Reasoning About Sound Programs

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Joint work with O. Hermant & P. Jouvelot
MINES ParisTech, PSL Research University, France

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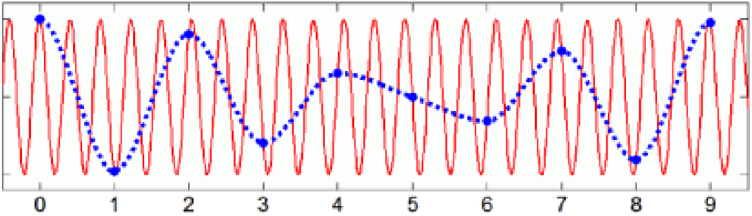
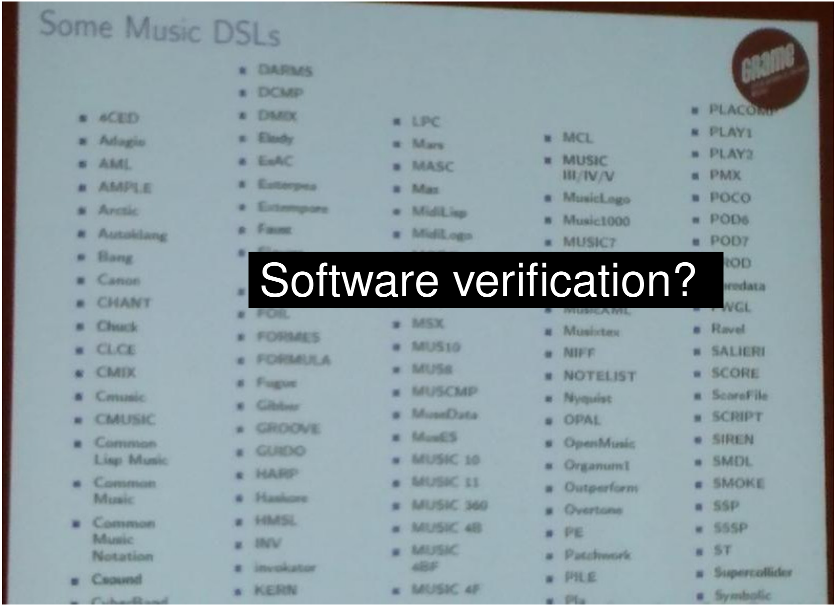


Some Music DSLs

- 4CED
- Adagio
- AML
- AMPLE
- Arctic
- Autoklang
- Bang
- Canon
- CHANT
- Chuck
- CLCE
- CMIX
- Cmusic
- CMUSIC
- Common Lisp Music
- Common Music
- Common Music Notation
- Csound
- CyberBand
- DARMS
- DCMP
- DMIX
- Elody
- EsAC
- Euterpea
- Extempore
- Faust
- Flavors Band
- Fluxus
- FOIL
- FORMES
- FORMULA
- Fugue
- Gibber
- GROOVE
- GUIDO
- HARP
- Haskore
- HMSL
- INV
- invokator
- KERN
- LPC
- Mars
- MASC
- Max
- MidiLisp
- MidiLogo
- MODE
- MOM
- Moxc
- MSX
- MUS10
- MUS8
- MUSCMP
- MuseData
- MusES
- MUSIC 10
- MUSIC 11
- MUSIC 360
- MUSIC 4B
- MUSIC 4BF
- MUSIC 4F
- MCL
- MUSIC III/IV/V
- MusicLogo
- Music1000
- MUSIC7
- Musictex
- MUSIGOL
- MusicXML
- Musixtex
- NIFF
- NOTELIST
- Nyquist
- OPAL
- OpenMusic
- Organum1
- Outperform
- Overtone
- PE
- Patchwork
- PILE
- Pla
- PLACOMP
- PLAY1
- PLAY2
- PMX
- POCO
- POD6
- POD7
- PROD
- Puredata
- PWGL
- Ravel
- SALIERI
- SCORE
- ScoreFile
- SCRIPT
- SIREN
- SMDL
- SMOKE
- SSP
- SSSP
- ST
- Supercollider
- Symbolic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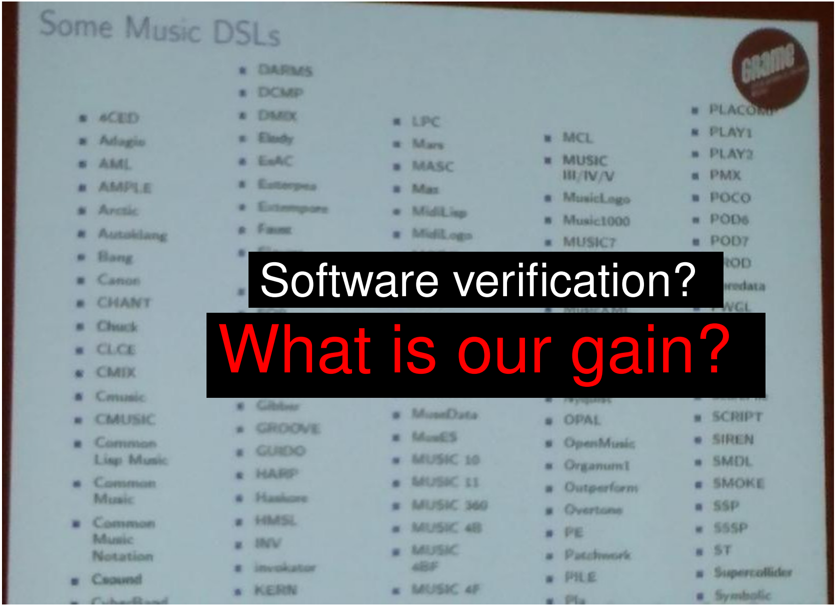
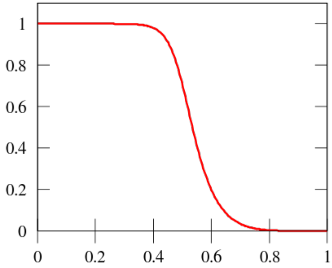


Software verification?

What is our gain?

Let's assume a simple IIR filter:

$$\text{smooth}_n = (1 - c) \cdot x_n + c \cdot \text{smooth}_{n-1}$$



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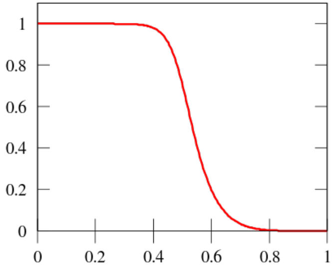

What would we like to know about it?

$$\text{smooth}_n = (1-c)\cdot x_n + c\cdot \text{smooth}_{n-1}$$

Natural questions are:

- Frequency response;
- Stability;
- Linearity/Time Invariance.

<u>Standard DSP theory gives answers.</u>

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We dive into the realm of PL theory!

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Paradigm shift!

Faust

- Functional PL for digital signal processing.
- Synchronous paradigm, geared towards audio.
- Programs: circuits/block diagrams with feedback.
- Semantics: streams of samples.
- *Efficiency is crucial*.
- Created in 2000 by Yann Orlarey et al. at GRAME.
- Mature, compiles to more than 14 platforms.

Faust's Ecosystem

Users:

- Grame: Multiple projects, main developer.
- Stanford: Class/books on signal processing, STK instrument toolkit, Faust2android, Mephisto. . .
- Ircam: Acoustic libraries, effects libraries,. . .
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Recent Events:

- Faust day at Stanford, LAC 2015.
- Faust program competition (€2,000).
- FEEVER project :)

Syntax and Well-Formedness

$$\textsc{Term}\ \frac{}{\vdash ! : 1 \to 0} \qquad \textsc{Id}\ \frac{}{\vdash \_ : 1 \to 1}$$

$$\textsc{Par}\ \frac{\vdash f_1 : i_1 \to o_1 \qquad \cdots \qquad \vdash f_n : i_n \to o_n}{\vdash (f_1, \ldots, f_n) : \sum_j^n i_j \to \sum_j^n o_j}$$

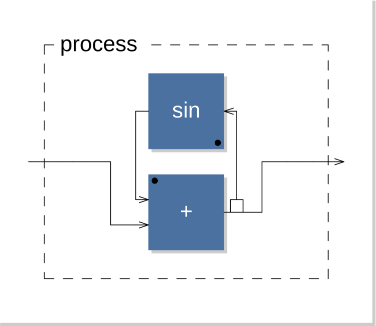
$$\textsc{Comp}\ \frac{\vdash f : i \to k \qquad \vdash g : k \to o}{\vdash (f : g) : i \to o}$$

$$\textsc{Pan}\ \frac{\vdash f : i \to k \qquad \vdash g : k * n \to o \qquad 0 < k \wedge 0 < n}{\vdash f <: g : i \to o}$$

Feedback

$$\text{FEED}\ \frac{\vdash f : o_g + i_f \rightarrow i_g + o_f \qquad \vdash g : i_g \rightarrow o_g}{\vdash f \sim g : i_f \rightarrow i_g + o_f}$$

Diagram for `+ ~ sin`:

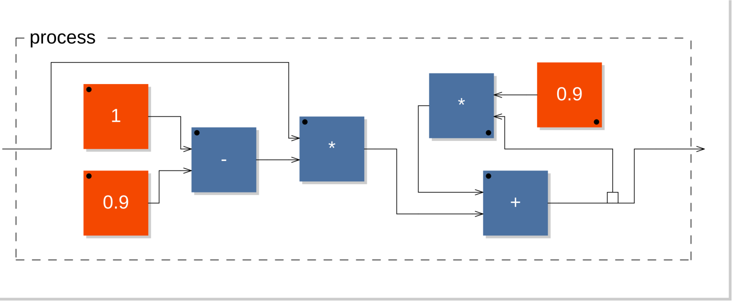


Back to the Filter

$$\text{smooth}_n = (1 - c)x_n + c \cdot \text{smooth}_{n-1}$$

Using Faust:

```
smooth(c) = *(1-c) : + ~ *(c)
```



[For c = 0.9]

Feedback Delay Network

```
fdnrev(N, dp, freqs, durs, loopgainmax)
  = delaylines ~ (delayfilters : feedbackmatrix)
where
  delaylines     = rep(N,i,delay(dp[i])));
  delayfilters   = rep(N,filter(freqs,durs));
  feedbackmatrix = bhadamard(N);
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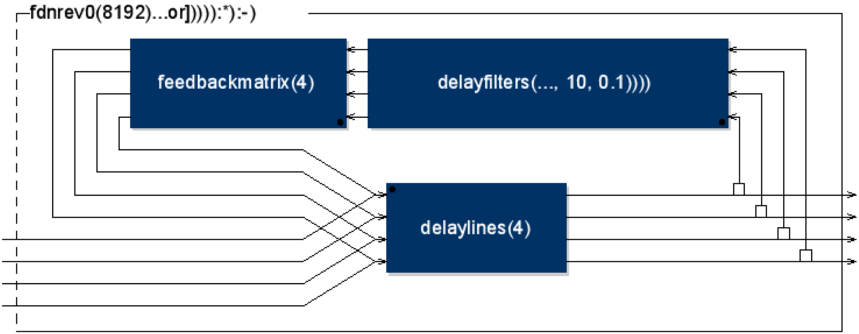

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  See next week's talk!
- Functional Reactive Programming/Arrows.
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Some related DSL: VOBLA, Ziria, Halide, Darkroom, Julia.

DSP & Faust

- Real-time Linear Processing.
- Real-time Non-linear Processing.
- Frequency Domain Processing.
- Non-necessarily causal.
- Filters, Feedback Networks, Interpolation.
- Windowing!
- Numerical issues.
- Nyquist/precision/aliasing.

Verification in DSP/Faust

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Problems with Audio:

bad sound, stability/glitches, under/overflows, time, safety/security, remote distribution.

We need more!

A Case Study: Stability

Test-bed: use Coq

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Smooth is stable when $c \in]0, 1[$. Formally:

$$\forall i \in [a, b],\ c \in]0, 1[\rightarrow \mathit{smooth}(c)\ i \in [a, b]$$

Let's build a mechanized constructive proof.

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4. Define a logic to simplify reasoning.
5. Verify!

Mechanized Semantics for Streams

- Coinductive semantics [Boulmé, et al]: problematic.
- Didn't consider PACO, etc. . . .
- Our wish: Sequences $\mathbb{S}$ of a base type **R** [Auger2013]

Mechanized Semantics for Streams

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Soundness needs stronger semantics (also [Guatto2014]):

$$[\![\vdash f : i \to o]\!]^n : [\![\underbrace{\mathbf{R} \times \ldots \times \mathbf{R}}_{i}]\!]^n \to [\![\underbrace{\mathbf{R} \times \ldots \times \mathbf{R}}_{o}]\!]^n$$

Index by number of steps; equality of streams more intensional wrt to $(\mathbb{N} \to \mathbf{R})$.

The Second Piece: Real Analysis

What about the base type **R**?

- Reals not in Mathcomp – algebraic structures good enough for most of our experiments so far.
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Not practical.

To remedy this, we define a *program logic* for sample-level properties.

Sampled-Level Predicates

Definition (Sample-Level Property)

A property $P : \mathbb{S} \to \mathbb{B}$ is sample-level if there exists a characteristic predicate $\varphi : \mathbf{R} \to \mathbb{B}$ such that for all streams s:

$$P(s) \iff \forall n. \varphi(s[n])$$

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Boundedness $x \in [a, b]$ is a sample-level property!
Properties can be made sample-level by self-composition, e.g: ratio:

$$f \Rightarrow \langle f, f' \rangle : /$$

We can also prove this way equivalence of filter implementation.

A Sampled Logic

Definition (Sampled Judgment)

Given two characteristic predicates φ, ψ, we write

$$\{\varphi\}\ f\ \{\psi\}$$

"for all input i meeting φ, the fi satisfies ψ."

Example

The stability judgment for smooth is written as:

$$\{x \in [a, b]\}\ \text{smooth}\ \{x \in [a, b]\}$$

Rules for The Sampled Logic

$$\frac{\forall i_1, i_2, (\varphi_1(i_1) \wedge \varphi_1(i_2)) \implies \psi(i_1 + i_2)}{\{\varphi_1, \varphi_2\} + \{\psi\}}\,\textit{Prim}$$

$$\frac{\{\varphi\}\ f\ \{\theta\} \qquad \{\theta\}\ g\ \{\psi\}}{\{\varphi\}\ f : g\ \{\psi\}}\,\textit{Comp}$$

$$\frac{\models \psi(x_0) \qquad \{\theta, \varphi\}\ f\ \{\psi\} \qquad \{\psi\}\ g\ \{\theta\}}{\{\varphi\}\ f \sim g\ \{\psi\}}\,\textit{Feed}$$

Soundness of the Logic

Definition (Validity)

$$[\![\{\varphi\}\ f\ \{\psi\}]\!] \equiv \forall i.(\forall t.\varphi(i(t))) \implies (\forall t, \psi([\![f]\!])(i)(t))$$

Theorem (Soundness)

For any program f of type $i \rightsquigarrow o$, if

$$\{\varphi_1, \ldots, \varphi_i\}\ f\ \{\psi_1, \ldots, \psi_o\}$$

is derivable then,

$$[\![\{\varphi_1, \ldots, \varphi_i\}\ f\ \{\psi_1, \ldots, \psi_o\}]\!]$$

is valid.

Stability Proof for Smooth

$$\dfrac{\dfrac{\Box}{\{I_{ab}\}\ *(1-c)\ \{I_{ab\bar{c}}\}} \qquad \dfrac{\dfrac{\Box}{\{I_{abc}, I_{ab\bar{c}}\} + \{I_{ab}\}} \qquad \dfrac{\Box}{\{I_{ab}\}\ *(c)\ \{I_{abc}\}}}{\{I_{ab\bar{c}}\} + \sim *(c)\ \{I_{ab}\}}}{\{i \in [a,b]\}\ *(1-c) : + \sim *(c)\ \{o \in [a,b]\}}$$

with:

$$\begin{aligned}
I_{ab}(x) &\equiv x \in [a, b] \\
I_{abc}(x) &\equiv x \in [a * c, b * c] \\
I_{ab\bar{c}}(x) &\equiv x \in [a * (1-c), b * (1-c)]
\end{aligned}$$

Stability of Smooth

Three main VC in the proof:

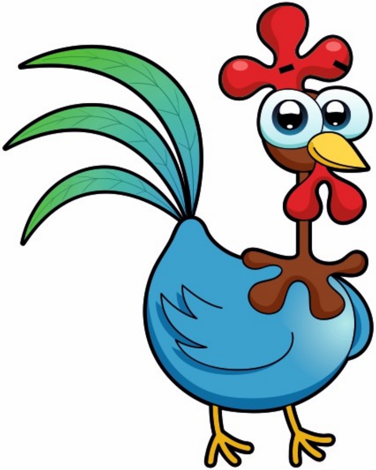
```
(* (1 - c) * i \in [(1 - c) * a, (1 - c) * b] *)
by rewrite ?ler_wpmul2r ?ler_subr_addr ?add0r.

have Ha: a = a * c + a * (1 - c)
  by rewrite -mulrDr addrC addrNK mulr1.
have Hb: b = b * c + b * (1 - c)
  by rewrite -mulrDr addrC addrNK mulr1.
by rewrite Ha Hb !ler_add.

(* c * i \in [c * a, c * b] *)
by rewrite ?ler_wpmul2r.
```

We pushed the VCs to Why3 with success.
Interval technique ready to go into the main compiler.

Stability Proof



One Step Beyond

Extending the logic

Allow predicates to refer to windows.

$$\varphi(i) \equiv \{i/i_{\square} = 0.8\}$$

where $i_{\square}$ is the sample produced in the execution step.

Linear System Theory

Consider the following subset of Faust:

$*(c)$	scaling by *c*
$+$	addition
$:$	composition
$\sim$	addition

Then every Faust program is LTI. Very related to [Bonchi et al. 2015]
A consequence of that is that every program can be viewed as a polynomial.

Two Poles IIR Filter

```
twopole = fir : + ~ feedback
where
  fir(x)      = (x - x'') * g * (1-RR) / 2;
  feedback(v) = 2*R*cos(T) * v - RR * v';
  ....
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  ....
```

Get and verify its transfer function:

$$H(z) = \frac{1 - z^{-2}}{1 - 2R\cos(\Theta_c)z^{-1} + R^2 z^{-2}}$$

Ongoing: Frequency Domain Analysis

Recall the Fourier Matrix:

$$W = 1/\sqrt{N}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

or:

$$W = \left(\frac{\omega^{jk}}{\sqrt{N}}\right)_{j,k=0,\dots,(N-1)}$$

where ω the nth-root of the unity. Then the DFT can be expressed as:

$$X = Wx$$

Fourier Properties Formally

Linearity, shifting and scaling follow from lemmas already in the MathComp linear algebra library!
Parseval's theorem is work in progress:

$$\sum_{n=0}^{N-1} |x_n|^2 = \sum_{n=0}^{N-1} |X_n|^2$$

Transfer Functions

- We can use a similar approach for the certification of transfer functions.
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- C.f: Algebraic Signal Processing [Puesel, Moura]

Paper with our adventures coming end of month.

Conclusions

- It was an interesting exercise; we learned a lot!
- The full Faust language is basically done.
- So far verification has been about math verification.
- Floating point issues ignored...
- Help from audio people. What are important things to certify?
- Non-Linear systems.
- We are investigating a different approaches to certification beyond program logics.
- Verified FFT/DSP computation. Trying CoqEAL.
- Improving the language for spectral processing.
- Non-linear Wave Filter, Scattered Delays Networks.