Emptiness Problems for Distributed Automata

Fabian Reiter

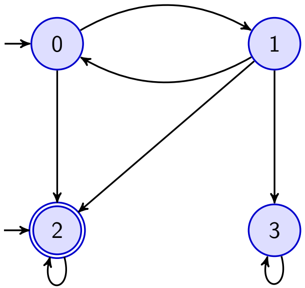
IRIF, Université Paris Diderot

September 21, 2017

Joint work with Antti Kuusisto

Distributed automata

Distributed automata



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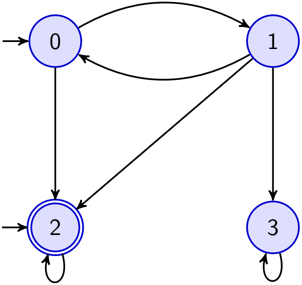

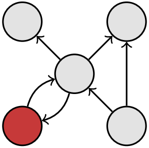

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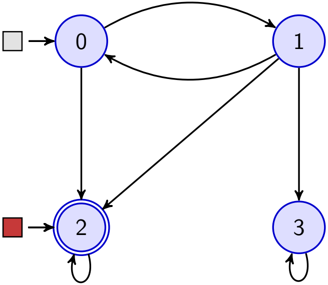

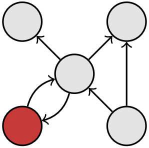

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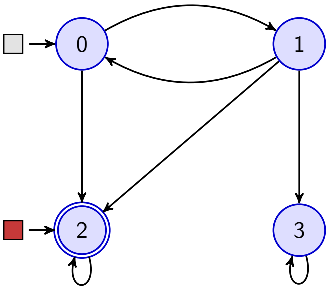
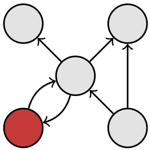


Transition function:
$\delta\colon Q \times 2^Q \to Q$
(Q: set of states)



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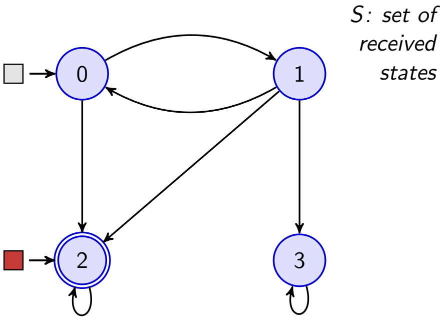

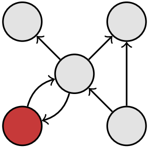

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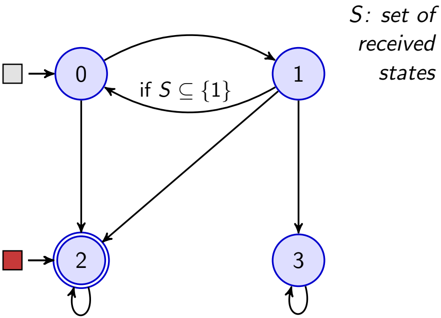

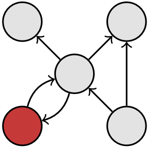

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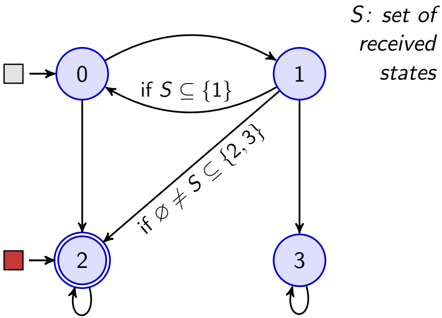

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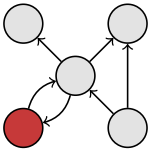

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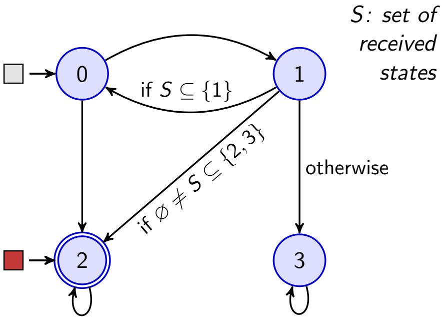

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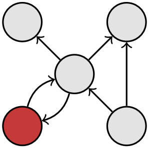

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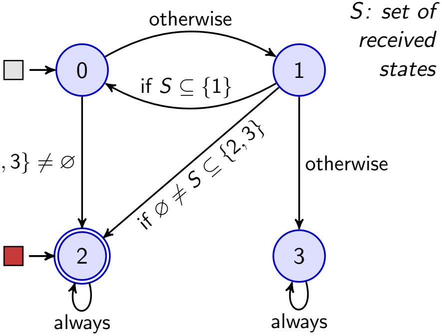

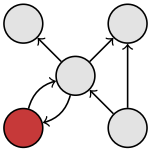

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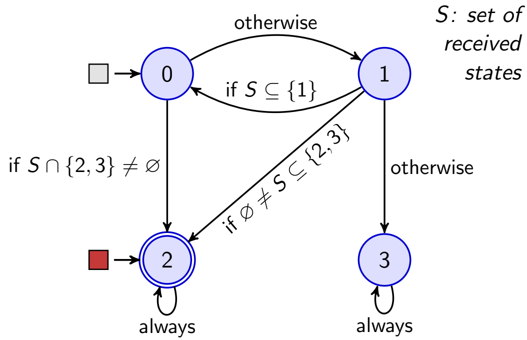
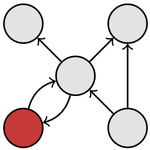


Synchronous run:



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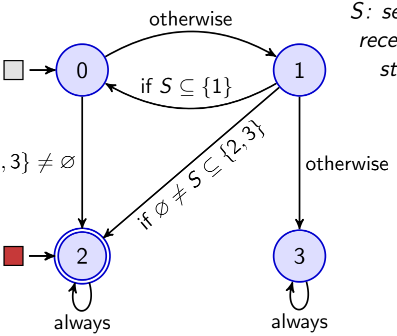

S: set of received states

Transition function:
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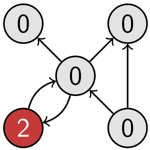

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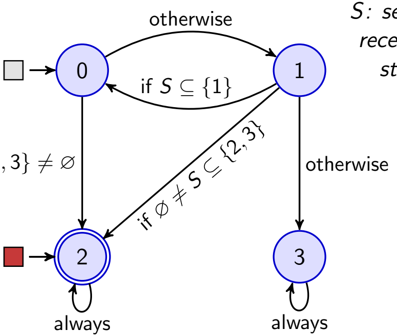

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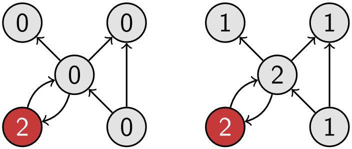

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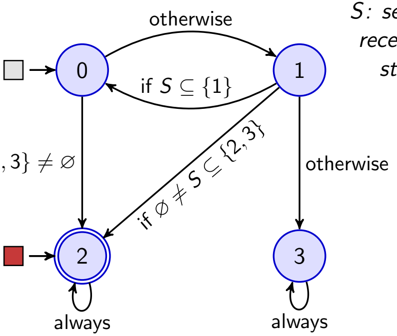

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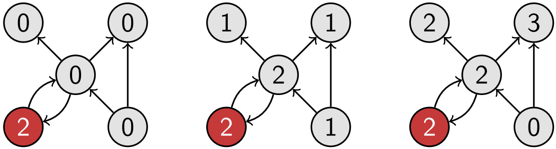

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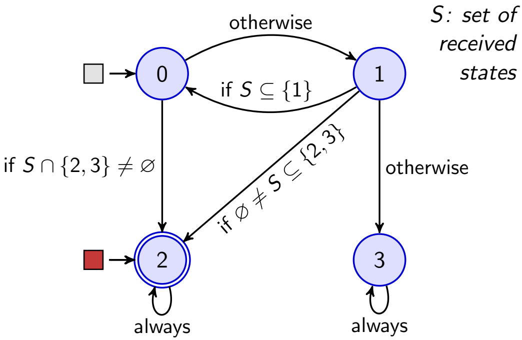

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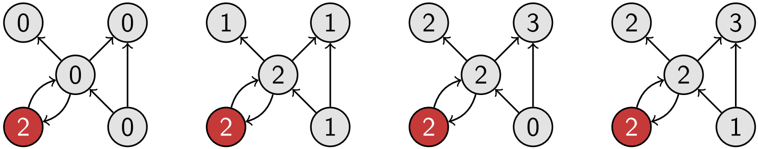

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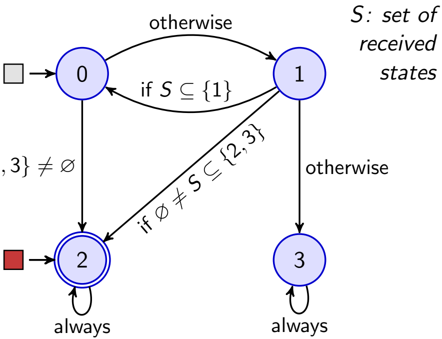

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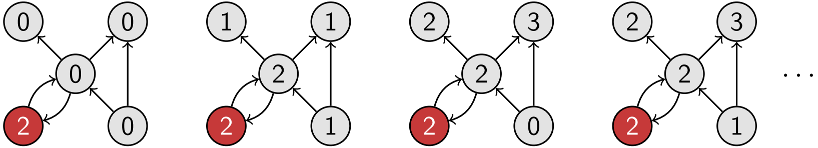

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Automaton A *accepts* digraph G on node $v \in G$
iff v visits an accepting state at some time $t \in \mathbb{N}$.

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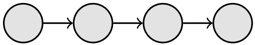
Simple reduction:

undecidable on dipaths



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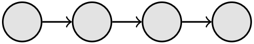
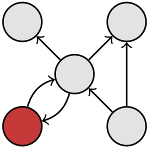
Emptiness problem:

Does automaton A accept on some node in some digraph?

Simple reduction:

undecidable on dipaths $\implies$ undecidable on digraphs





Simulating a Turing machine

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Would be easy on doubly linked dipaths:

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Turing machine

with alphabet $\{\square, \blacksquare\}$
and state set $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

Simulating a Turing machine

Would be easy on doubly linked dipaths:

space $\longrightarrow$

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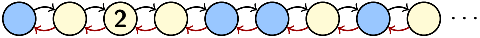

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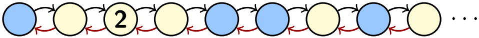

Distributed automaton

with state set
$\{\square, \blacksquare\} \times \{\epsilon, \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

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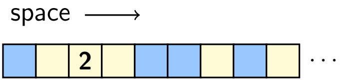

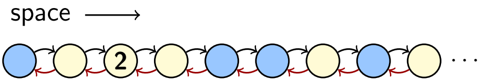

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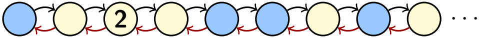

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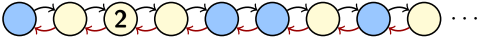

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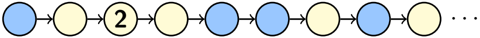

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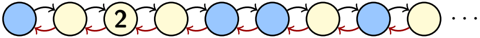

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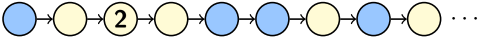


One-way communication ☹

Exchanging space and time

Exchanging space and time

Turing machine

alphabet : $\{\square, \blacksquare\}$

state set : $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

Exchanging space and time

Turing machine

space $\longrightarrow$

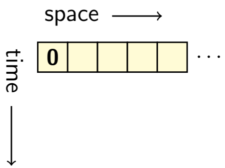
time $\longrightarrow$

alphabet : $\{\square, \square\}$

state set : $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

Exchanging space and time

Turing machine



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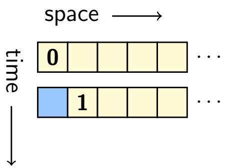

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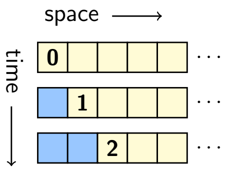

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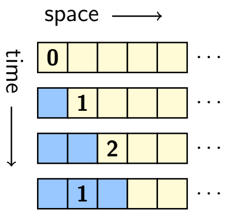

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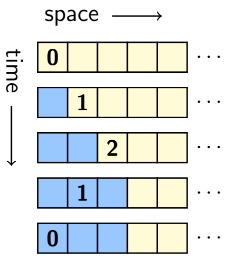

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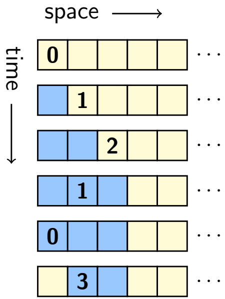

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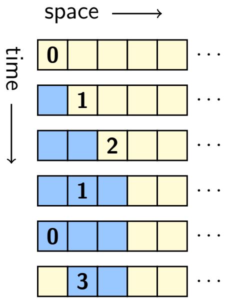

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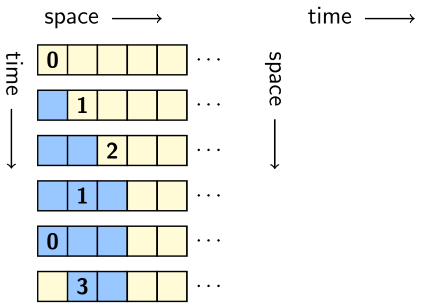

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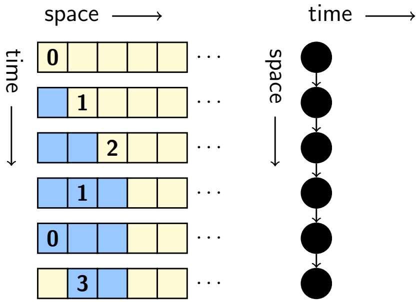

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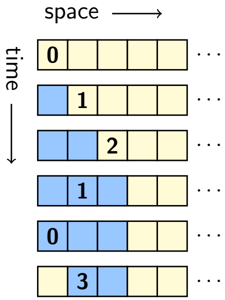

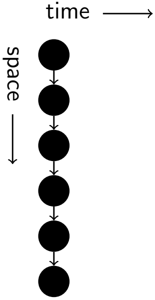

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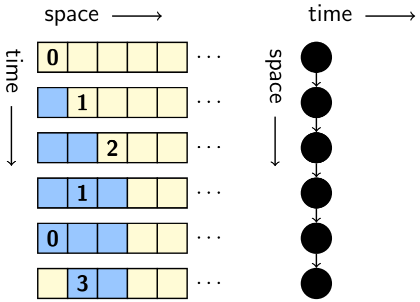
● : waiting node

Exchanging space and time

Turing machine | **Distributed automaton**



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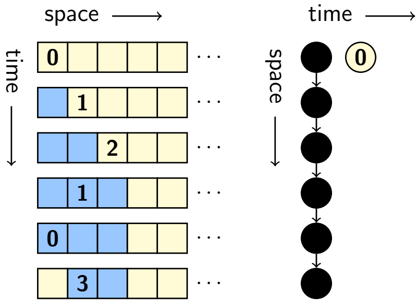
● : waiting node

: nodes “visiting” a Turing cell

Exchanging space and time

Turing machine **Distributed automaton**



alphabet : {□, ■}

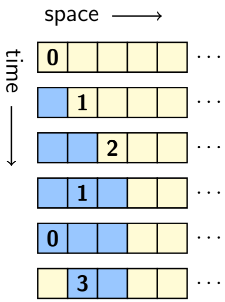
state set : {**0**, **1**, **2**, **3**}

● : waiting node

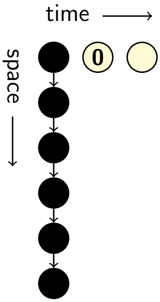
● (1) ○ : nodes "visiting" a Turing cell

Exchanging space and time

Turing machine



Distributed automaton



alphabet : {□, ■}

state set : {**0**, **1**, **2**, **3**}

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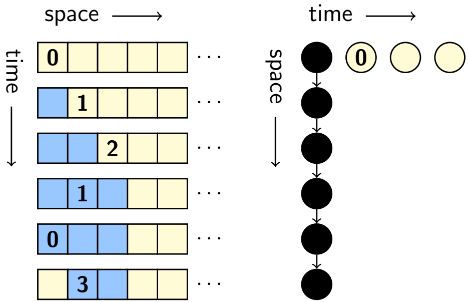
Exchanging space and time

Turing machine **Distributed automaton**



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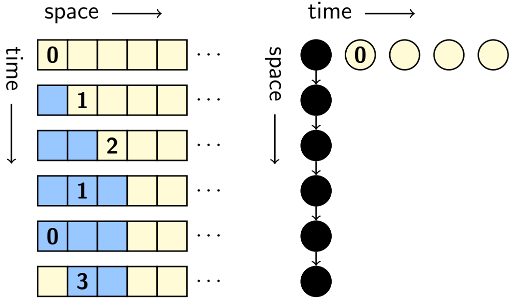
Exchanging space and time

Turing machine | **Distributed automaton**



alphabet : {□, □}

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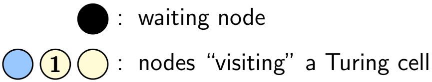
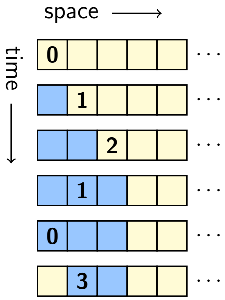
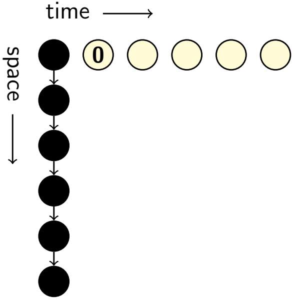


Exchanging space and time

Turing machine



Distributed automaton



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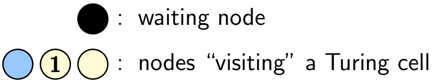

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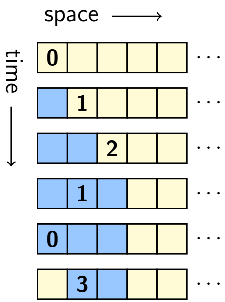
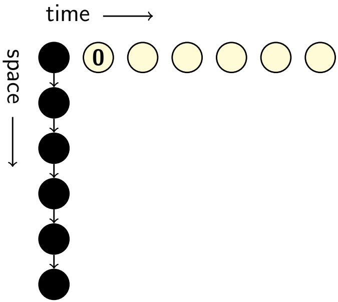


Distributed automaton



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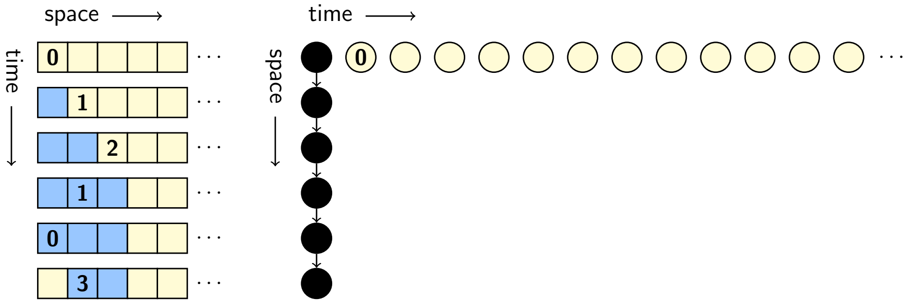
Exchanging space and time

Turing machine **Distributed automaton**



alphabet : {□, □}

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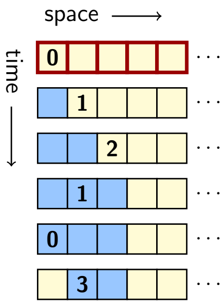
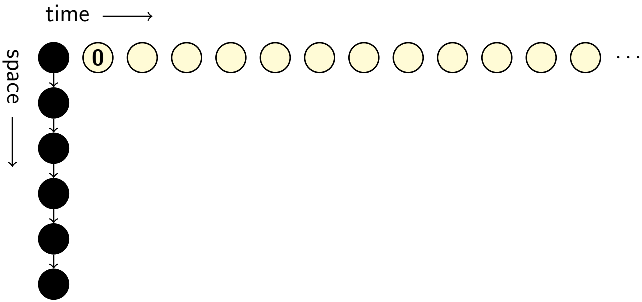
Exchanging space and time

Turing machine



Distributed automaton



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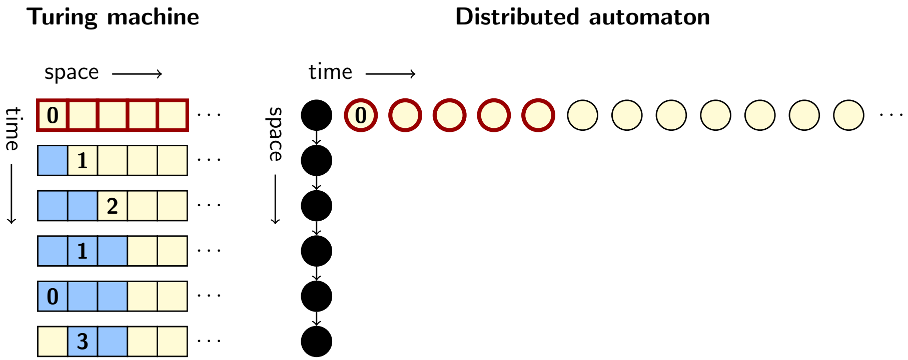

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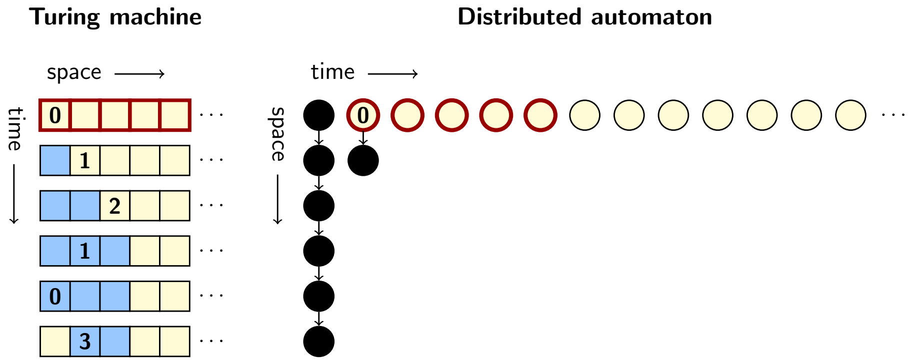

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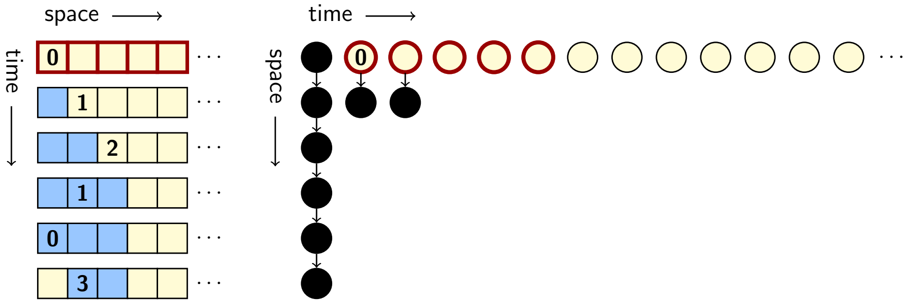
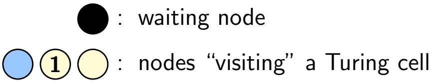
Exchanging space and time

Turing machine | **Distributed automaton**



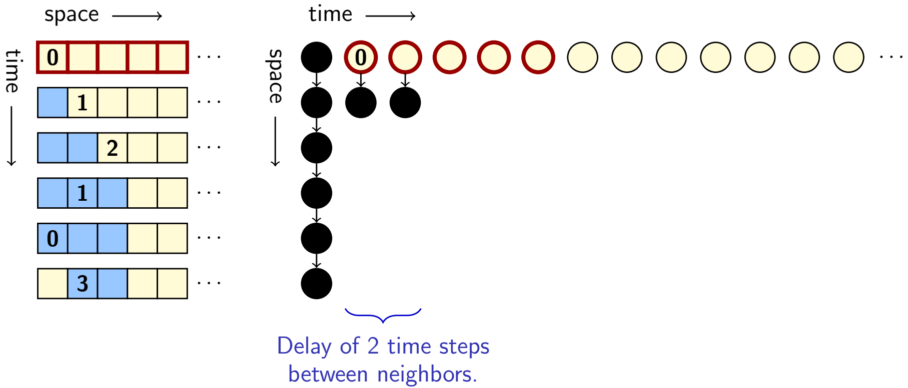
alphabet : {□, ■}

state set : $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$



Exchanging space and time

Turing machine **Distributed automaton**



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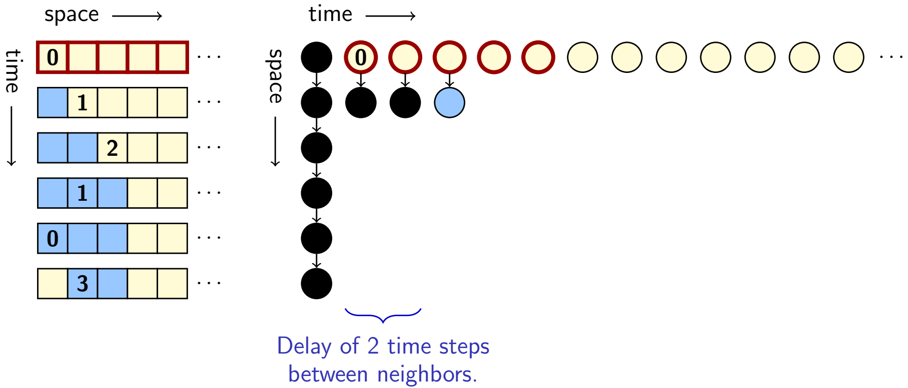
Exchanging space and time

Turing machine **Distributed automaton**



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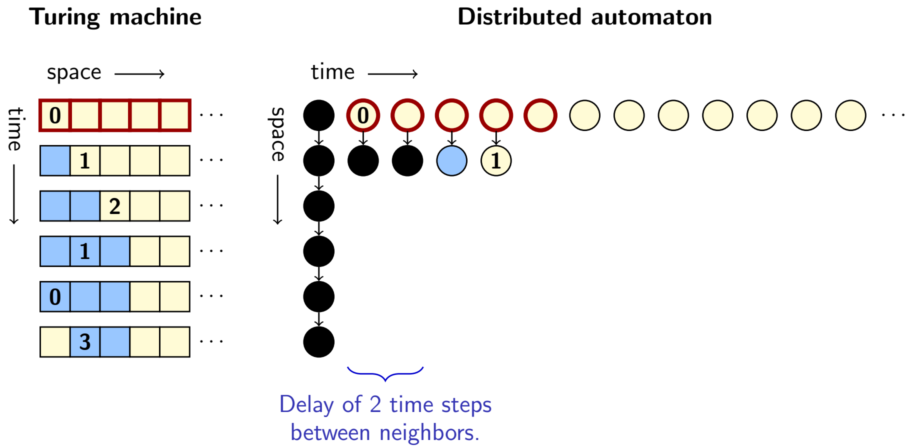
○ ① ○ : nodes "visiting" a Turing cell

Exchanging space and time



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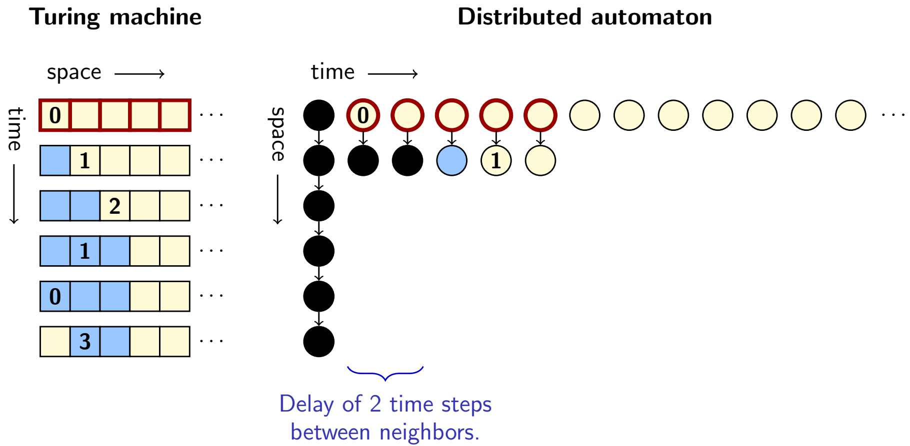
Exchanging space and time



alphabet : {□, □}

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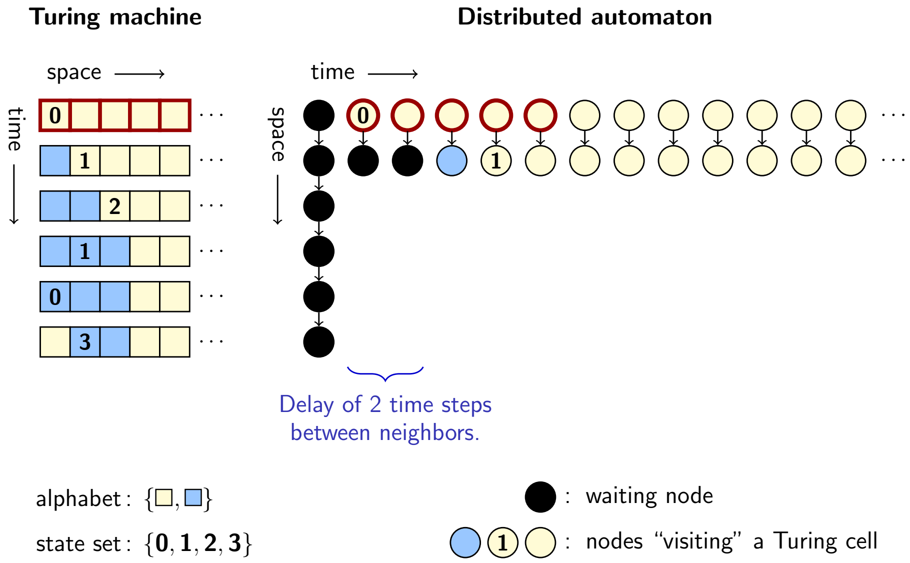
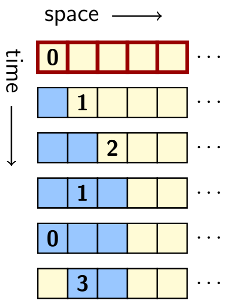
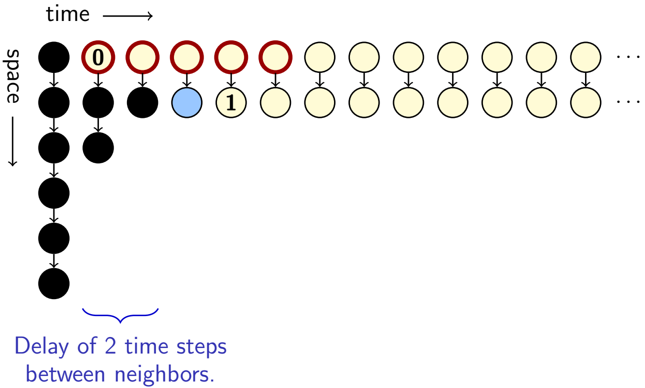


Exchanging space and time

Turing machine



Distributed automaton



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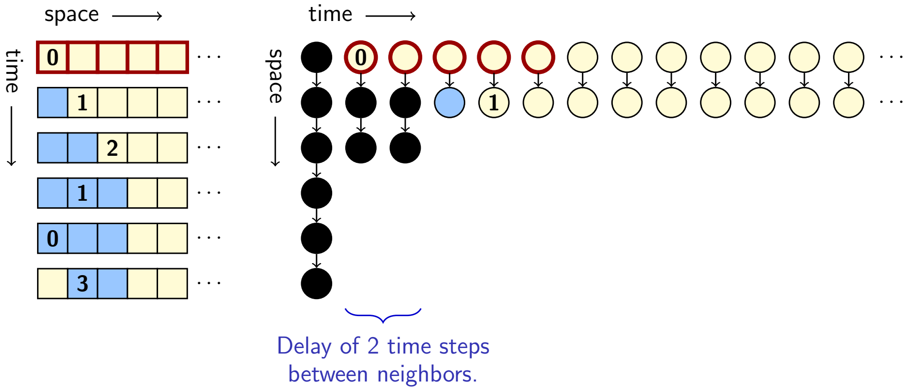
Exchanging space and time

Turing machine | **Distributed automaton**



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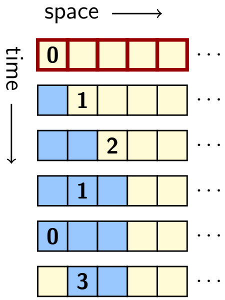
Exchanging space and time

Turing machine



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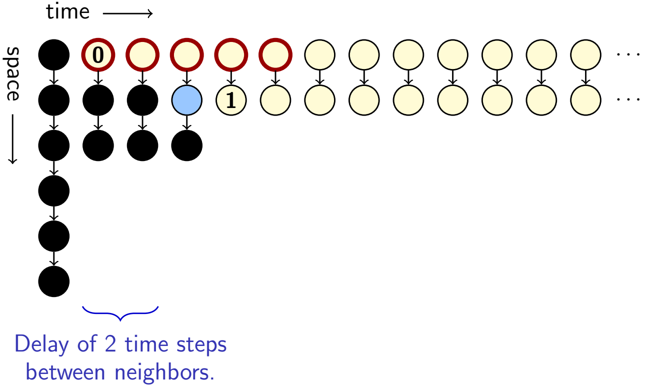

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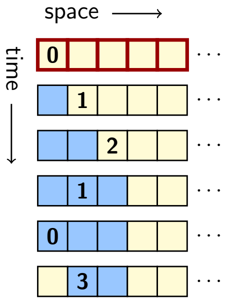
○ (1) ○ : nodes "visiting" a Turing cell

Exchanging space and time

Turing machine



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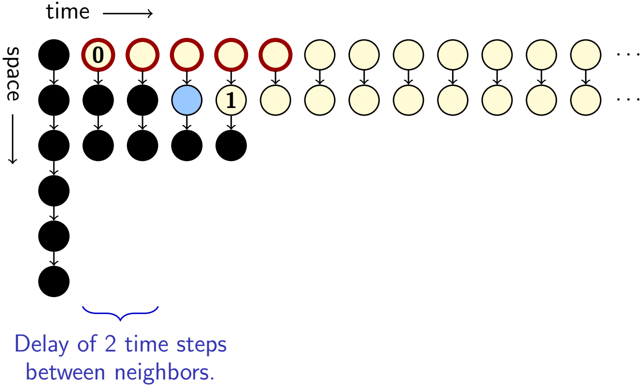

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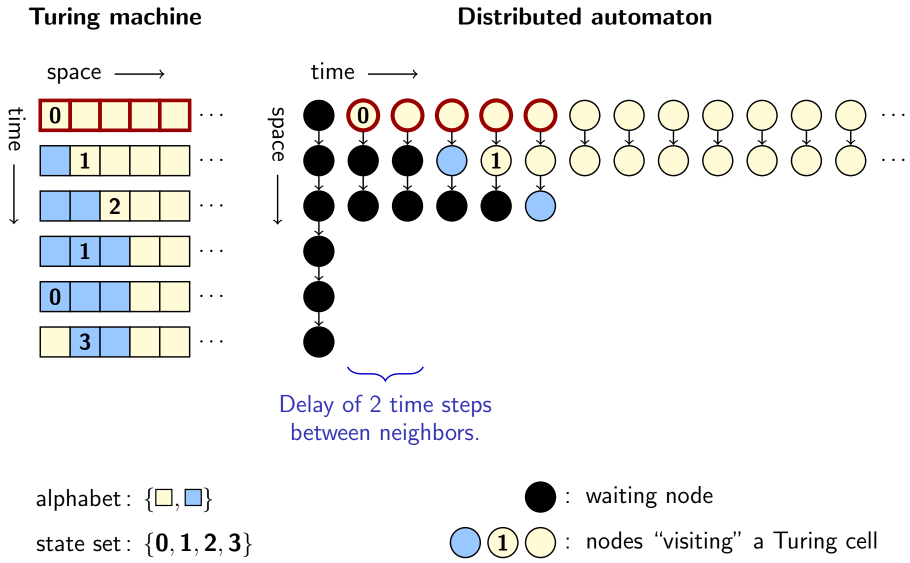

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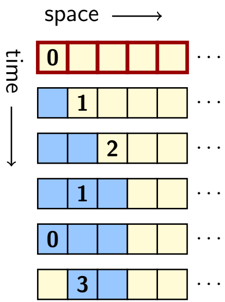

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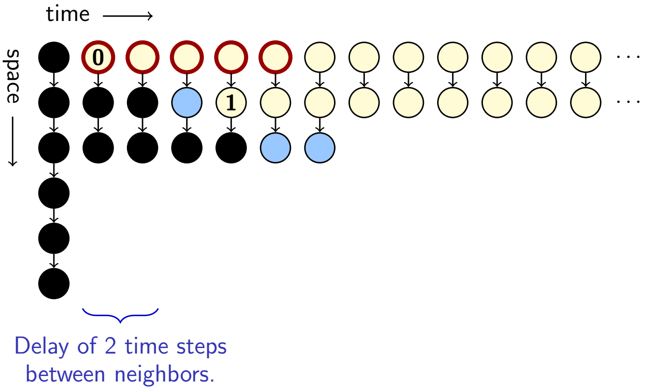

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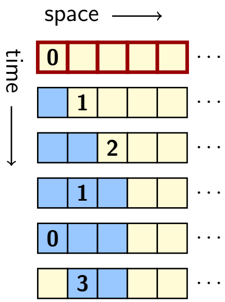
Exchanging space and time

Turing machine



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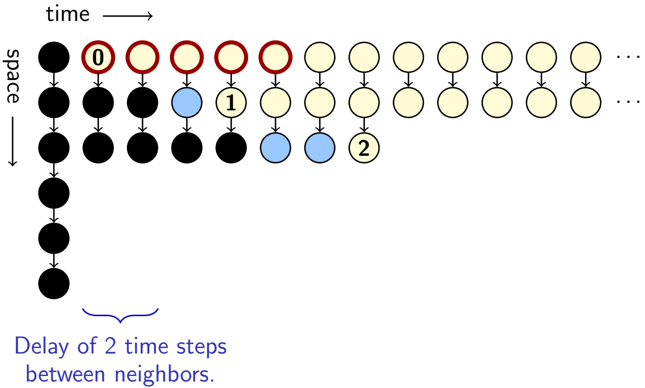

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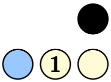

: waiting node

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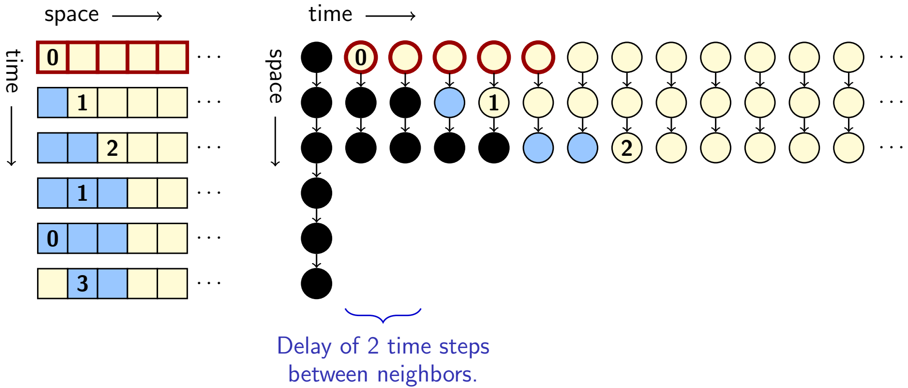
Exchanging space and time

Turing machine | **Distributed automaton**



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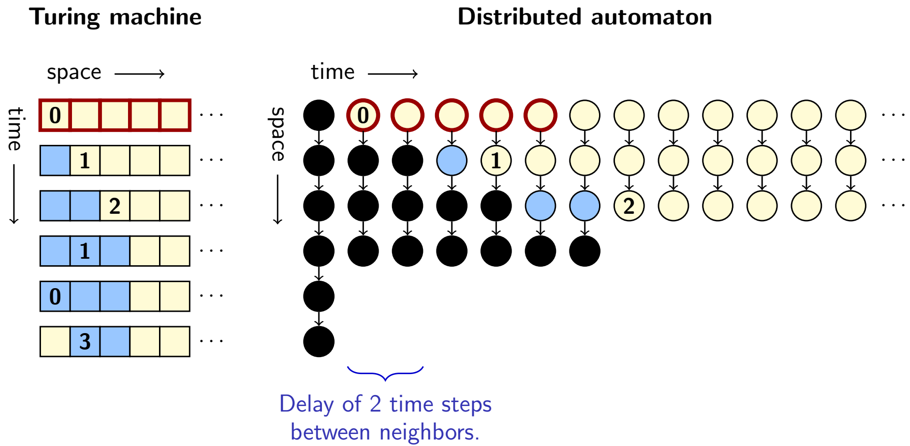

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Exchanging space and time

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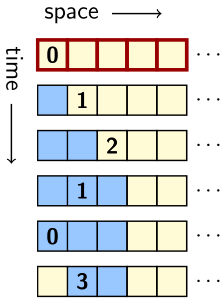

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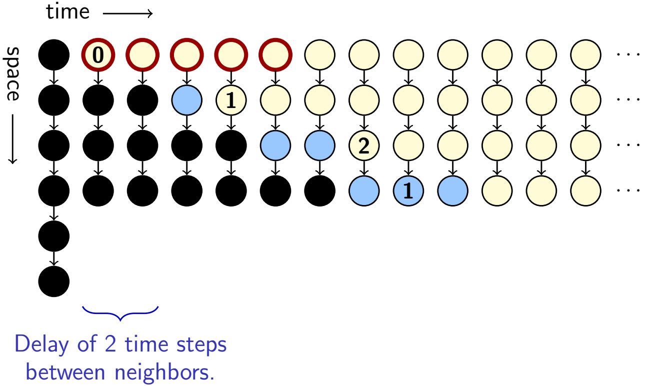

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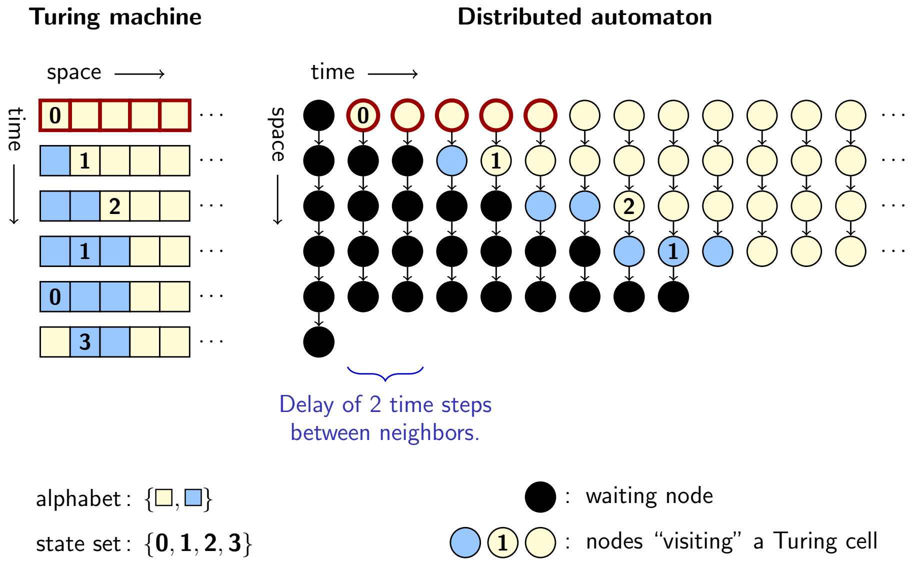
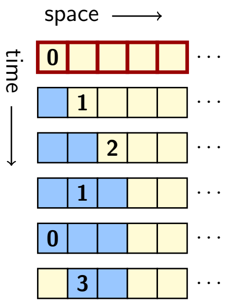


Exchanging space and time

Turing machine



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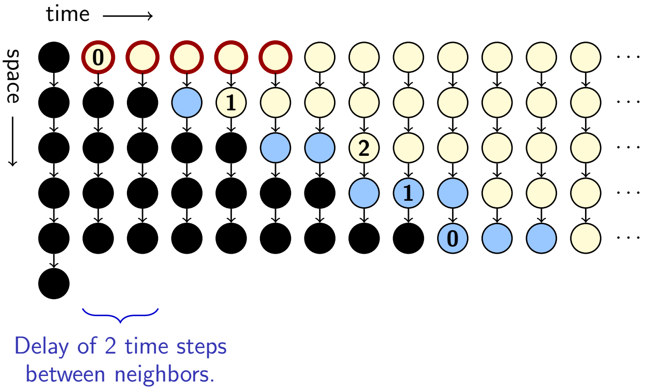

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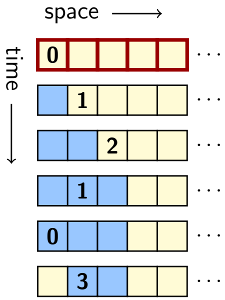

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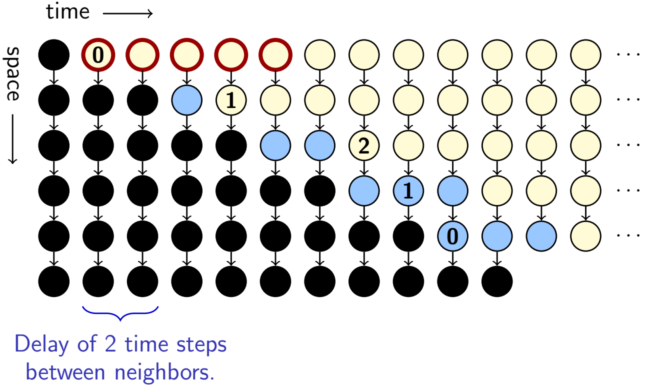

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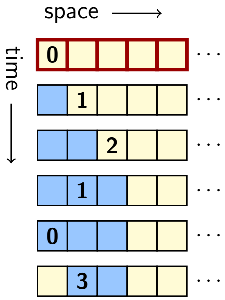
(blue) (1) (yellow) : nodes “visiting” a Turing cell

Exchanging space and time

Turing machine



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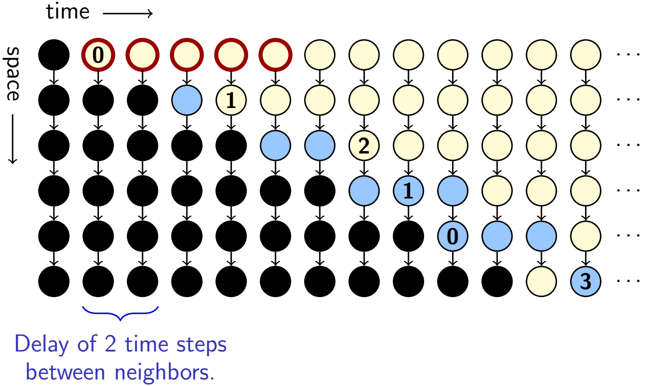

alphabet : {□, □}

state set : $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

: waiting node

: nodes “visiting” a Turing cell

Results

Results

Emptiness problem undecidable:

Results

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- In general.

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- For *quasi-acyclic* automata.

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Emptiness problem decidable in LOGSPACE:

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Emptiness problem decidable in LOGSPACE:

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Emptiness problem decidable in LOGSPACE:

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  ↖ Nodes cannot remember their own state.

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State diagrams acyclic except for self-loops.

Emptiness problem decidable in LOGSPACE:

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Nodes cannot remember their own state.

On words:	MSO logic	=	forgetful automata

Results

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On words:	MSO logic	$=$	forgetful automata
On trees:	———	$\subsetneq$	———

Results

Emptiness problem undecidable:

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  ↖ State diagrams acyclic except for self-loops.

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On words:	MSO logic	$=$	forgetful automata
On trees:	——	$\subsetneq$	——
On digraphs:	——	$\not\subseteq$ $\not\supseteq$	——

Thank you!