SSReflect in Coq 8.10 New intro patterns and support for rewriting under binders

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3 Tactic to rewrite under binders





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SSReflect in a nutshell

SSR is a *proof language* (a bit more than a list of tactics)

- Way past break-in period: 4C Thm, Odd Order Thm, ...
- Backward compatible (e.g. MathComp 1.9 works on Coq 8.7 ightarrow 8.10)
- Integrated in Coq since version 8.7 (Require Import ssreflect.)
- Enables SSR formalization style, but does not force it



Small Scale Reflection formalization style

The name: reflecting decideable propositions to bool...But it is more than that, too much for one slide.

Focus: easy to repair scripts = scripts that break early and locally

• basic bricks are dumb, predictable and do fail

• explicit naming of context items (bookkeeping discipline) Example:

• rewrite [in RHS]leq_ab vs. rewrite {35}H16

In this talk we focus on intro patterns and rewriting



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Intro patterns by examples: working the goal stack

```
Lemma test : \forall a b, a \leq b \Rightarrow G. Proof. move \Rightarrow a? leq_ab.
 a, b : nat
 leq ab : a <= b
 G
Lemma test : \forall a b, a \leq b \Rightarrow G. Proof. move \Rightarrow a [|b] leq_ab.
 a : nat
                    a, b : nat
 leq_ab : a <= 0 leq_ab : a <= b.+1</pre>
                       ------
_____
 G
                       G
Lemma test : \forall a b, a \leq b \Rightarrow G. Proof. move \Rightarrow a b / leqW; move: a b.
_____
 ∀a b, a <= b.+1 → G
```

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Block introduction: case

Destructuring an inductive type using standard names.

```
Inductive i :=
    | K1 (a : T)
    | K2 (_ : U) (b : T). (* these names are kept by Coq *)
Lemma test (x : i) : G.
Proof.
case: x \Rightarrow [^ y_ ].
    _____?__ : U
    y_a : T    y_b : T
    _____
G     G
```

Names are predictable (derived by simple concatenation) and unique (you choose a prefix/suffix that must not generate clashes).



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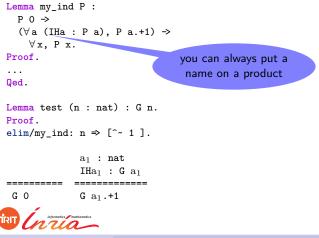
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Block introduction: elim

Destructuring also happens as a result of an induction.



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Fast and temporary introduction

Skip to the fist assumption with the > intro pattern

```
Lemma test : ∀a b, a <= b → G.

Proof.

move⇒ >leq_ab

_a_, _b_ : nat

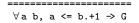
leq_ab : _a_ <= _b_

______

G
```

Introduce now and revert at the end of the intro pattern

```
Lemma test: \forall a b, a \le b \Rightarrow G.
move\Rightarrow + + / leqW.
```





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When the developer replies DIY...

```
Notation "'dupP'" := ltac:(code to duplicate an hypothesis) : ssripat_scope.
```

```
Lemma test x : x = 3 \Rightarrow G x.
move\Rightarrow /dupP def_x \Rightarrow.
```

```
x : nat
def_x : x = 3
G 3
```

Ltac views

Bonus: dupP could take arguments!



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Big operators in a nutshell

• Formalization of $\sum_{\substack{i \in A \\ P(i)}} F(i)$, $\prod_{\substack{i \in A \\ P(i)}} F(i)$, $\bigcap_{\substack{i \in A \\ P(i)}} F(i)$, $\bigcup_{\substack{i \in A \\ P(i)}} F(i)$, $\max_{\substack{i \in A \\ P(i)}} F(i)$...

 $\bullet\,$ Implem: higher-order iterator applied to some lambda for P and F



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Big operators in a nutshell

• Formalization of
$$\sum_{\substack{i \in A \\ P(i)}} F(i)$$
, $\prod_{\substack{i \in A \\ P(i)}} F(i)$, $\bigcap_{\substack{i \in A \\ P(i)}} F(i)$, $\bigcup_{\substack{i \in A \\ P(i)}} F(i)$, $\max_{\substack{i \in A \\ P(i)}} F(i)$...

 $\bullet\,$ Implem: higher-order iterator applied to some lambda for P and F

Example

 $\sum_{\substack{i=1\\i \text{ odd}}}^{4} i^2 \text{ can be formally written as: } sum_(1 \le i \le 5 | \text{ odd i}) i^2,$ that is to say: $big[addn/0]_(1 \le i \le 5 | \text{ odd i}) i^2,$ which expands to: $bigop _ 0 \text{ (index_iota 1 5) (fun i : nat } BigBody _ i addn (odd i) (i ^ 2))$



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Higher-order iterators? Need for rewriting under binders...

From mathcomp Require Import bigop. ---- provides congruence lemmas to be applied by hand

 $\begin{array}{l} \mathsf{eq_big}: (* \text{ main congruence lemma for bigops }*) \\ \forall (\texttt{R}: \texttt{Type}) (\texttt{idx}:\texttt{R}) (\texttt{op}:\texttt{R} \rightarrow \texttt{R} \rightarrow \texttt{R}) (\texttt{I}:\texttt{Type}) (\texttt{r}:\texttt{seq I}), \\ \forall (\texttt{P1 P2}:\texttt{pred I}) (\texttt{F1 F2}:\texttt{I} \rightarrow \texttt{R}), \\ (\forall\texttt{i}:\texttt{I},\texttt{P1 i} = \texttt{P2 i}) \rightarrow (\forall\texttt{i}:\texttt{I},\texttt{P1 i} \rightarrow \texttt{F1 i} = \texttt{F2 i}) \rightarrow \\ & \texttt{big[op/idx]_(i <-r \mid \texttt{P1 i}) \texttt{F1 i} = \texttt{big[op/idx]_(i <-r \mid \texttt{P2 i}) \texttt{F2 i}.} \end{array}$



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Higher-order iterators? Need for rewriting under binders...

From mathcomp Require Import bigop. ---- provides congruence lemmas to be applied by hand

eq_big : (* main congruence lemma for bigops *) \forall (R : Type) (idx : R) (op : R \rightarrow R \rightarrow R) (I : Type) (r : seq I), \forall (P1 P2 : pred I) (F1 F2 : I \rightarrow R), (\forall i : I, P1 i = P2 i) \rightarrow (\forall i : I, P1 i \rightarrow F1 i = F2 i) \rightarrow $\big[op/idx]_(i <-r \mid P1 i)$ F1 i = $\big[op/idx]_(i <-r \mid P2 i)$ F2 i.

Running example

rewrite subnn. (* Error: The LHS of subnn, (_ - _), does not match any subterm of the goal *)
rewrite eq_big. (* Error: Unable to find an instance for the variables P2, F2. *)

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Higher-order iterators? Need for rewriting under binders...

From mathcomp Require Import bigop. ---- provides congruence lemmas to be applied by hand

```
eq_big : (* main congruence lemma for bigops *)

\forall (R : Type) (idx : R) (op : R \rightarrow R \rightarrow R) (I : Type) (r : seq I),

\forall (P1 P2 : pred I) (F1 F2 : I \rightarrow R),

(\foralli : I, P1 i = P2 i) \rightarrow (\foralli : I, P1 i \rightarrow F1 i = F2 i) \rightarrow

\big[op/idx]_(i <-r \mid P1 i) F1 i = \big[op/idx]_(i <-r \mid P2 i) F2 i.
```

Running example

Tactic to rewrite under binders

The under tactic - I

One-liner (a.k.a. batch) mode

n : nat

 $\sum_{0 \le k \le n} dk \ (k != 1)) \ (k - k) = 0$

under eq_big do [| rewrite subnn].



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The under tactic - I



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The under tactic - II

Interactive mode (without do clause)

under eq_big \Rightarrow [i | i /andP[i_odd i_neq1]].



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The under tactic - II

Interactive mode (wi	thout do clause)	
n : nat		
under eq_big ⇒[i i /andP[i_odd i_neq1]].		
n, i : nat	n, i : nat i_odd : odd i i_neq1 : i != 1 n : nat	
'Under[odd i && (i != 1	L)] 'Under[i - i] \sum_(0 <= i < n ?P2 i) ?F2 i = 0	



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The under tactic - II

Interactive mode (without do clause)		
n : nat		
under eq_big ⇒[i i /andP[i_odd i_neq1]].		
n i . nat	n, i : nat i_odd : odd i	
n, i : nat	i_neq1 : i != 1 n : nat	
'Under[odd i && (i != 1)]	'Under[i - i] \sum_(0 <= i < n ?P2 i) ?F2 i = 0	
\downarrow	\downarrow	
over.	rewrite subnn. over.	



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The under tactic - III

• Batch mode: can be viewed as a shortcut for interactive mode + dispatch:

```
under eq_big ⇒[i_1 | i_2] do [tac1 | tac2].

≡
(under eq_big)⇒[i_1 | i_2 | ]; [tac1; over | tac2; over | ].
```

see also https://github.com/math-comp/math-comp/blob/master/CONTRIBUTING.md#proof-style



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The under tactic - III

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- Some even shorter syntax is available (with automatic introduction):

```
under eq_big do [ | rewrite subnn].
    is the defective form for:
under eq_big ⇒[* | *] do [ | rewrite subnn].
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The under tactic - III

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- Some even shorter syntax is available (with automatic introduction):

```
under eq_big do [ | rewrite subnn].
    is the defective form for:
under eq_big ⇒[* | *] do [ | rewrite subnn].
```

- Interactive mode: useful to debug/repair a broken proof script
- Choice between batch & interactive versions? mostly a matter of style¹

see also https://github.com/math-comp/math-comp/blob/master/CONTRIBUTING.md#proof-style



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The under tactic - IV

• The tactic also supports occurrences switches and contextual patterns, which are both optional:

under {2}[in RHS]eq_lem.



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The under tactic - IV

• The tactic also supports occurrences switches and contextual patterns, which are both optional:

under {2}[in RHS]eq_lem.

• Intro patterns are optional, but recommended:

```
under eq_big ⇒ [i|i ?].
under eq_bigl ⇒ i.
```



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The under tactic - IV

• The tactic also supports occurrences switches and contextual patterns, which are both optional:

under {2}[in RHS]eq_lem.

• Intro patterns are optional, but recommended:

```
under eq_big \Rightarrow [i|i ?].
under eq_bigl \Rightarrow i.
```

(notably as under attempts to preserve the name of bound variables from the first branch, as we'll see in the demo)



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The under tactic - V

Design decisions

- Implemented in OCaml to avoid Ltac1 limitations^a
- Give a protected context 'Under[_] for evars
- Name all bound variables
- Compatibility with SSReflect's intro patterns
- Compatibility with precedence level of tacticals ";" and "do"



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^aa prototype was first coded in Ltac [mid-2016]: github.com/erikmd/ssr-under-tac

What about setoid_rewrite?

setoid_rewrite

- + automatic way to rewrite a bunch of occurrences
- not precise enough: doesn't allow to specify contextual patterns for the desired rewrite

under

- + more flexibility (one can choose the congruence lemma to follow and precisely select the redex to rewrite), can be nested
- ++ ability to perform conditional rewrites
 - + compatible with registered Setoid equalities [\rightsquigarrow Coq 8.11]

[Demo]



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Concluding remarks & perspectives

SSReflect

- In Coq since version 8.7, documented [1, 2], stable proof language
- Since Coq 8.10:
 - Fast, temporary, block, DIY intro patterns: [ssr] extended intro patterns https://github.com/coq/coq/pull/6705
 - Rewriting under binders: [ssr] Add tactics under and over https://github.com/coq/coq/pull/9651
- In the pipeline for Coq 8.11:
 - Make under support equivalence relations other than "=": [ssr] Generalize tactics under and over to any Setoid relation https://github.com/coq/coq/pull/10022



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References

[1] The Coq Development Team.

The Coq Proof Assistant, version 8.10.0, August 2019. URL: https://coq.inria.fr/distrib/current/refman/ proof-engine/ssreflect-proof-language.html.

 [2] Assia Mahboubi and Enrico Tassi. Mathematical Components. draft, v1-183-gb37ad7, 2018. URL: https://math-comp.github.io/mcb.

