$$\delta : Q \times 2^Q \to Q$$

Distributed Automata and Logic

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12 December 2017

INSTITUT DE RECHERCHE EN INFORMATIQUE FONDAMENTALE

université PARIS DIDEROT PARIS 7

USPC Université Sorbonne Paris Cité

Ultimate objective

Descriptive complexity theory

for

Distributed computing

Fagin's theorem (1974)

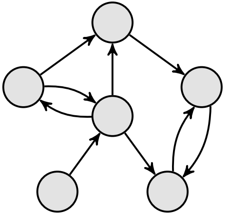
Fagin's theorem (1974)

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC

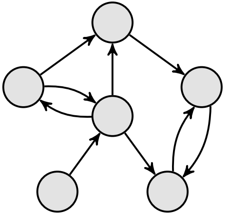
Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



Example: Hamiltonian path

Fagin's theorem (1974)

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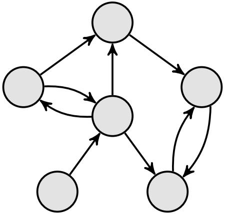
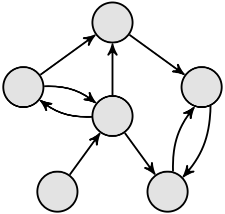


Example: Hamiltonian path

$\exists R\,(\qquad\qquad)$

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



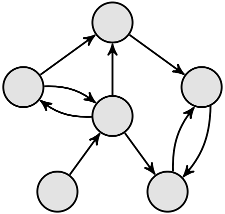
Example: Hamiltonian path

$\exists R\,\big($ "R is a strict total order" $\wedge$

$\big)$

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC

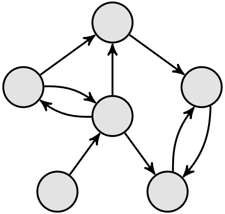


Example: Hamiltonian path

$\exists R\,\big($ "R is a strict total order" $\wedge$
"R-successors are adjacent" $\big)$

Fagin's theorem (1974)

$\exists$ SECOND-ORDER LOGIC



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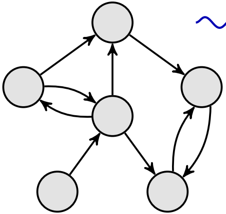
NP TURING MACHINES

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC

NP TURING MACHINES



encoding

Example: Hamiltonian path

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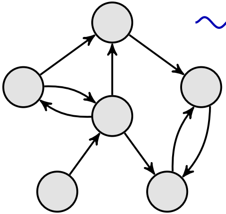
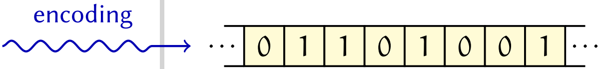
NP TURING MACHINES





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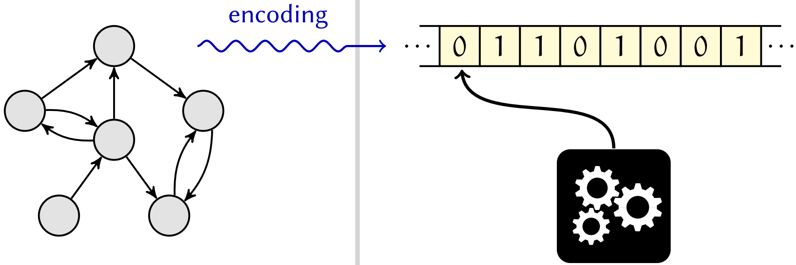
∃ SECOND-ORDER LOGIC | NP TURING MACHINES



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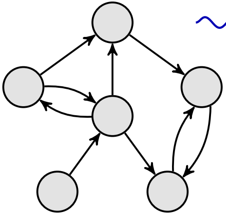

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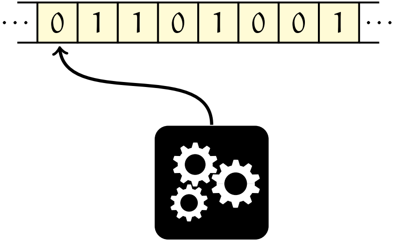
NP TURING MACHINES



Example: Hamiltonian path

$\exists R\,\big($ "R is a strict total order" $\wedge$
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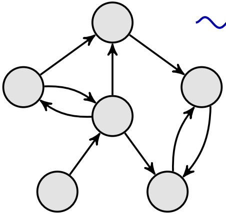
- Nondeterministic moves

Fagin's theorem (1974)

∃ SECOND-ORDER LOGIC



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$\exists R\,\big($ "R is a strict total order" $\wedge$
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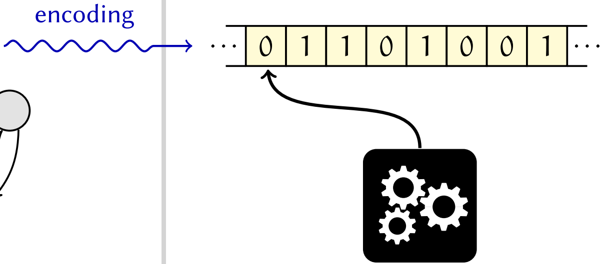
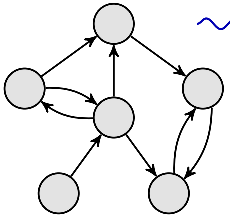
NP TURING MACHINES



- Nondeterministic moves
- Polynomial running time

Fagin's theorem (1974)

$\exists$ SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ NP TURING MACHINES



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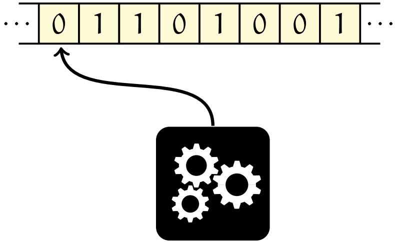

Example: Hamiltonian path

$\exists R \big($ "R is a strict total order" $\wedge$
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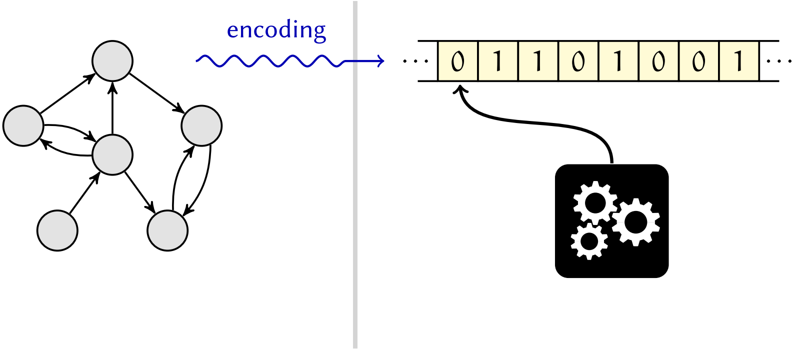
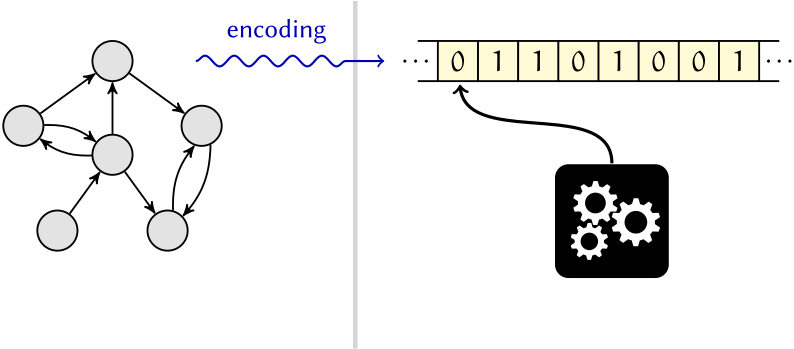
- Nondeterministic moves
- Polynomial running time

Descriptive complexity



Descriptive complexity

SOME LOGICAL FORMALISM



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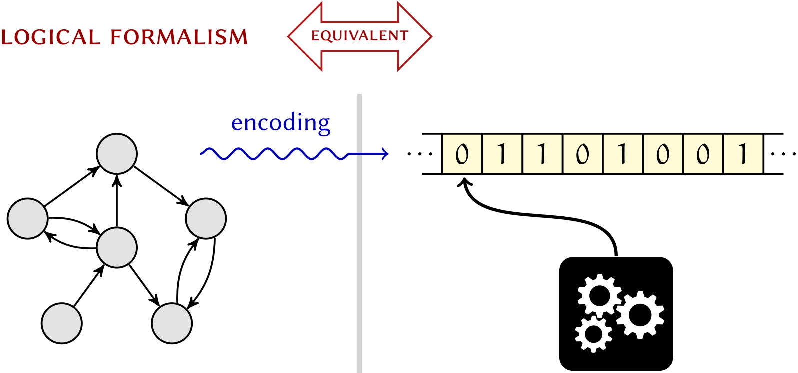
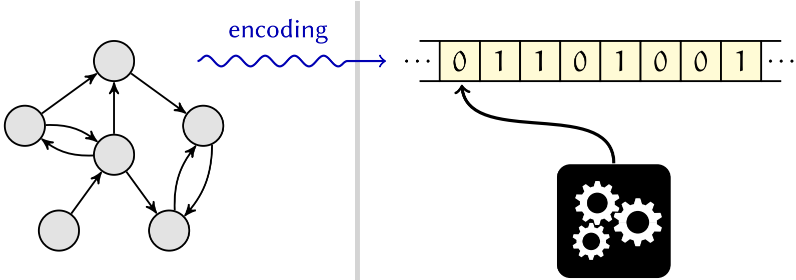
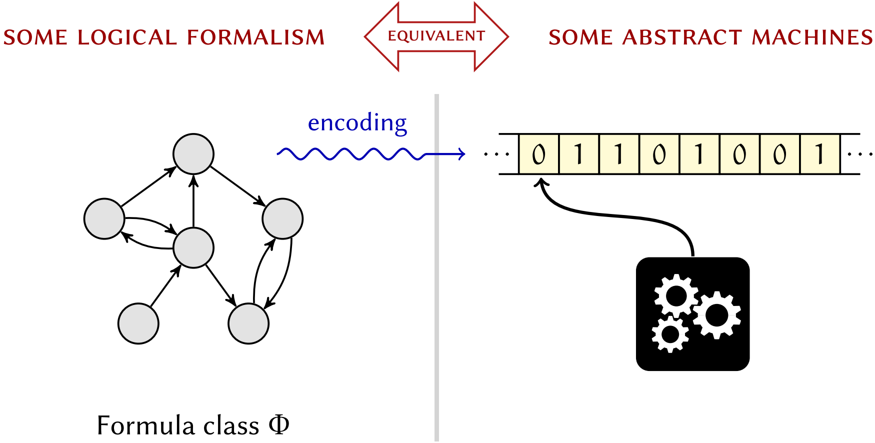


Descriptive complexity

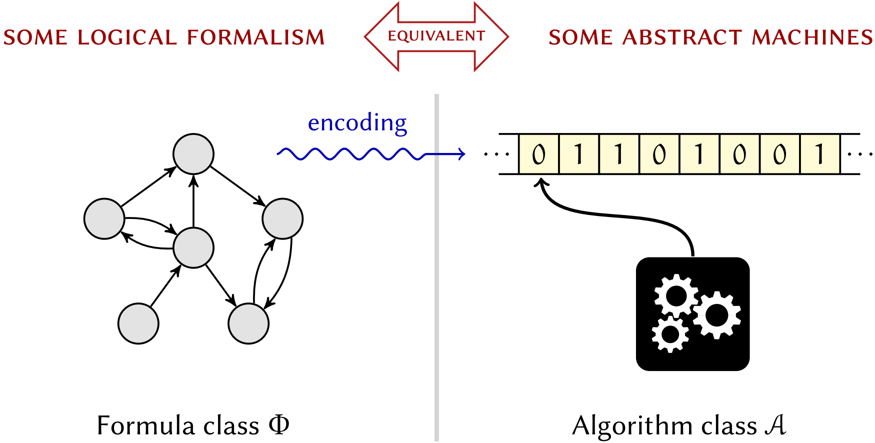
SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ SOME ABSTRACT MACHINES



Descriptive complexity



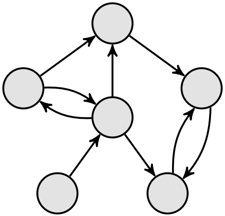
Descriptive complexity



Descriptive distributed complexity

Descriptive distributed complexity

SOME LOGICAL FORMALISM

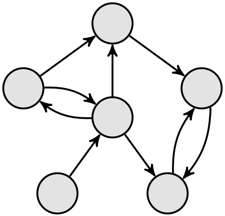


Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM

EQUIVALENT

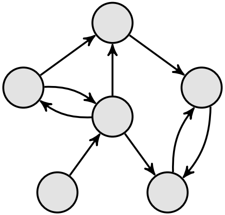


Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM

COMMUNICATING MACHINES

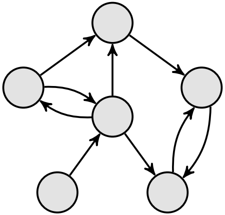


Formula class Φ

Descriptive distributed complexity

SOME LOGICAL FORMALISM

COMMUNICATING MACHINES



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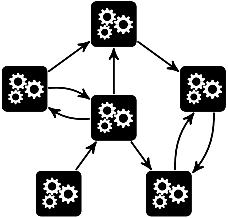
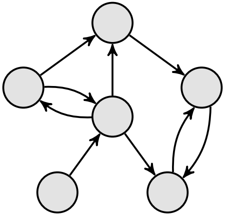


Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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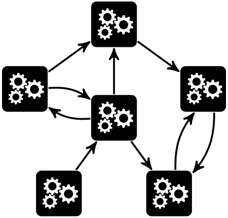
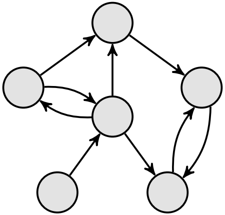


Distributed algorithm class $\mathcal{A}$

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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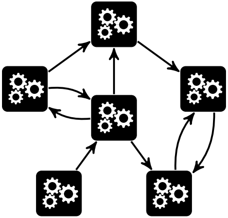
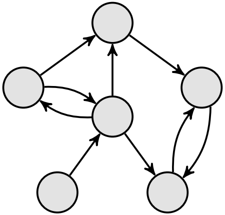


Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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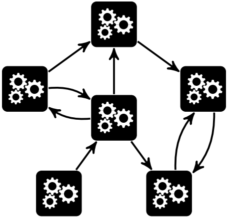


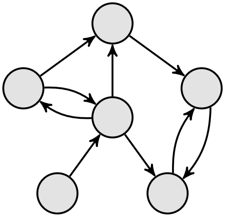
Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

- The graph is not encoded.

Descriptive distributed complexity

SOME LOGICAL FORMALISM ⟺ EQUIVALENT ⟺ COMMUNICATING MACHINES



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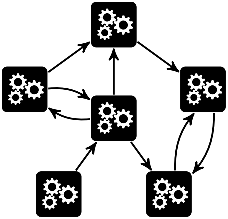
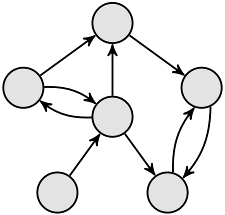


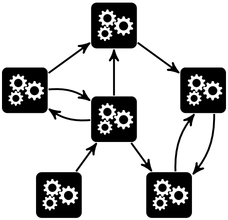
Distributed algorithm class $\mathcal{A}$

Unlike the sequential case:

- The graph is not encoded.
- It does not have to be finite.

The “Helsinki-Tampere theorem” (2012)





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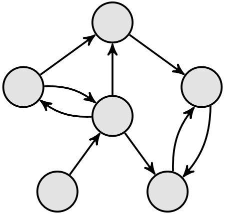

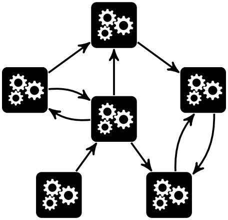

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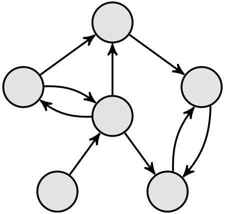
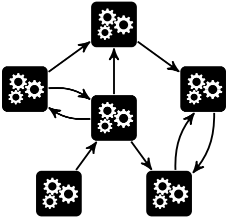
The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC





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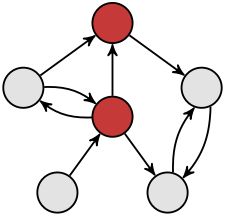

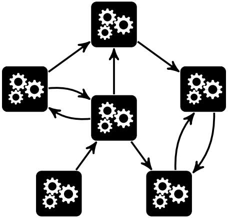

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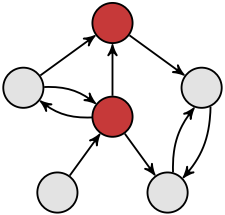
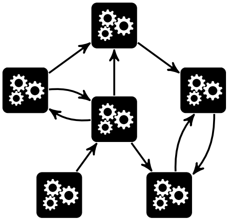
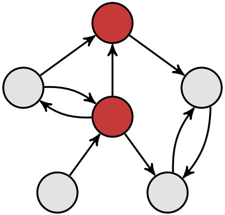




Example: $\bar{\Diamond}\big(\bar{\Box}\,\text{white} \vee \bar{\Box}\,\text{red}\big)$

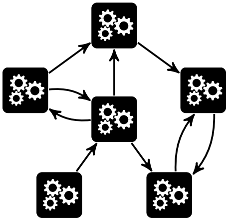
The "Helsinki-Tampere theorem" (2012)

BACKWARD MODAL LOGIC



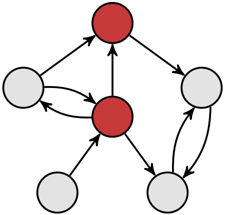
Example: $\bar{\Diamond}(\bar{\Box}\,\text{white} \vee \bar{\Box}\,\text{red})$

"I have an in-neighbor whose
in-neighbors are all white or all red."



The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC | EQUIVALENT | LOCAL DISTRIB. AUTOMATA



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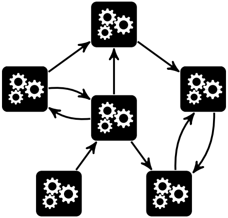

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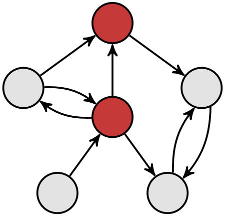

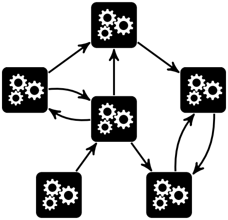

Example: $\overline{\Diamond}(\overline{\Box}\,\text{white} \vee \overline{\Box}\,\text{red})$

“I have an in-neighbor whose
in-neighbors are all white or all red.”

 : Finite-state machine

$\delta\colon Q \times 2^Q \to Q$

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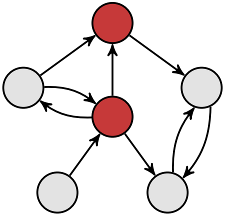
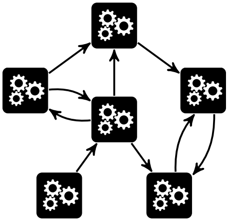
The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC ⟺ EQUIVALENT ⟺ LOCAL DISTRIB. AUTOMATA





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“I have an in-neighbor whose
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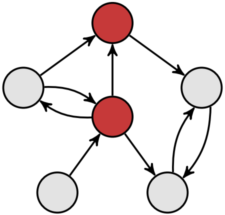
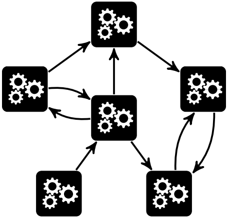
 : Finite-state machine
$\delta\colon Q \times 2^{Q} \to Q$

- Synchronous execution

The “Helsinki-Tampere theorem” (2012)

BACKWARD MODAL LOGIC | EQUIVALENT | LOCAL DISTRIB. AUTOMATA





Example: $\bar{\Diamond}(\bar{\Box}\,\text{white} \lor \bar{\Box}\,\text{red})$

“I have an in-neighbor whose in-neighbors are all white or all red.”

 : Finite-state machine

$$\delta \colon Q \times 2^{Q} \to Q$$

- Synchronous execution
- Constant running time

Contributions

Contributions

MONADIC SECOND-ORDER LOGIC

Contributions

MONADIC SECOND-ORDER LOGIC

$$\forall Z \left(\exists x, y \big(Z(x) \wedge \neg Z(y) \big) \rightarrow \cdots \right)$$

Contributions

MONADIC SECOND-ORDER LOGIC — EQUIVALENT — ALTERNATING LOCAL AUTOMATA

$$\forall Z \left(\exists x, y \big(Z(x) \land \neg Z(y) \big) \to \cdots \right)$$

Contributions

MONADIC SECOND-ORDER LOGIC

EQUIVALENT

ALTERNATING LOCAL AUTOMATA

$$\delta \colon Q \times 2^Q \to 2^Q$$

$$\forall Z \left(\exists x, y \big(Z(x) \wedge \neg Z(y) \big) \to \cdots \right)$$

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

$$\forall Z \left(\exists x, y \big(Z(x) \land \neg Z(y) \big) \to \cdots \right)$$

$$\delta \colon Q \times 2^Q \to 2^Q$$

+ Alternation

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

$$\forall Z\left(\exists x, y\big(Z(x) \wedge \neg Z(y)\big) \rightarrow \cdots\right)$$

$$\delta\colon Q \times 2^Q \rightarrow 2^Q$$

+ Alternation
+ Global acceptance

Contributions

MONADIC SECOND-ORDER LOGIC

EQUIVALENT

ALTERNATING LOCAL AUTOMATA

$$\forall Z\left(\exists x, y\big(Z(x) \land \neg Z(y)\big) \to \cdots\right)$$

$$\delta\colon Q \times 2^{Q} \to 2^{Q}$$

+ Alternation
+ Global acceptance

THE BACKWARD μ-FRAGMENT

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

$$\forall Z\,\Big(\exists x,y\big(Z(x)\wedge\neg Z(y)\big)\rightarrow\cdots\Big)$$

$$\delta\colon Q\times 2^{Q}\rightarrow 2^{Q}$$

+ Alternation
+ Global acceptance

THE BACKWARD μ-FRAGMENT

$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R\wedge Y)\vee\bar{\Diamond}X\\ \bar{\Box}Y\end{pmatrix}$$

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

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- \+ Alternation
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THE BACKWARD μ-FRAGMENT ⟺ EQUIVALENT ⟺ ASYNCHRONOUS AUTOMATA *with quasi-acyclic diagrams*

$$\mu\begin{pmatrix} X \\ Y \end{pmatrix}.\begin{pmatrix} (R \land Y) \lor \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

Contributions

MONADIC SECOND-ORDER LOGIC

EQUIVALENT

ALTERNATING LOCAL AUTOMATA

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- $+$ Global acceptance

THE BACKWARD μ-FRAGMENT

EQUIVALENT

ASYNCHRONOUS AUTOMATA

with quasi-acyclic diagrams

$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R \wedge Y) \vee \overline{\Diamond}X\\ \overline{\Box}Y\end{pmatrix}$$

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Contributions

MONADIC SECOND-ORDER LOGIC

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EQUIVALENT

ASYNCHRONOUS AUTOMATA
with quasi-acyclic diagrams

$$\delta\colon Q \times 2^Q \rightarrow Q$$

- \+ Unbounded running time

Contributions

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+ Unbounded running time
− Asynchronous execution

Contributions

MONADIC SECOND-ORDER LOGIC

EQUIVALENT

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Other contributions:

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

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Other contributions:

- Emptiness problems for deterministic nonlocal automata.

Contributions

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Other contributions:

- Emptiness problems for deterministic nonlocal automata.
- Connections to classical automata on words and trees.

Contributions

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+ Unbounded running time
− Asynchronous execution

Other contributions:

- Emptiness problems for deterministic nonlocal automata.
- Connections to classical automata on words and trees.
- Set quantifier alternation hierarchies in modal logic.

Contributions

MONADIC SECOND-ORDER LOGIC ⇔ EQUIVALENT ⇔ ALTERNATING LOCAL AUTOMATA

$$\forall Z \left(\exists x, y \big(Z(x) \land \neg Z(y) \big) \to \cdots \right)$$

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Contributions

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EQUIVALENT

ALTERNATING LOCAL AUTOMATA

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THE BACKWARD μ-FRAGMENT

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \land Y) \lor \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

EQUIVALENT

ASYNCHRONOUS AUTOMATA

with quasi-acyclic diagrams

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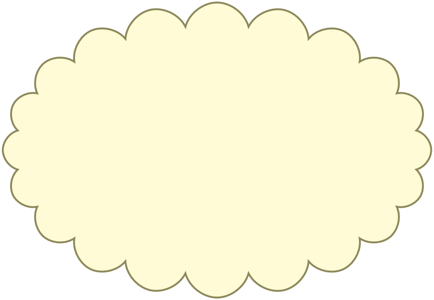
Monadic second-order logic (MSOL)

Monadic second-order logic (MSOL)

Example: weakly connected digraph

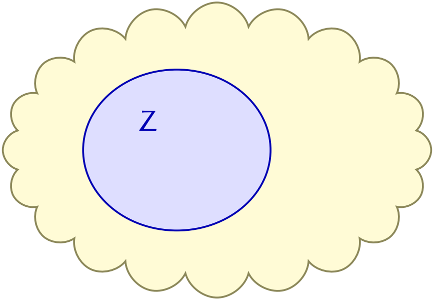
Monadic second-order logic (MSOL)

Example: weakly connected digraph



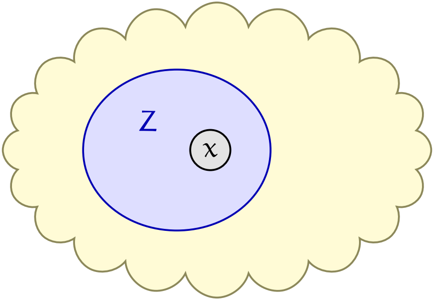
Monadic second-order logic (MSOL)

Example: weakly connected digraph



Monadic second-order logic (MSOL)

Example: weakly connected digraph



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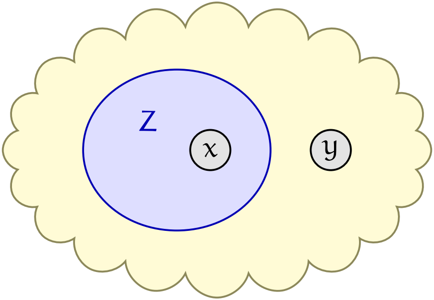
Example: weakly connected digraph



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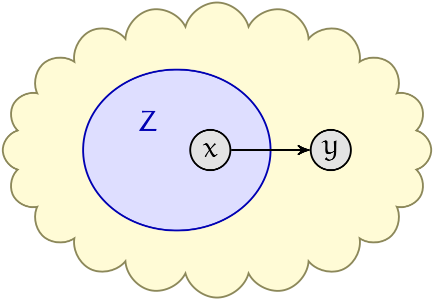
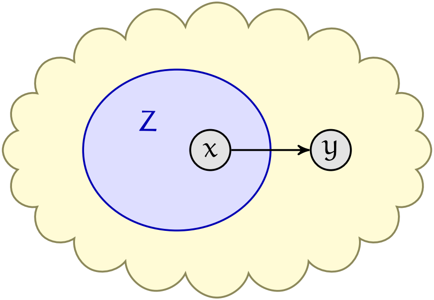
Example: weakly connected digraph



Monadic second-order logic (MSOL)

Example: weakly connected digraph

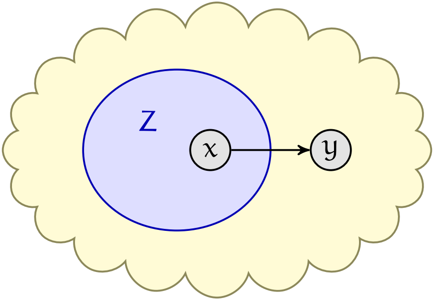
$$\forall Z \Big(\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Big)$$



Monadic second-order logic (MSOL)

Example: weakly connected digraph

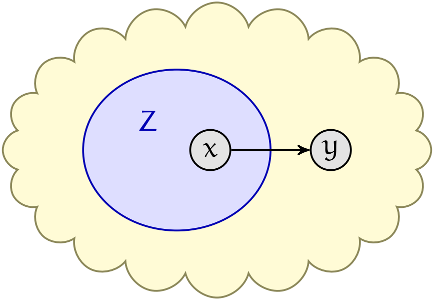
$$\forall Z \Big(\underbrace{\qquad\qquad\qquad\qquad}_{Z \text{ is a nontrivial subset.}} \qquad\qquad\qquad\qquad\qquad\qquad \Big)$$



Monadic second-order logic (MSOL)

Example: weakly connected digraph

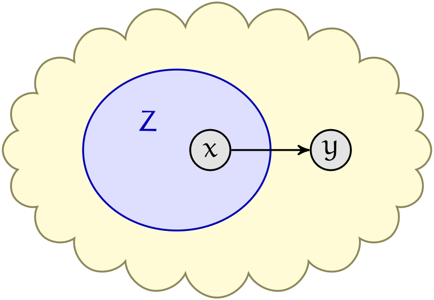
$$\forall Z \Big(\underbrace{\qquad\qquad\qquad}_{Z \text{ is a nontrivial subset.}} \rightarrow \underbrace{\qquad\qquad\qquad\qquad}_{Z \text{ is connected to its complement.}} \Big)$$



Monadic second-order logic (MSOL)

Example: weakly connected digraph

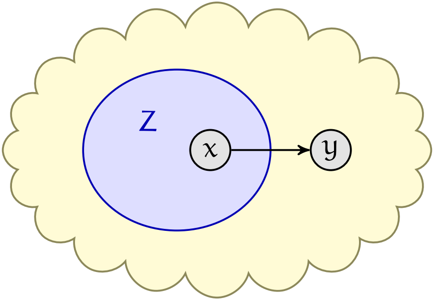
$$\forall Z \Big(\underbrace{\exists x, y \big(Z(x) \wedge \neg Z(y) \big)}_{\text{Z is a nontrivial subset.}} \rightarrow \underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Z is connected to its complement.}} \Big)$$



Monadic second-order logic (MSOL)

Example: weakly connected digraph

$$\forall Z \Big(\underbrace{\exists x, y \big(Z(x) \wedge \neg Z(y) \big)}_{Z \text{ is a nontrivial subset.}} \to \underbrace{\exists x, y \big((Z(x) \leftrightarrow \neg Z(y)) \wedge E(x, y) \big)}_{Z \text{ is connected to its complement.}} \Big)$$



Contributions

MONADIC SECOND-ORDER LOGIC

EQUIVALENT

ALTERNATING LOCAL AUTOMATA

$$\forall Z \big(\exists x, y (Z(x) \land \neg Z(y)) \rightarrow \cdots \big)$$

$$\delta \colon Q \times 2^Q \rightarrow 2^Q$$

+ Alternation
+ Global acceptance

THE BACKWARD μ-FRAGMENT

EQUIVALENT

ASYNCHRONOUS AUTOMATA

with quasi-acyclic diagrams

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \land Y) \lor \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

$$\delta \colon Q \times 2^Q \rightarrow Q$$

+ Unbounded running time
− Asynchronous execution

Alternating local automata

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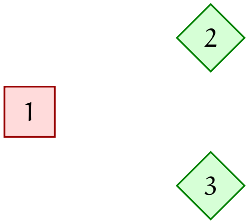
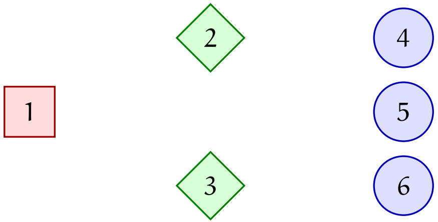
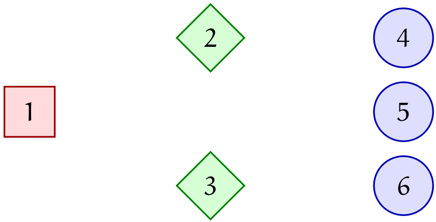


Alternating local automata

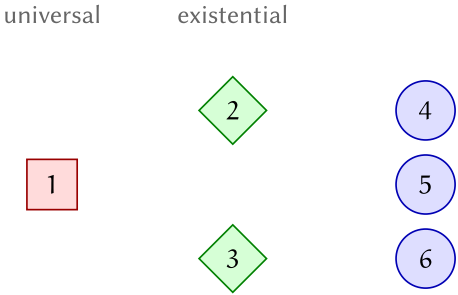


Alternating local automata

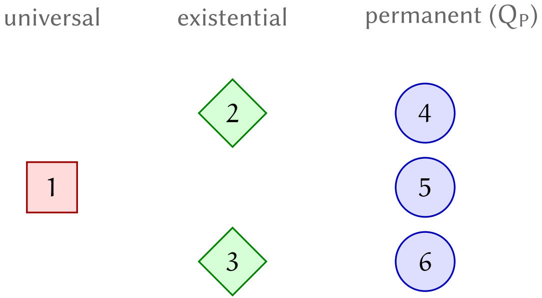
universal



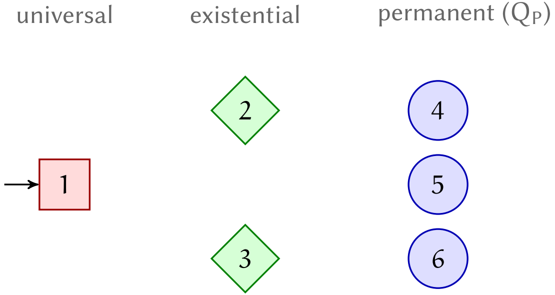
Alternating local automata



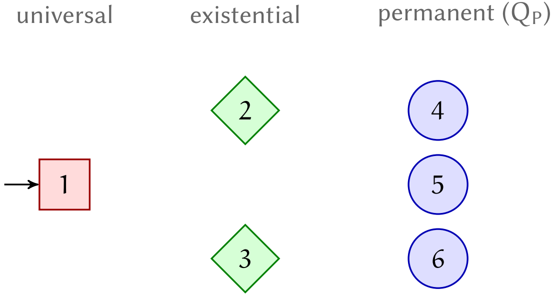
Alternating local automata



Alternating local automata



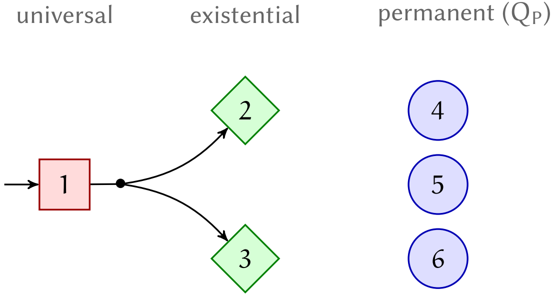
Alternating local automata



$$\delta\colon Q \times 2^Q \to 2^Q$$

(transition)

Alternating local automata

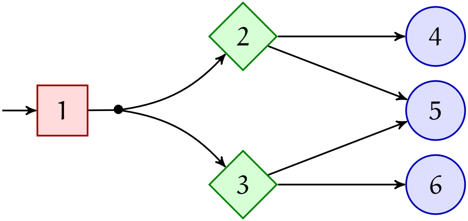


$$\delta\colon Q \times 2^Q \to 2^Q$$

(transition)

Alternating local automata

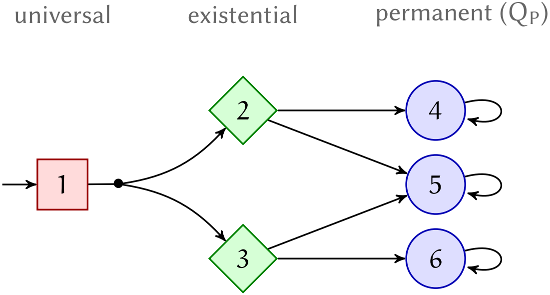
universal   existential   permanent (Q_P)



$\delta \colon Q \times 2^Q \to 2^Q$

(transition)

Alternating local automata



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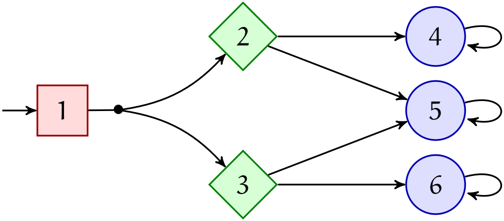
(transition)

Alternating local automata

universal existential permanent (Q_P)



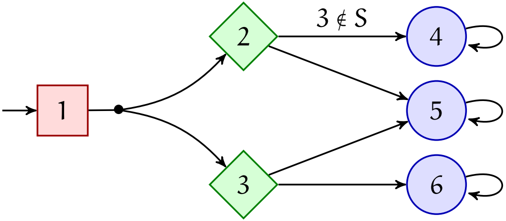
S: set of received states

$\delta\colon Q \times 2^Q \to 2^Q$

(transition)

Alternating local automata

universal existential permanent (Q_P)



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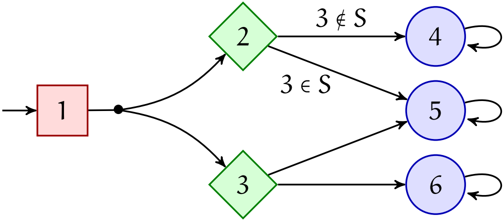
(transition)

Alternating local automata

universal existential permanent (Q_P)



S: *set of received states*

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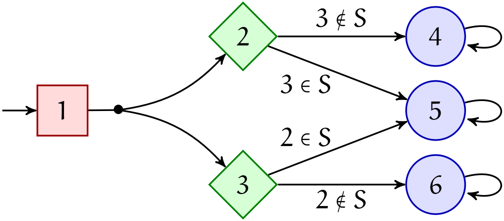
(transition)

Alternating local automata

universal existential permanent (Q_P)

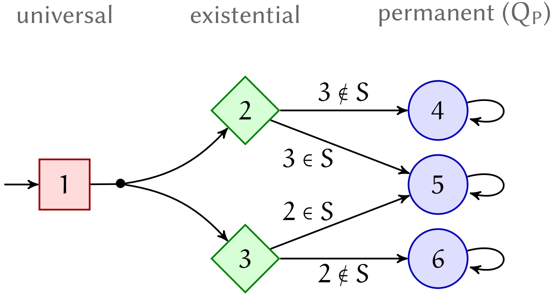


S: set of received states

$\delta\colon Q \times 2^Q \to 2^Q$

(transition)

Alternating local automata



S: set of received states

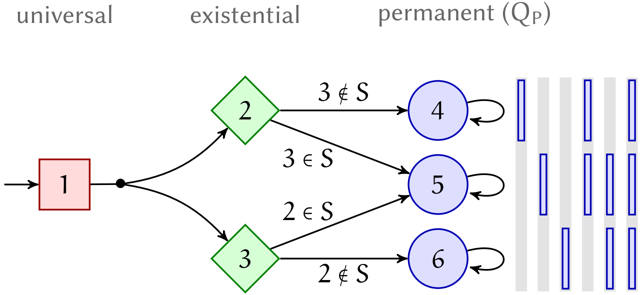
$\delta\colon Q \times 2^Q \to 2^Q$

(transition)

$\mathcal{F} \subseteq 2^{Q_P}$

(global acceptance)

Alternating local automata



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$\mathcal{F} \subseteq 2^{Q_P}$

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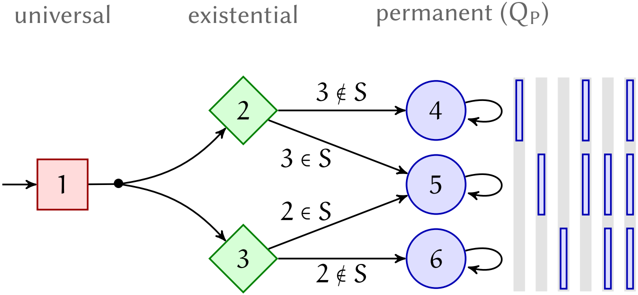
Alternating local automata



S: set of received states

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(transition)

$\mathcal{F} \subseteq 2^{Q_P}$

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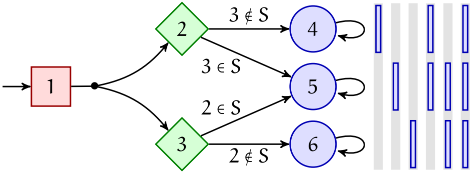
Same example: weakly connected digraph

Alternating run



S: set of received states

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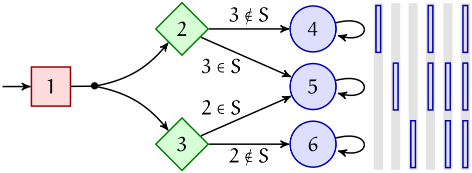

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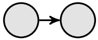
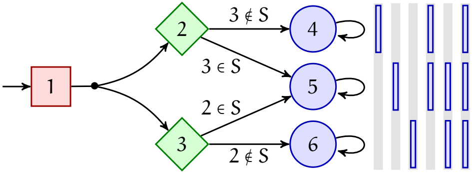


Alternating run



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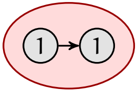

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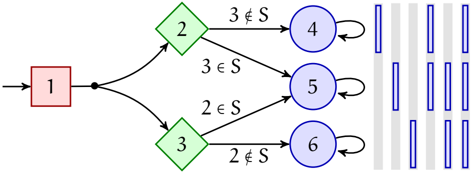

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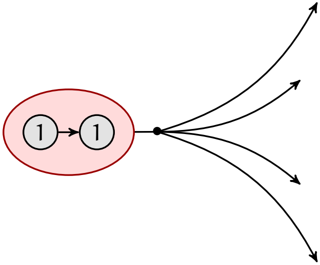

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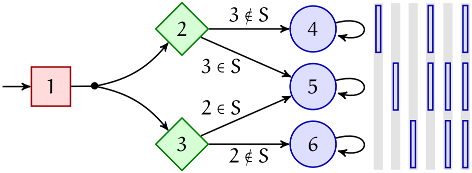

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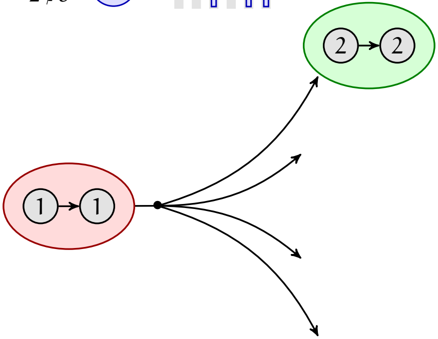
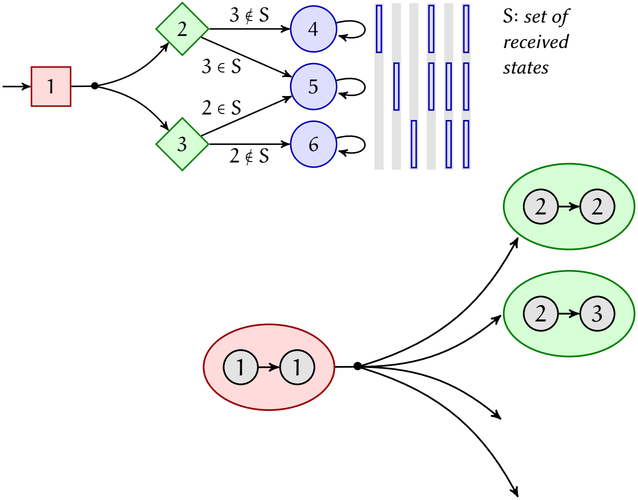


Alternating run



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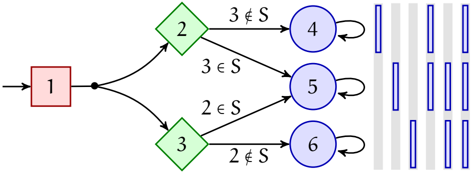

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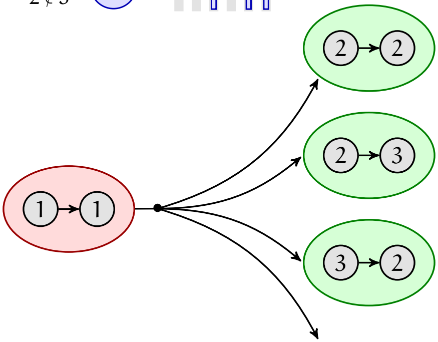

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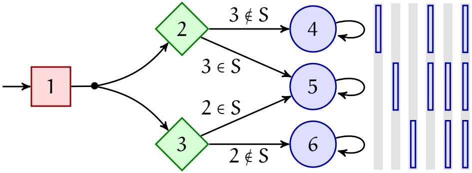
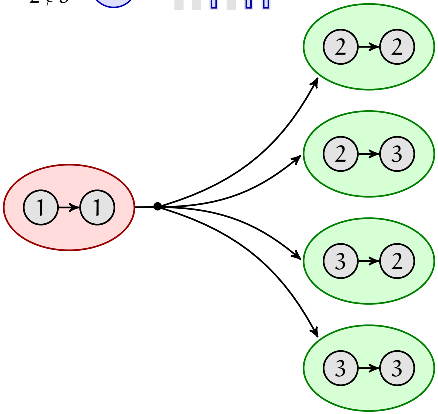
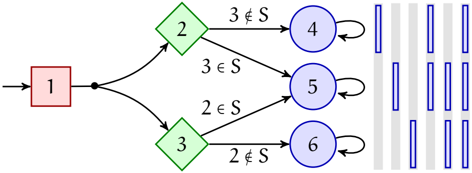


S: set of received states



Alternating run



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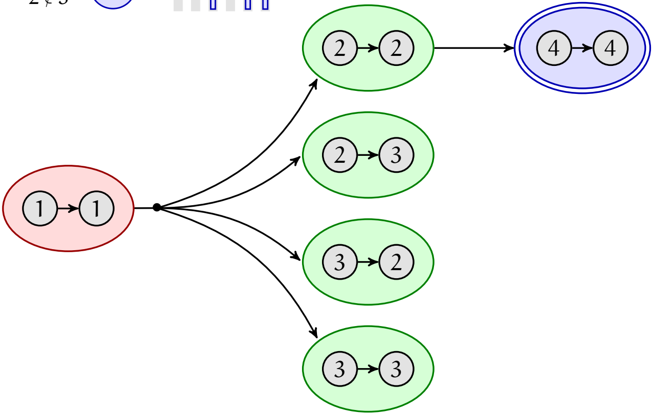
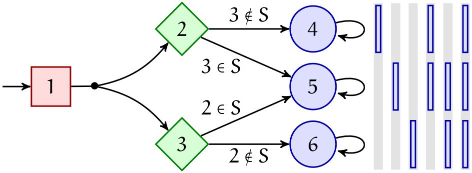


Alternating run



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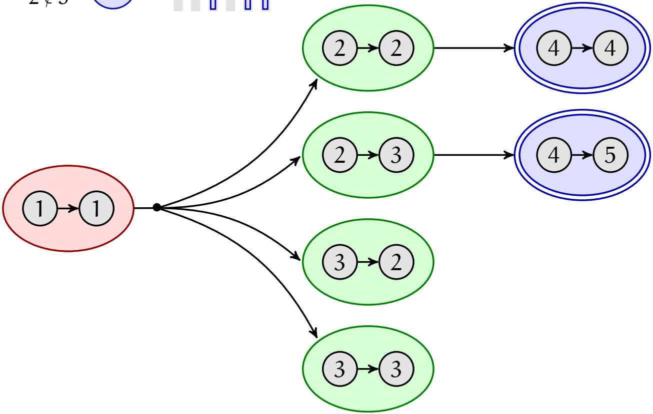
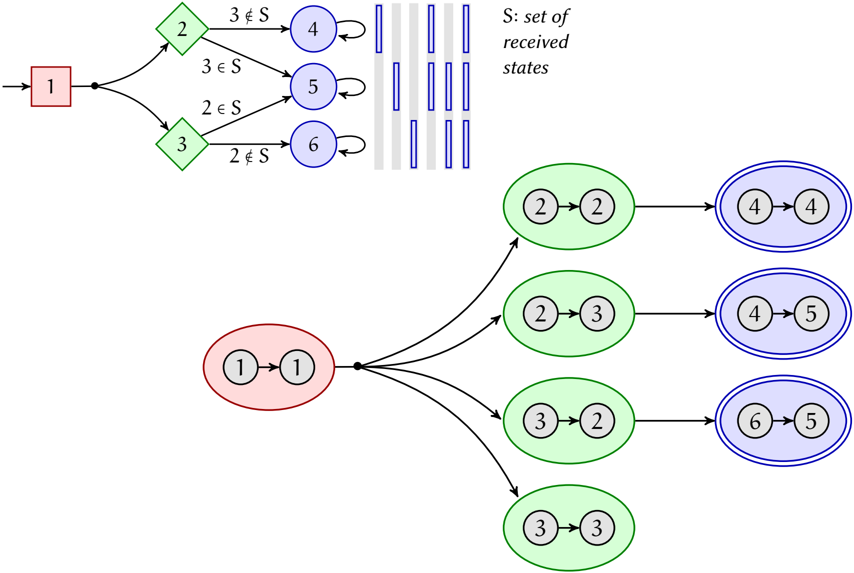
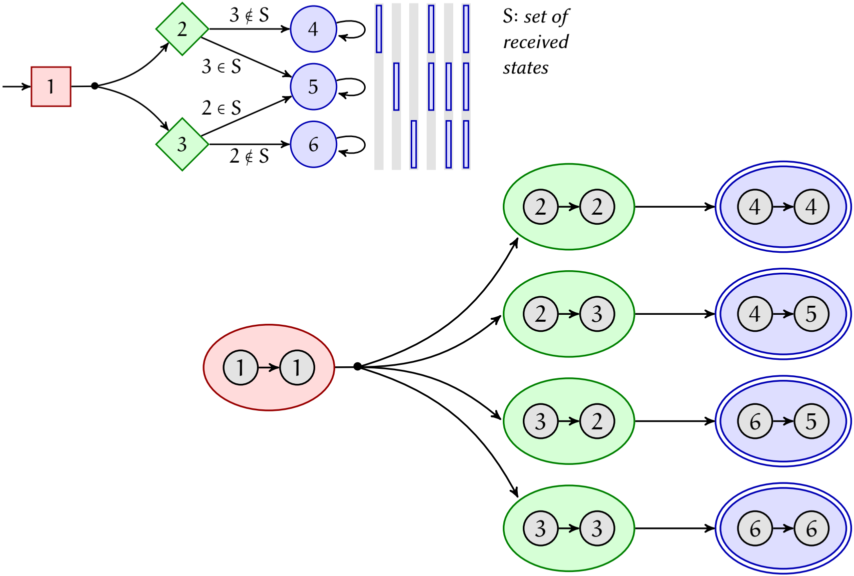


Alternating run



Alternating run



Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ ALTERNATING LOCAL AUTOMATA

$$\forall Z \left(\exists x, y \left(Z(x) \wedge \neg Z(y) \right) \rightarrow \cdots \right)$$

$$\delta\colon Q \times 2^Q \rightarrow 2^Q$$

+ Alternation
+ Global acceptance

THE BACKWARD μ-FRAGMENT ⟺ EQUIVALENT ⟺ ASYNCHRONOUS AUTOMATA *with quasi-acyclic diagrams*

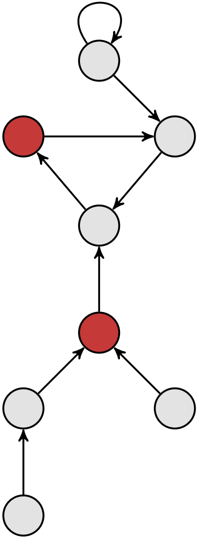
$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

$$\delta\colon Q \times 2^Q \rightarrow Q$$

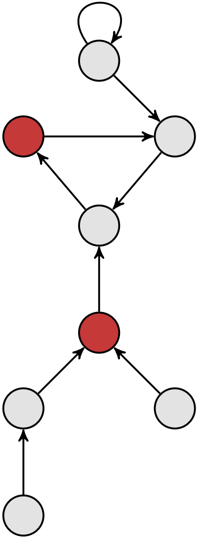
+ Unbounded running time
− Asynchronous execution

The backward μ-fragment

The backward μ-fragment

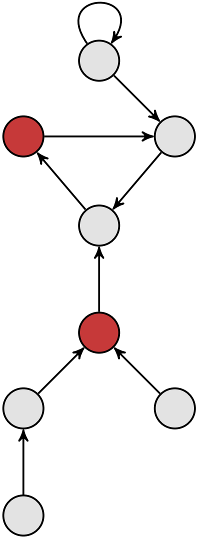


The backward μ-fragment



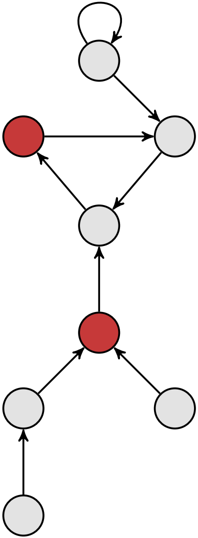
$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\left(\qquad\qquad\qquad\right)$$

The backward μ-fragment



$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \\ \end{pmatrix}$$

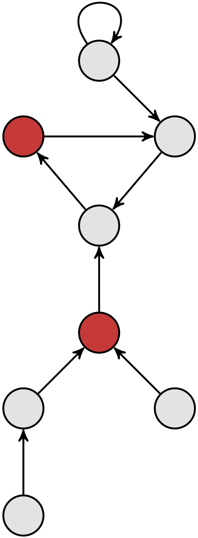
The backward μ-fragment



constant

$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R \wedge Y) \vee \bar{\Diamond} X\\ \\ \end{pmatrix}$$

The backward μ-fragment

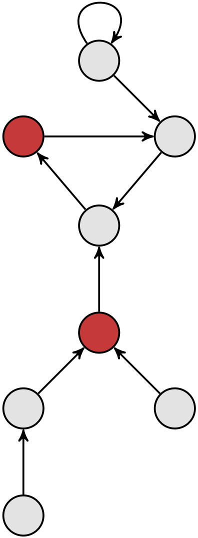


unnegated variable

constant

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \\ \end{pmatrix}$$

The backward μ-fragment



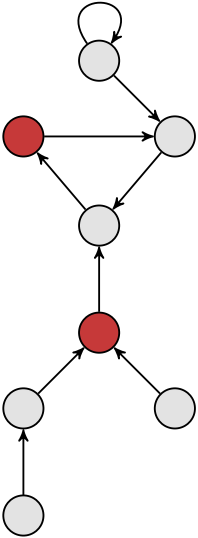
unnegated variable

constant

∃ incoming neighbor

$$\mu\begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \\ \end{pmatrix}$$

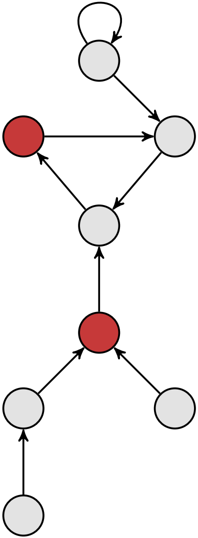
The backward μ-fragment



constant — unnegated variable — ∃ incoming neighbor

$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R \wedge Y) \vee \overline{\Diamond}X\\ \overline{\Box}Y\end{pmatrix}$$

The backward μ-fragment



constant

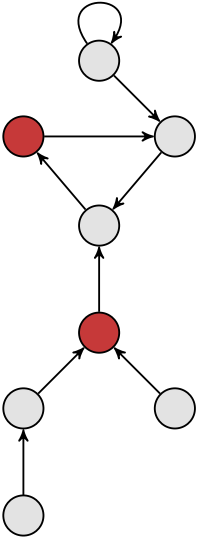
unnegated variable

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

The backward μ-fragment



unnegated variable

constant

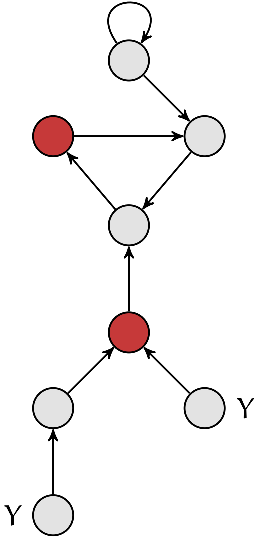
∃ incoming neighbor

$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R \wedge Y) \vee \bar{\Diamond}X\\ \bar{\Box}Y\end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

The backward μ-fragment



constant

unnegated variable

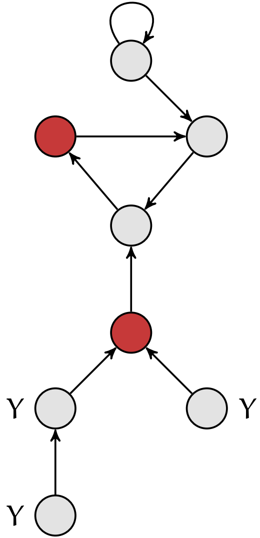
∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \land Y) \lor \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

The backward μ-fragment



constant

unnegated variable

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$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

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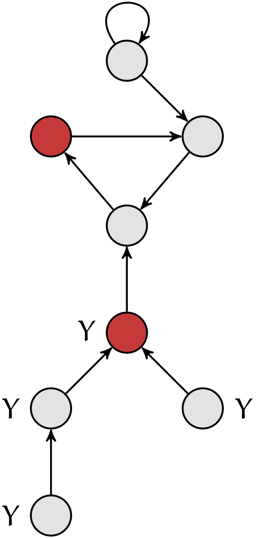
The backward μ-fragment



constant

unnegated variable

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

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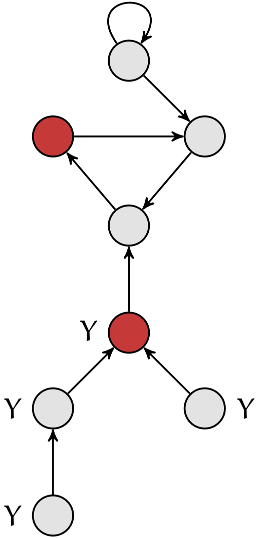
Compute the simultaneous least fixpoint.

The backward μ-fragment



constant

unnegated variable

∃ incoming neighbor

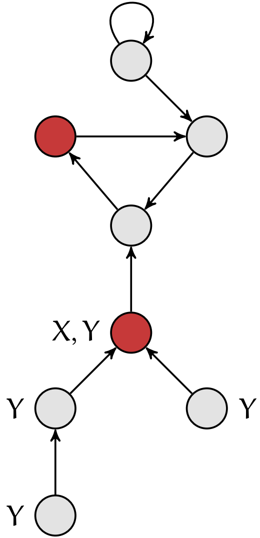
$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \land Y) \lor \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

The backward μ-fragment



unnegated variable

constant

∃ incoming neighbor

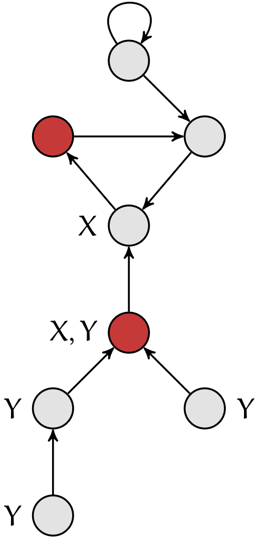
$$\mu\begin{pmatrix} X \\ Y \end{pmatrix}.\begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

Y: “Going backwards, we cannot reach any directed cycle (only dead-ends).”

The backward μ-fragment



constant

unnegated variable

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

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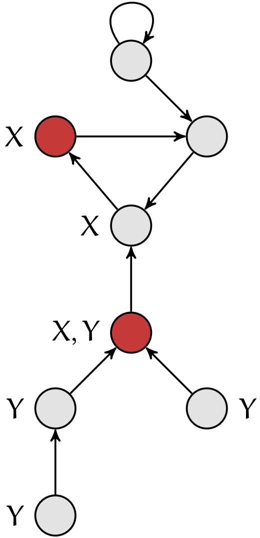
The backward μ-fragment



unnegated variable

constant

∃ incoming neighbor

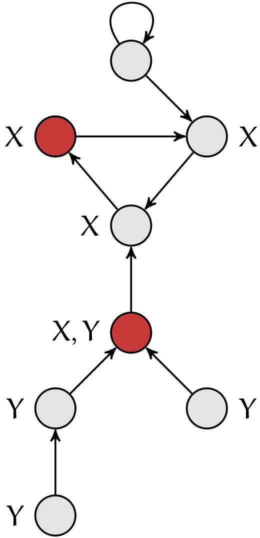
$$\mu\begin{pmatrix}X\\Y\end{pmatrix}.\begin{pmatrix}(R\wedge Y)\ \vee\ \bar{\Diamond}X\\ \bar{\Box}Y\end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

Y: “Going backwards, we cannot reach any directed cycle (only dead-ends).”

The backward μ-fragment



constant

unnegated variable

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \bar{\Diamond} X \\ \bar{\Box} Y \end{pmatrix}$$

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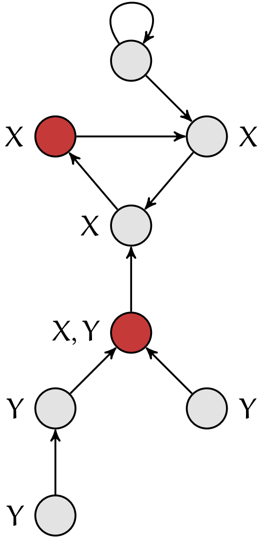
Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

The backward μ-fragment



constant

unnegated variable

∃ incoming neighbor

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

∀ incoming neighbors

Compute the simultaneous least fixpoint.

Y: "Going backwards, we cannot reach any directed cycle (only dead-ends)."

X: "Going backwards, we can reach a red node from which no directed cycle is reachable."

Contributions

MONADIC SECOND-ORDER LOGIC ⟺ EQUIVALENT ⟺ **ALTERNATING LOCAL AUTOMATA**

$$\forall Z \left(\exists x, y \left(Z(x) \wedge \neg Z(y)\right) \rightarrow \cdots\right)$$

$$\delta \colon Q \times 2^Q \rightarrow 2^Q$$

- \+ Alternation
- \+ Global acceptance

THE BACKWARD μ-FRAGMENT ⟺ EQUIVALENT ⟺ **ASYNCHRONOUS AUTOMATA** *with quasi-acyclic diagrams*

$$\mu \begin{pmatrix} X \\ Y \end{pmatrix} . \begin{pmatrix} (R \wedge Y) \vee \overline{\Diamond} X \\ \overline{\Box} Y \end{pmatrix}$$

$$\delta \colon Q \times 2^Q \rightarrow Q$$

- \+ Unbounded running time
- − Asynchronous execution

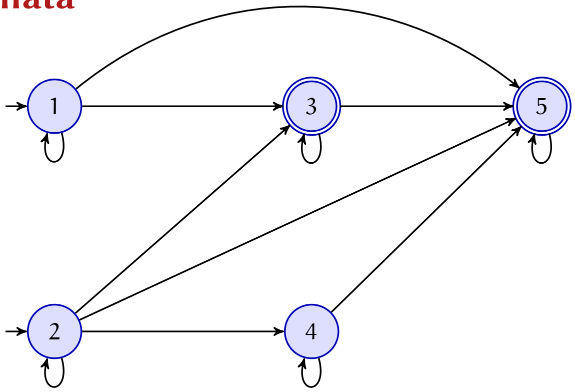
Asynchronous automata

Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Asynchronous automata

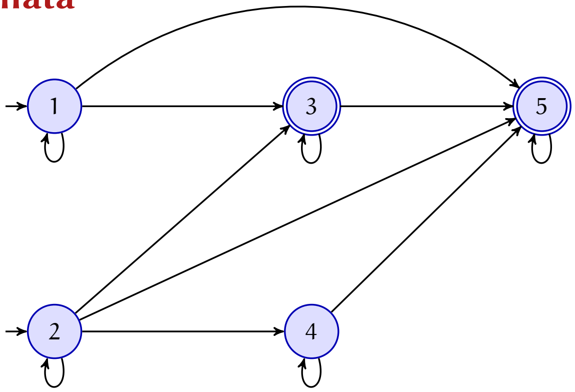
$\delta\colon Q \times 2^Q \to Q$



Asynchronous automata

$\delta\colon Q \times 2^{Q} \to Q$

Quasi-acyclic diagram.



Asynchronous automata

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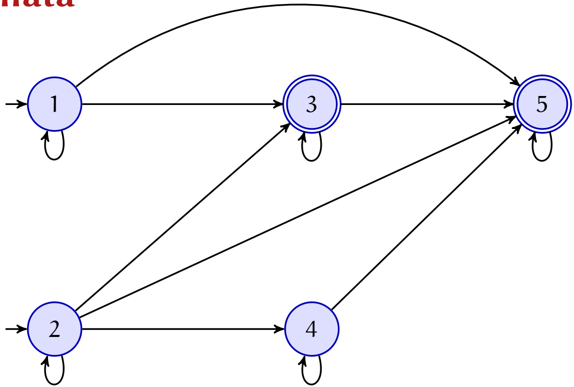
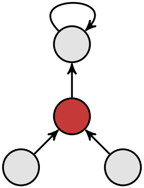
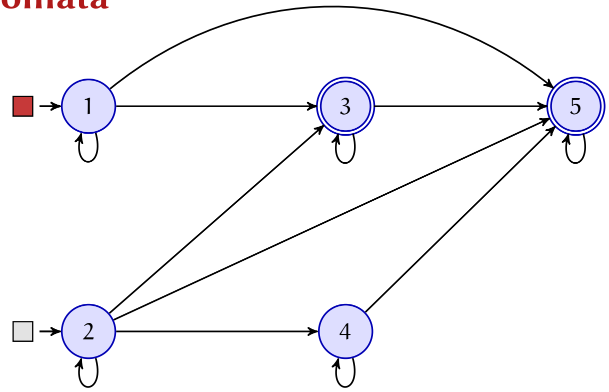


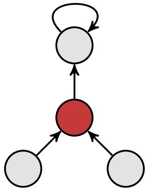


Asynchronous automata

$\delta \colon Q \times 2^Q \to Q$

Quasi-acyclic
diagram.

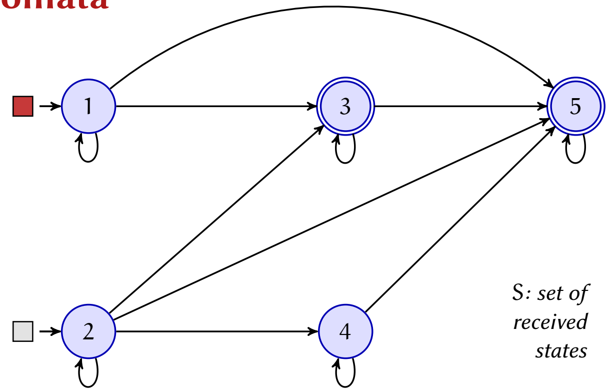




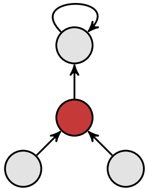
Asynchronous automata

$\delta\colon Q \times 2^{Q} \to Q$

Quasi-acyclic diagram.



S: *set of received states*



Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic
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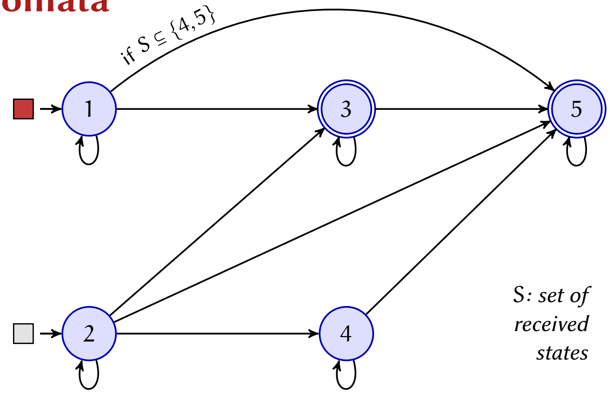
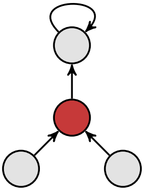
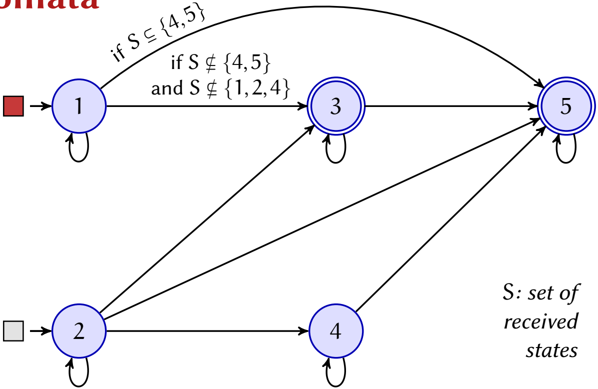


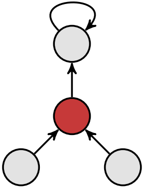


Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic diagram.

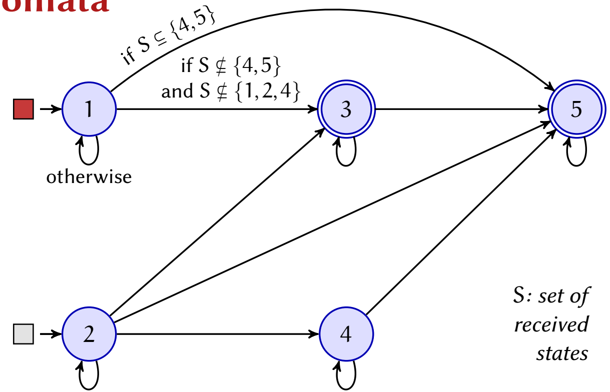




Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic
diagram.



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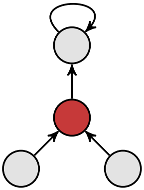
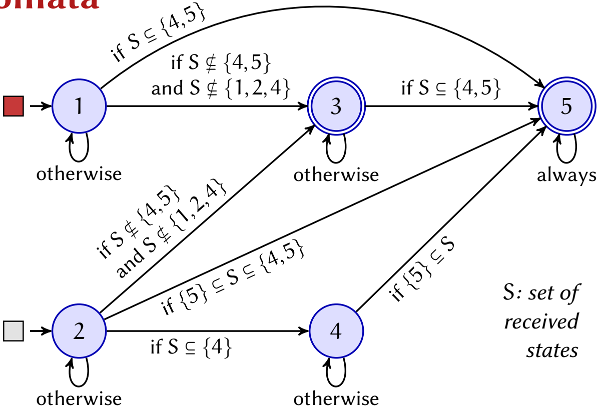


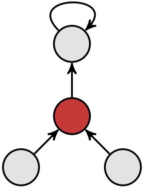
Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic diagram.



S: set of received states



Asynchronous automata

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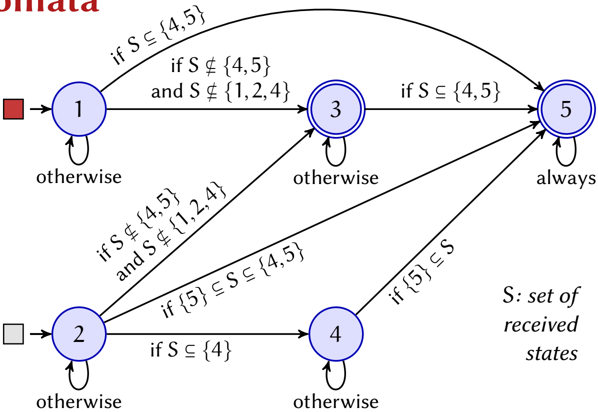
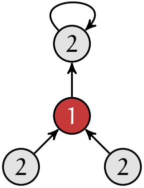
Quasi-acyclic
diagram.



S: set of received states

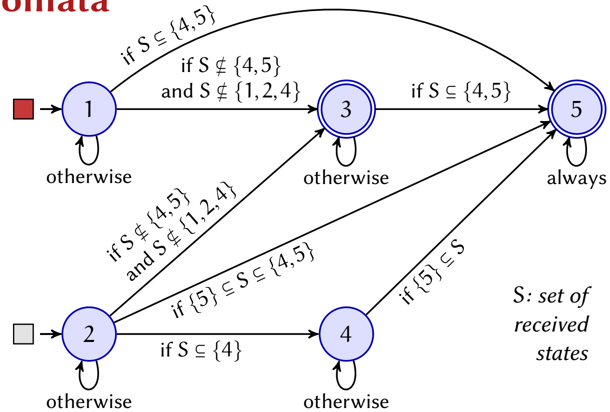


Asynchronous automata

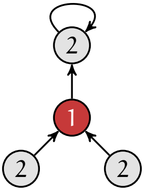
$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic diagram.

Nodes may sleep & miss messages.



S: set of received states



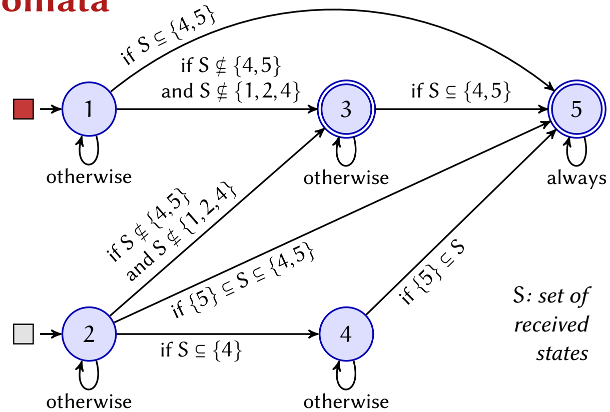
Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

Quasi-acyclic diagram.

Nodes may sleep & miss messages.

Messages may be delayed (FIFO).



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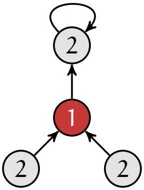

Asynchronous automata

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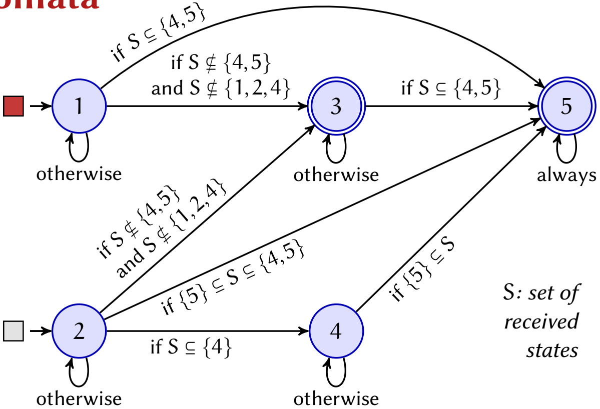

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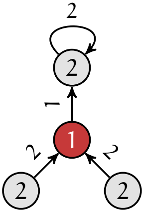

Asynchronous automata

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Quasi-acyclic diagram.

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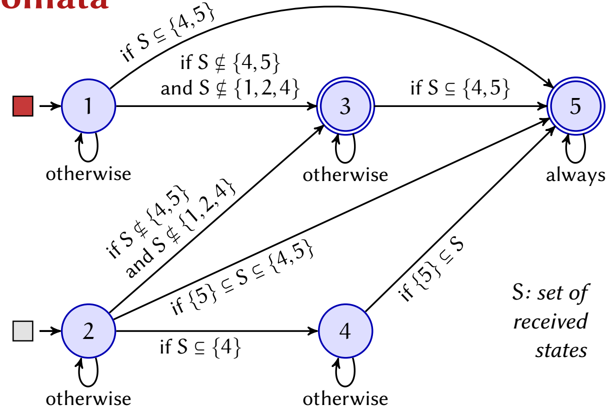

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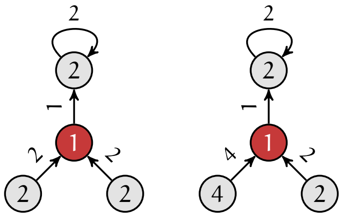

Asynchronous automata

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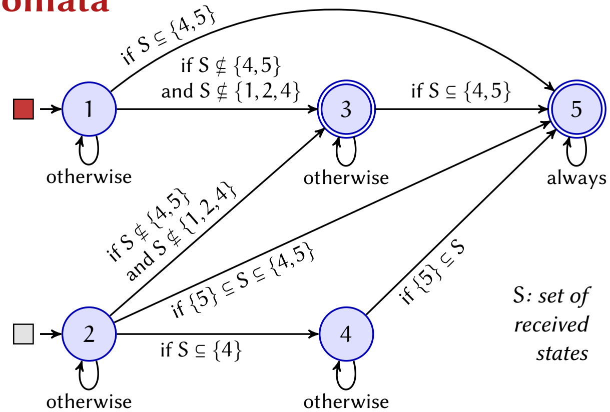
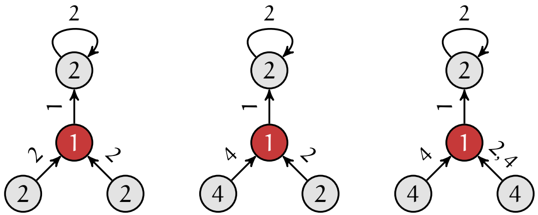




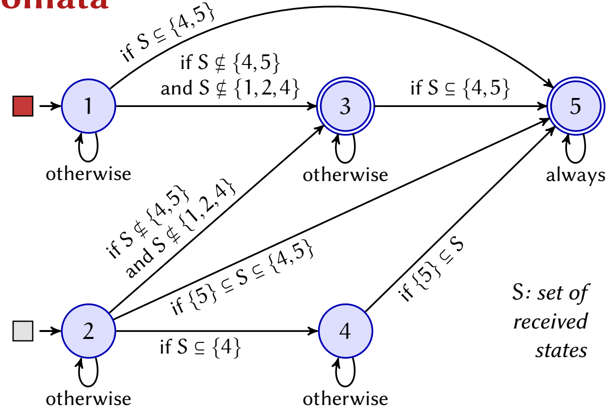
Asynchronous automata

$\delta\colon Q \times 2^Q \to Q$

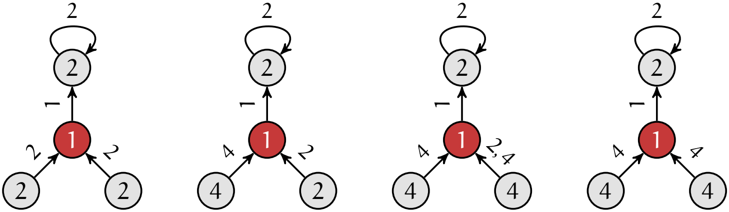
Quasi-acyclic diagram.

Nodes may sleep & miss messages.

Messages may be delayed (FIFO).



S: set of received states



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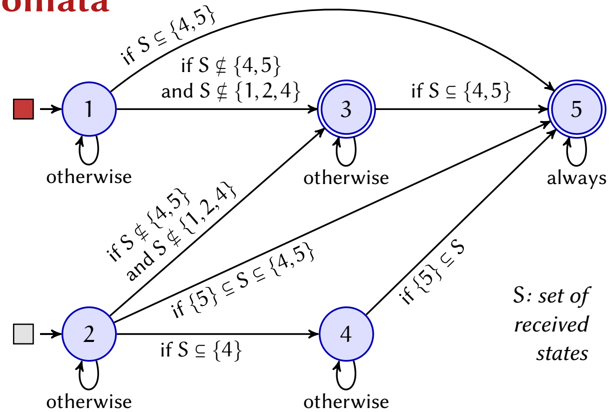

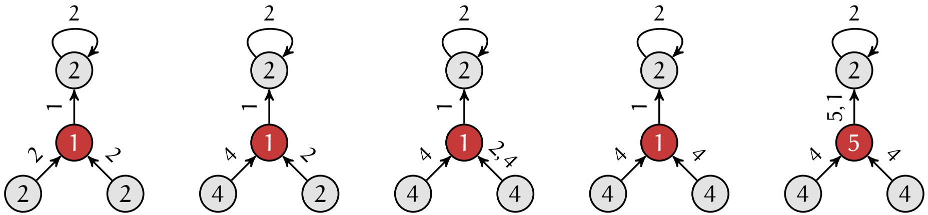

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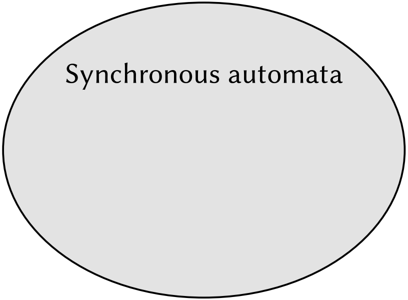

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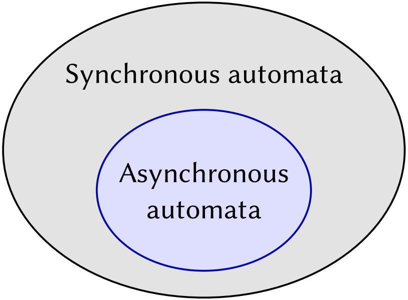

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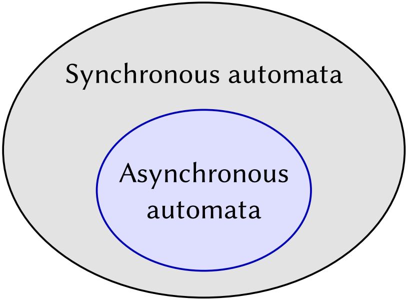

Asynchrony is an additional semantic property.

Perspectives

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Thanks!