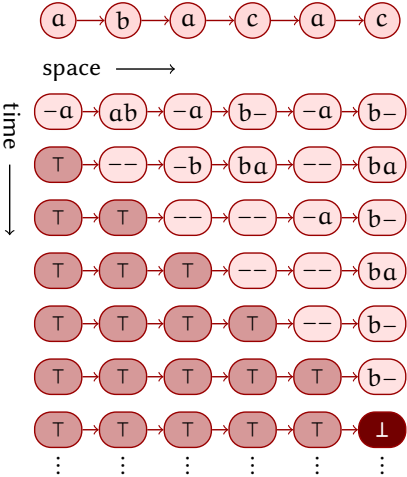
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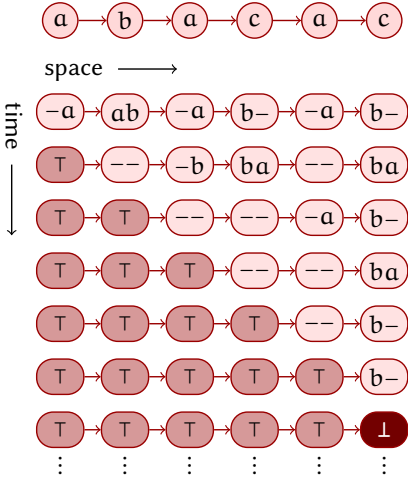
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The same as (reversed, one-dimensional) *one-way cellular automata*.

Our translations



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Copyless
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Quasi-acyclic
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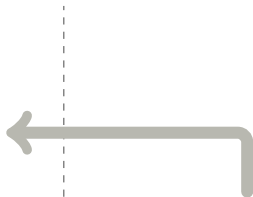
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Using time as space



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Quasi-acyclic
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time →

space ↓

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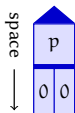
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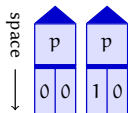
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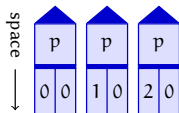
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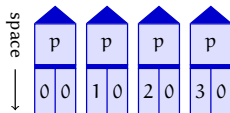
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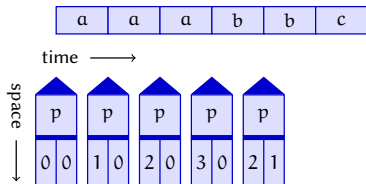


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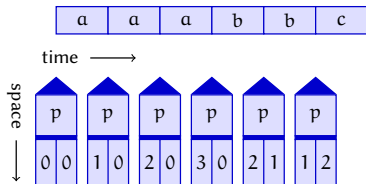


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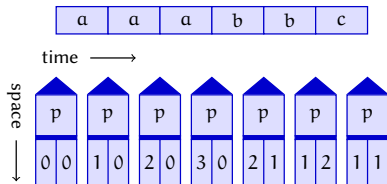
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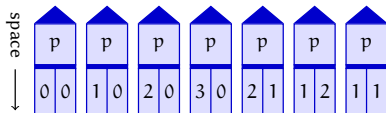


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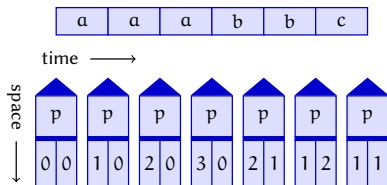
a a a b b c



time →



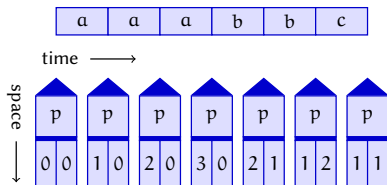
Using time as space



Without loss of generality:

- ▶ Nonnegative counter values.

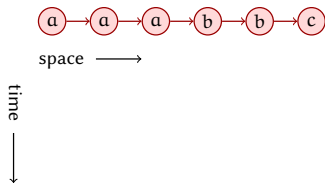
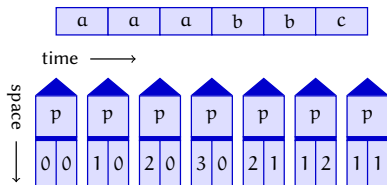
Using time as space



Without loss of generality:

- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

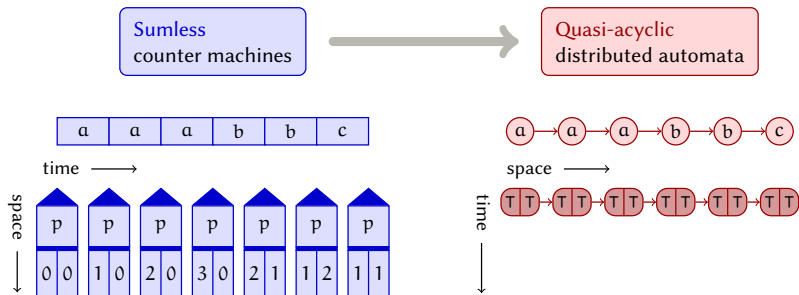
Using time as space



Without loss of generality:

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- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

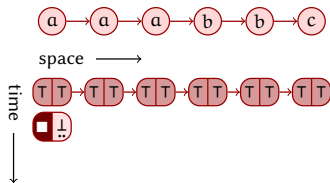
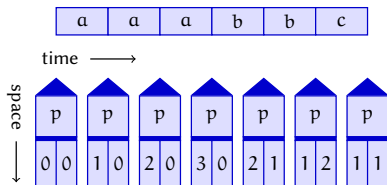
Using time as space



Without loss of generality:

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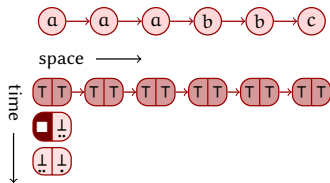
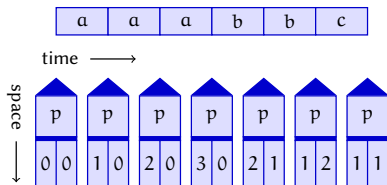
Using time as space



Without loss of generality:

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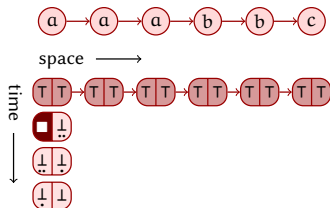
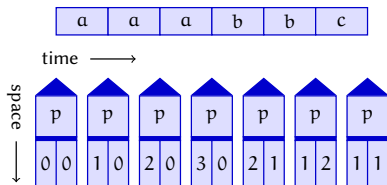
Using time as space



Without loss of generality:

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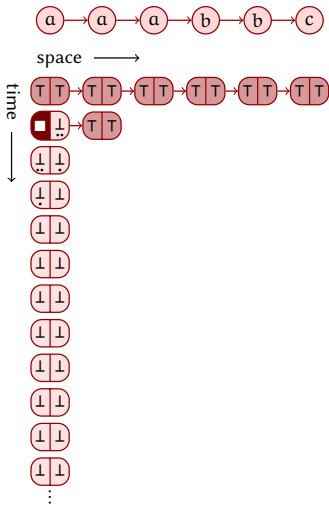
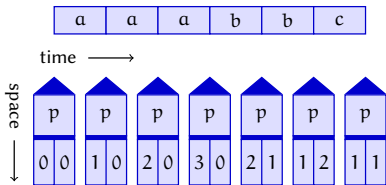
Using time as space



Without loss of generality:

- ▶ Nonnegative counter values.
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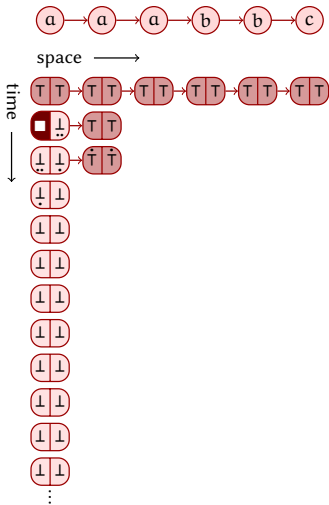
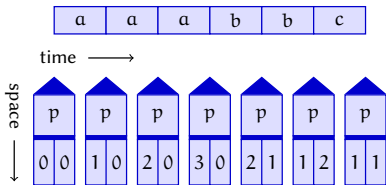
Using time as space



Without loss of generality:

- ▶ Nonnegative counter values.
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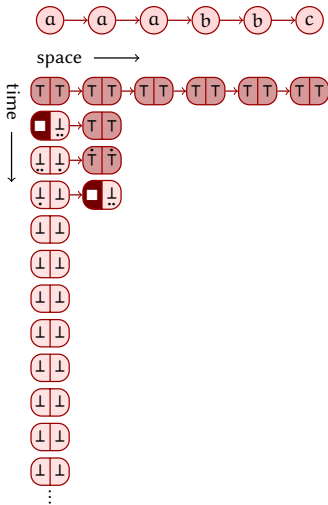
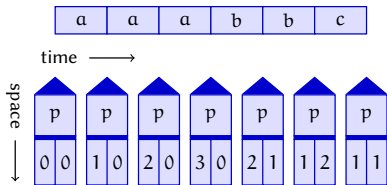
Using time as space



Without loss of generality:

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Using time as space



Without loss of generality:

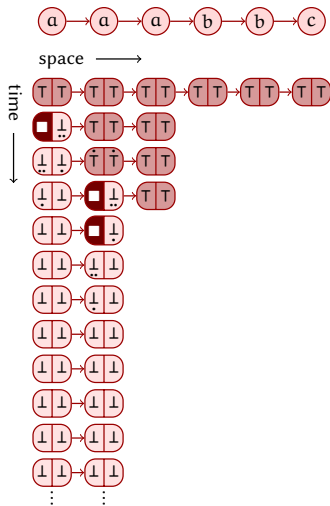
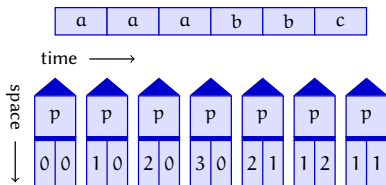
- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

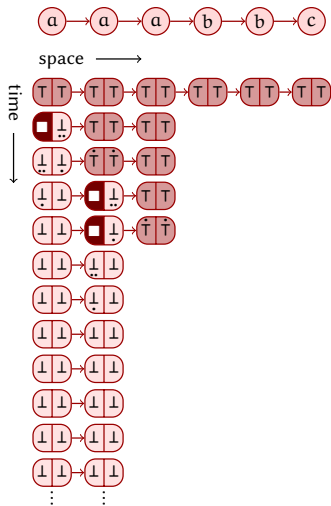
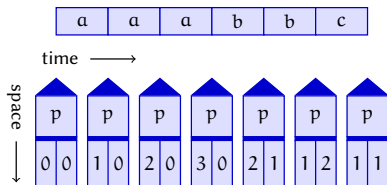
- ▶ Nonnegative counter values.
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Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

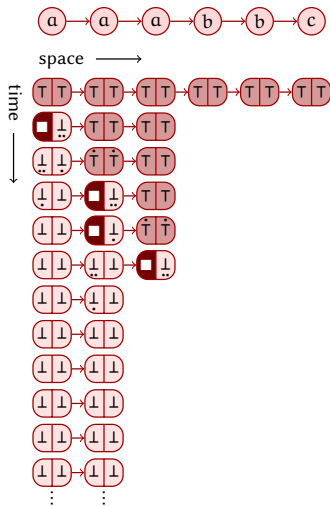
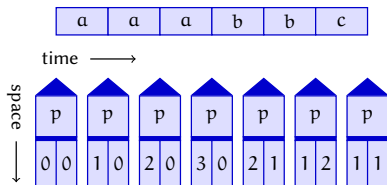
- ▶ Nonnegative counter values.
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Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

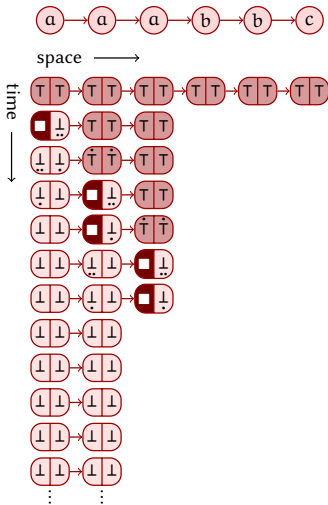
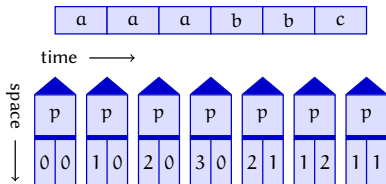
- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

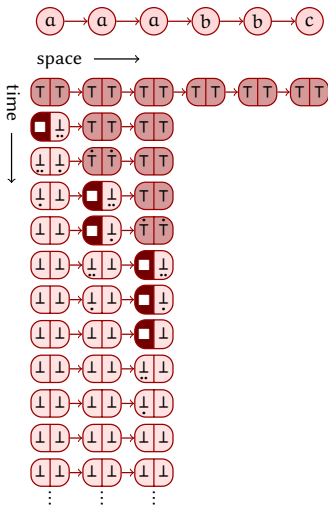
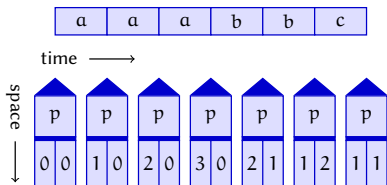
- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

Using time as space

Sumless counter machines



Quasi-acyclic distributed automata



Without loss of generality:

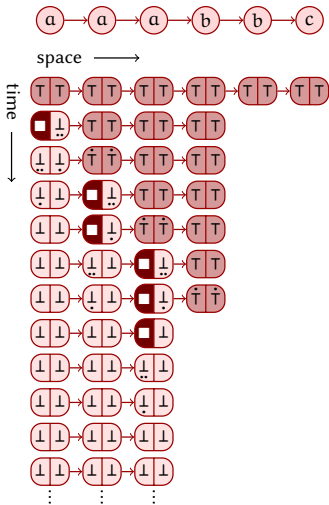
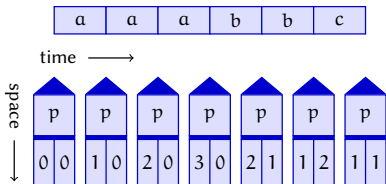
- ▶ Nonnegative counter values.
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Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

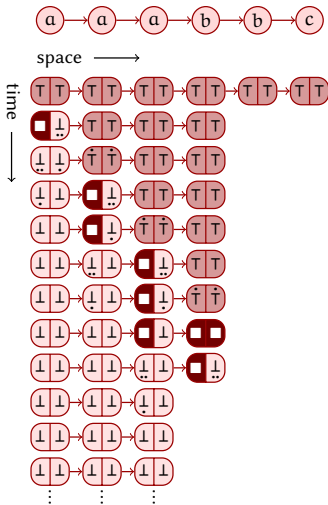
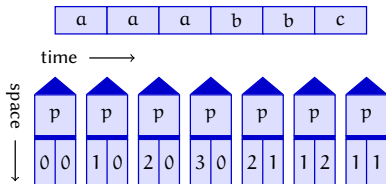
- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and **copied**.

Using time as space

Sumless
counter machines



Quasi-acyclic
distributed automata



Without loss of generality:

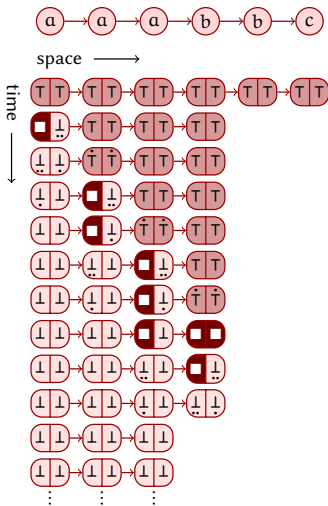
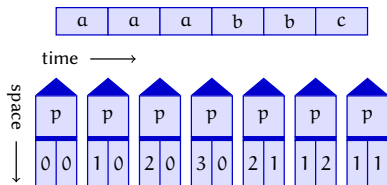
- ▶ Nonnegative counter values.
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Using time as space

Sumless
counter machines



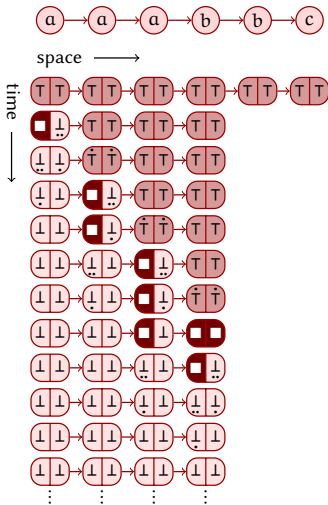
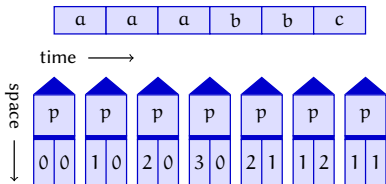
Quasi-acyclic
distributed automata



Without loss of generality:

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Using time as space



Without loss of generality:

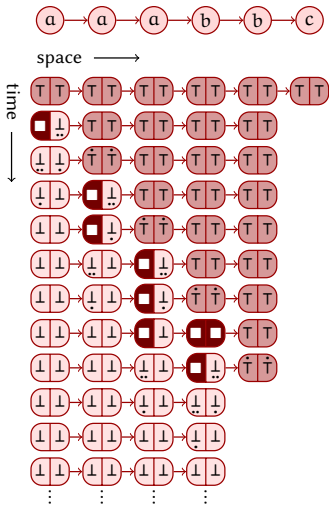
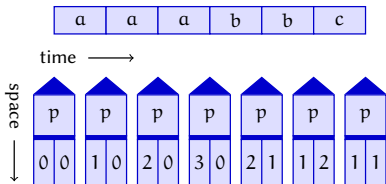
- ▶ Nonnegative counter values.
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Using time as space

Sumless counter machines



Quasi-acyclic distributed automata



Without loss of generality:

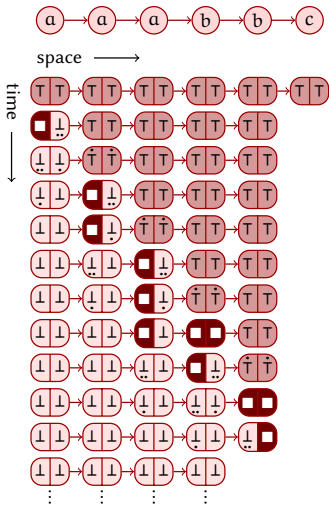
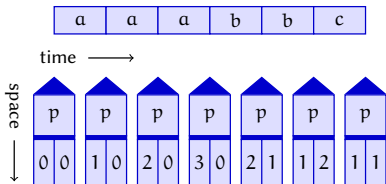
- ▶ Nonnegative counter values.
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Using time as space

Sumless counter machines



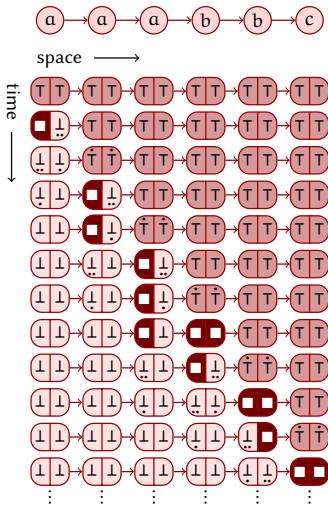
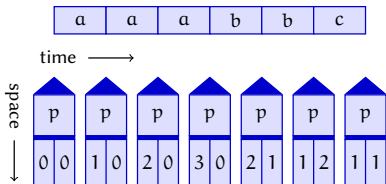
Quasi-acyclic distributed automata



Without loss of generality:

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Using time as space



Without loss of generality:

- ▶ Nonnegative counter values.
- ▶ Counters can only be tested for 0, incremented by $-1, 0, 1$, and copied.

Our translations

Counters can be summed up but not copied.

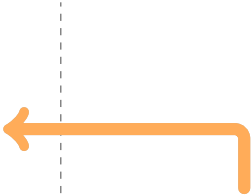
Copyless
counter machines



Sumless
counter machines

Counters can be copied but not summed up.

LINEAR SPACE



Quasi-acyclic
distributed automata



The state diagram has no directed cycles except for self-loops.

LINEAR TIME

Using space as time

Quasi-acyclic
distributed automata



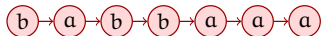
Copyless
counter machines

Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

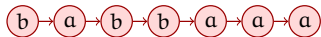


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines



space →

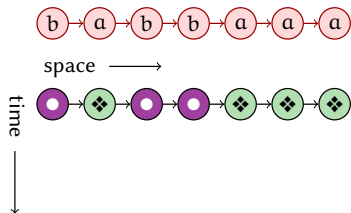
time ↓

Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

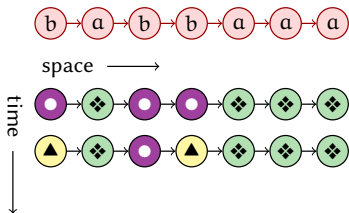


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

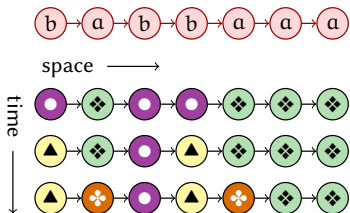


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

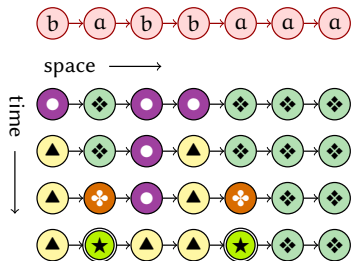


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

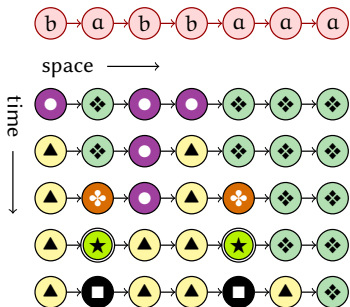


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

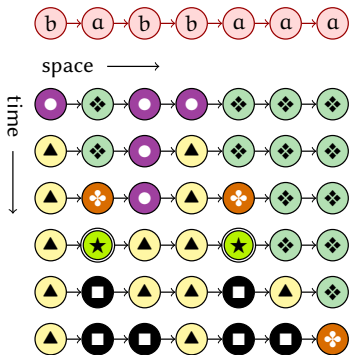


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

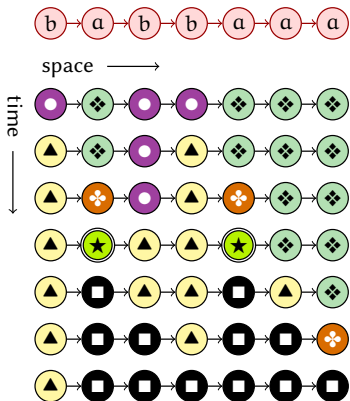


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

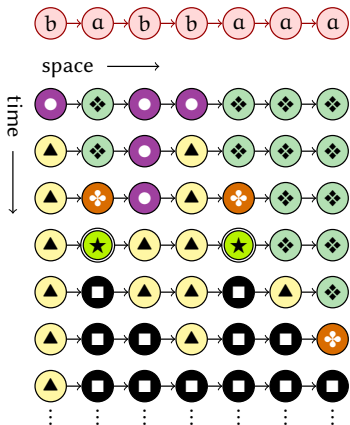


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

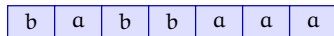
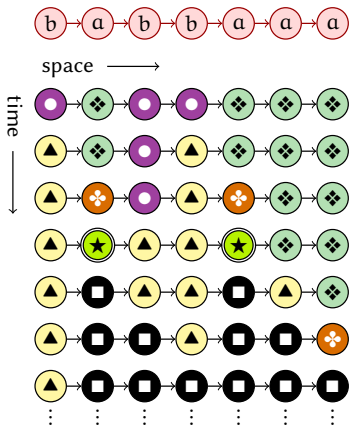


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

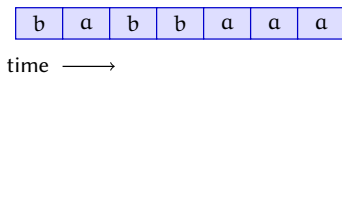
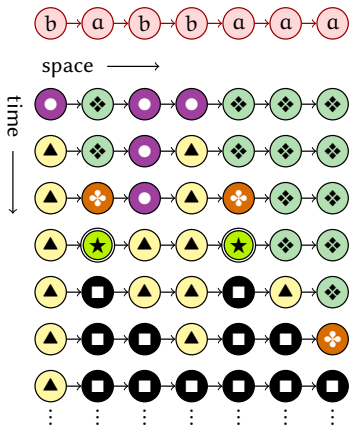


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

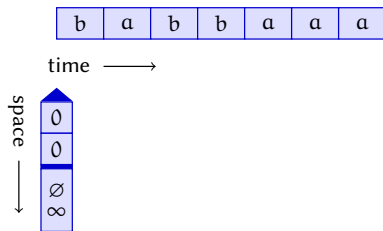
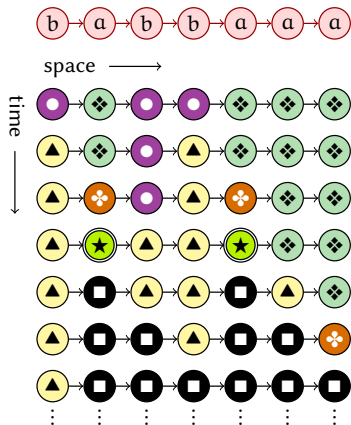


Using space as time

Quasi-acyclic
distributed automata



Copyless
counter machines

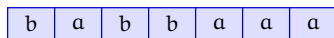
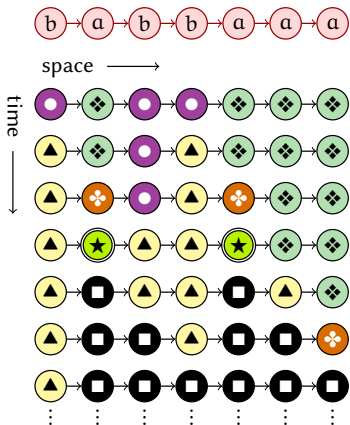


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines



time →



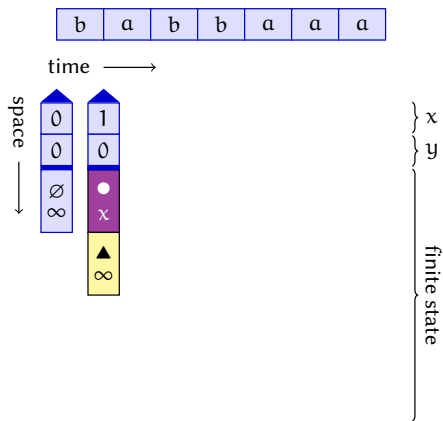
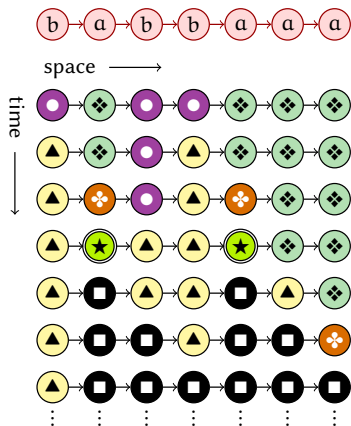
} x
y
finite state

Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

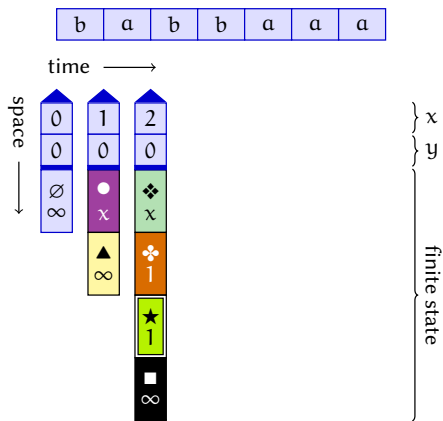
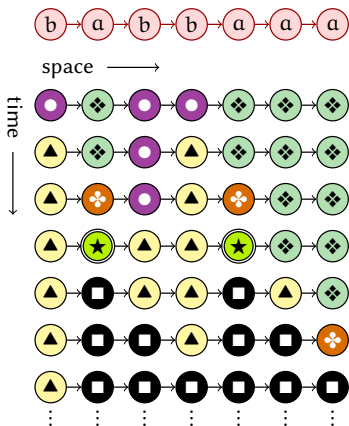


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

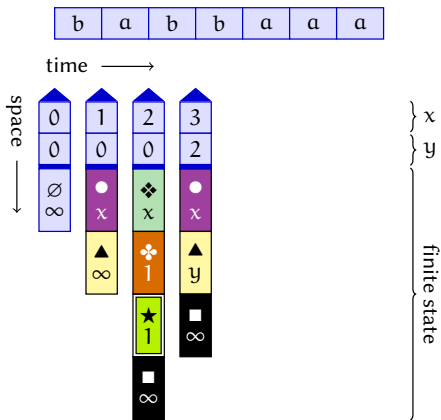
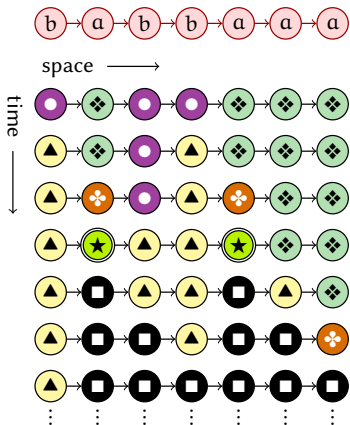


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

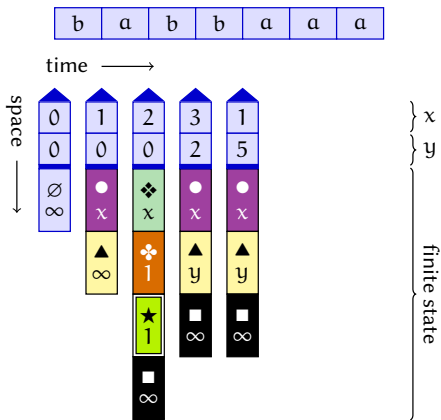
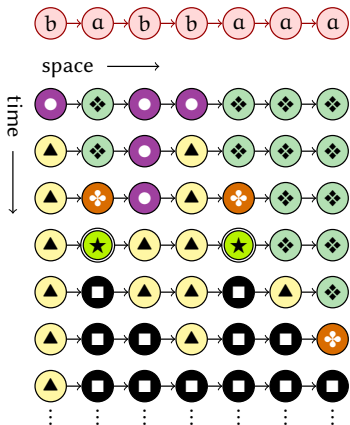


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

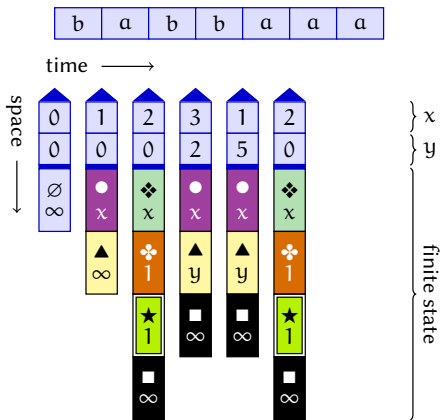
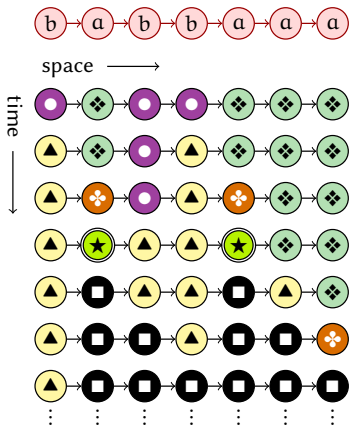


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

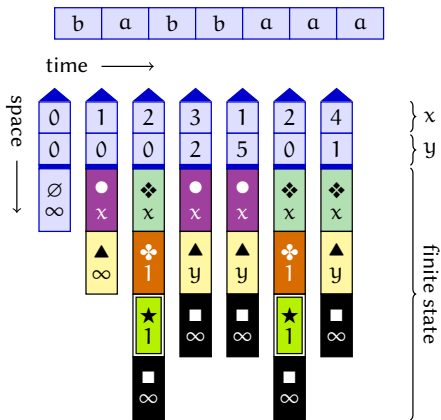
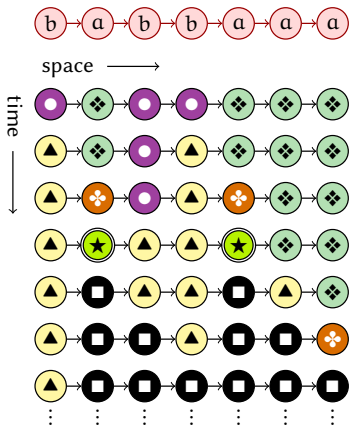


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines

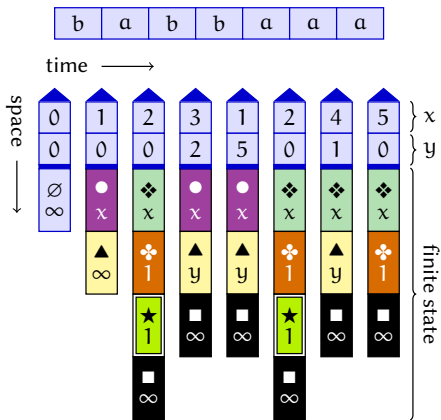
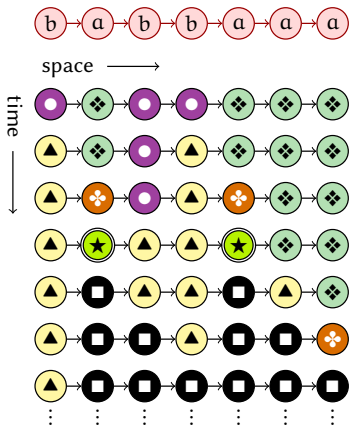


Using space as time

Quasi-acyclic distributed automata



Copyless counter machines



Our translations

Counters can be summed up but not copied.

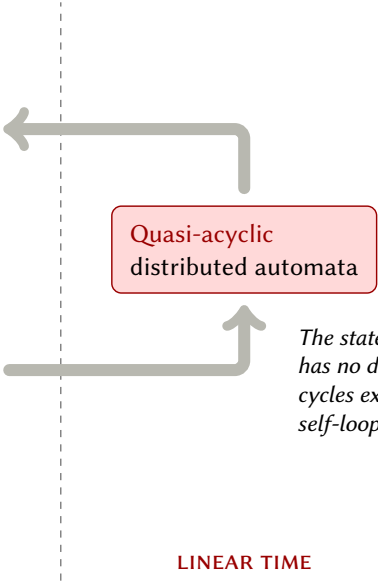
Copyless
counter machines



Sumless
counter machines

Counters can be copied but not summed up.

LINEAR SPACE



Quasi-acyclic
distributed automata

The state diagram has no directed cycles except for self-loops.

LINEAR TIME

Possible future work

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Thanks!