# THE USE OF FISHEYE-LENSES IN ClOSE-RANGE PHOTOGRAMMETRY

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## Abstract

Fisheye-lenses are optical systems with extremly wide angles, which do not have a central-perspective projection.

The photogrammetric use of Fisheye-lenses has been made possible through the use of computer aided methods. However, the projection equations have to be known.

The problem arsing when using Fisheye-projection are illustrated by means of a lens Distagon 30/Zeiss in combination with a Rolleiflex-camera ($6x6cm^2$).

A special calibration method and newly developed distortion functions are described. The object coordinates have been calculated by analytical methods.

Using examples it is shown, how the number of pictures or rather models can be reduced in contrast to conventional cameras. The accuracy is discussed here.

## Zusammenfassung

Fisheye-Objektive sind nicht-zentralperspektiv abbildende optische Systeme mit extremen Öffnungswinkeln (180 Grad). Ihre photogrammetrische Verwertbarkeit ist infolge rechnerunterstützter Auswerteverfahren sinnvoll geworden. Voraussetzung ist jedoch die Kenntnis des Abbildungsvorganges.

Das Problem der Fisheye-Abbildung wird an Hand eines Zeiss Distagon 30 Objektives in Verbindung mit einer Rolleiflex Mittelformatkamera ($6x6cm^2$) aufgezeichnet.

Ein spezielles Kalibrierverfahren wird angewendet. Für die Beschreibung der Verzeichnung sind entsprechende Funktionen entwickelt worden. Die Objektkoordinatenbestimmung erfolgt nach analytischen Verfahren.

An Hand von Beispielen wird die Bild- bzw. Modellreduzierung gegenüber herkömmlichen Kameras aufgezeigt und die erreichbare Genauigkeit dargestellt.

## 1. Introduction

The photogrammetric measurements of the interior of buildings, vaults etc. often poses a problem. Short distances between the camera and the object require an increased number of pictures or rather models. The photogrammetric method can thus be ineffective.

This problem can be overcome extensively by using fisheye-lenses: The extreme angle of view in fisheye-objectives usually covers the whole object (see fig. 5 and 7). However, in contrast to the advantage gained through the use of fisheye-lenses, there is a loss of the central-perspective projection. Projection equations have to be determined as a result of this. A suitable method for assessing the calibration above all for the outer edges of the photograph has yet to be found.

The use of fisheye-lenses in photogrammetry is only worthwhile when used in combination with analytical methods. Analogue projection can not be considered due to the extremely distorted values in the photographs.

## 2. Camera and Objective

All the investigations were carried out using a Zeiss fisheye-objective Distagon 30 in combination with a Rolleiflex-camera SL 66 (see fig. 1). The Rolleiflex-camera produces pictures with a format of 6x6cm². It is possible to use rollfilm as well as flat film.

The relative aperture of the lens Distagon 30 is 1 : 3.5, the maximum angle of view is 180° (diagonally).

The body of the Rolleiflex-camera combined with the fisheye-objective was attached to a fixed point with two clamps and an aluminium groundplate. This assembled camera is called 'FE - 66'.

The frame is secured connected to the camera in a similar way. The frame defines the photo coordinate system. There are moreover small areas along the edges of the frame which are marked by notches (ca. 1 mm). In these areas the measuring mark has to be placed in each case in one direction only, when making a photogrammetrical measurement of the picture. The corners and the centre of the picture result from the intersection of two lines in the frame. The centre of the picture can then be reproduced to within +- 4 mm).

## 3. Projection and Distortion

In order to illustrate minimum and maximum distortions with the 'ellipse of distortion', the image plane and the object plane must be arranged parallel to one another. If infinitesimal circles, arranged radially and äquidistantly to the centre of the picture, are observed on the image plane then there is a distorted relationship as illustrated in figure 2. The axis h and k of the ellipse of distortion become

$$h = \cos (T/2) \cdot \cos^2 T$$

$$k = 2 \cdot \sin (T/2) / \tan T$$

$$T \ldots \text{semi angle of view}$$

If the central perspective projection is $h = k = 1$ , then the radial äquidistance is retained.

Figure 2 shows this extreme distortion in a radial direction. Hence this is an unfavourable means of taking photographs of flat objects which lie parallel to the image plane and which require an extreme angle of view. The fisheye-objectiv therefore provides a suitable method for photographing objects, which curve inwards (see fig. 5).

One must detect the distortions and by using functions formulate them, namely by reproducing the photographed bundle of rays, if one is to employ by FE-66 for photogrammetric tasks. The distortion were determinded in a special test-field during the calibration. The spatial testfield at the University of Göttingen has a high density of points. Extra invar levelling rods were installed in the room for diagonals with extreme angle of view. The principal point of the photograph was determind separately from the other interior orientation elements.
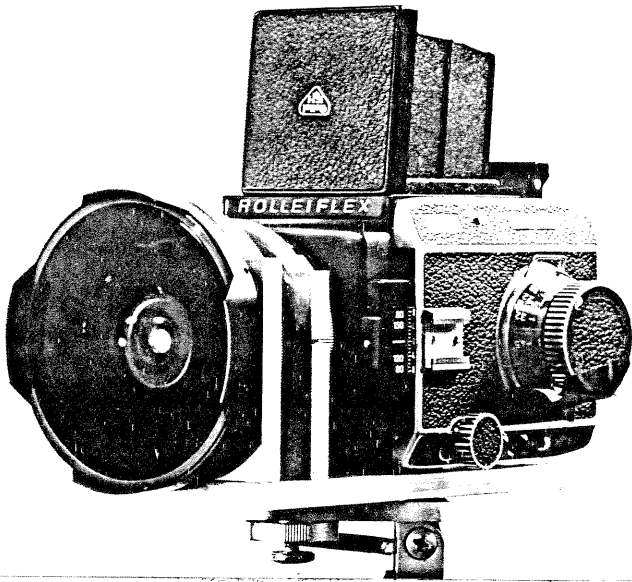


Fig. 1    Rolleiflex-camera (6x6cm²)
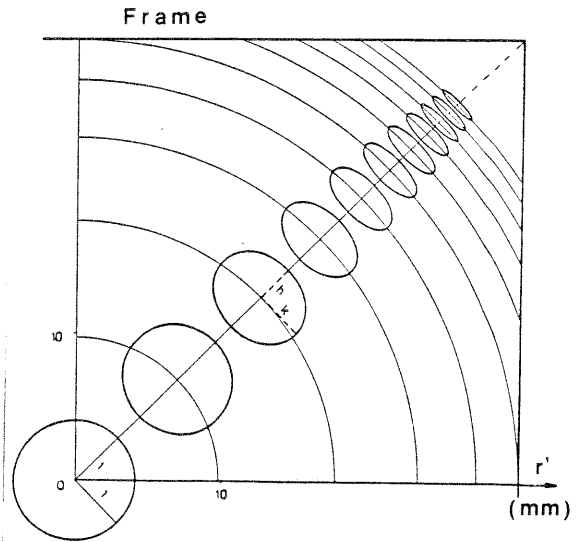with Fisheye-objective
Distagon 30



Fig. 2    Indikatrix Distagon 30

The distortions, with a constant calibrated focal length, can be described using three different functions. Better results can be obtained by dividing the functions into three groups. The curve of distortion with the area of interpolation is illustrated in figure 3. Distortion in the lower region was determined through use of fourth degree polynomial equations in $R^2$ (function A).A mathematical equation follows using the so-called 'polefunction' (function B) between r' = 17 mm and 37 mm. A main characteristic of this polefunction is that it is expected that the distortion presumably show an asymptotic process with increasing radial distance r'.
When $\lim_{\Delta r \to \infty} r' = P$ (Pole), the first part of the function in function B then becomes

$$F_1 = \frac{R_i^{(2n+1)}}{P^2 - R_i^2}$$

($R_i$ ... distance of radius to point of symmetry )

since           $R_i \longrightarrow P$

$\Longrightarrow$   $P^2 - R_i^2 \longrightarrow 0$

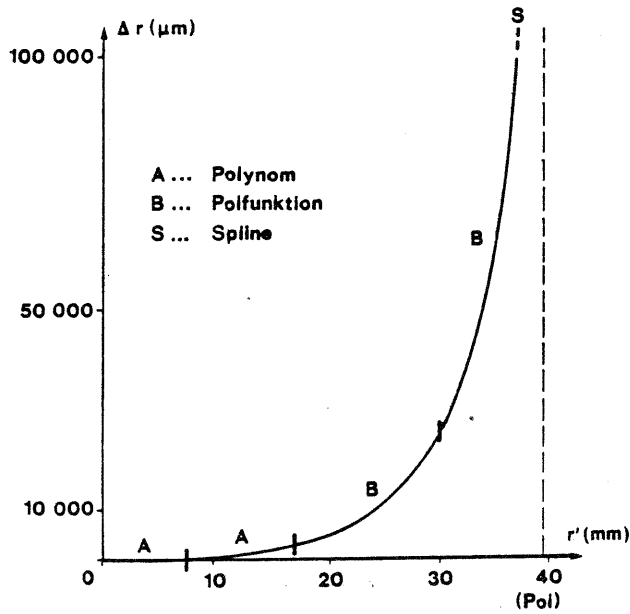$\Longrightarrow$   $r \longrightarrow \infty$

Fig. 3

Distortion Fisheye lense
Zeiss Distagon 30
Areas of Interpolation

Cubic splines are used for the extreme outer regions (function S, /2/). The components $\Delta x, \Delta y$ for the functions A and B become

## Funktion A

$$x_i' = xS_i \cdot \sum_{j=0}^{n} a_j \cdot R_i^{(2j-1)}$$

$$y_i' = yS_i \cdot \sum_{j=0}^{n} a_j \cdot R_j^{(2j-1)}$$

$$R_i = ((x_i' - xS)^2 + (y_i - y_s)^2)^{1/2}$$

$a_j, b_j$ ... polynomial coefficients
$S$ ... point of symmetry
$xS_i, yS_i$ ... image coordinates relating to S
$x_i', y_i'$ ... image coordinates relating to H'
$P$ ... Pole

## Funktion B

$$x_i' = xS_i \cdot \frac{R_i^2}{(P^2 - R_i^2)} \cdot (\sum_{j=0}^{n} bj \cdot R_i^{2j})^{-1}$$

$$y_i' = yS_i \cdot \frac{R_i^2}{(P^2 - R_i^2)} \cdot (\sum_{j=0}^{n} bj \cdot R_i^{2j})^{-1}$$

The accuracy of interpolation already achieved decreases with increasing radial distance. The level of accuracy lays by ca. +- 10 µm at a radial distance r' =

30 mm ca. +- 40 µm at the centre of the image.
The image coordinates used for determinating the object coordinates are conveyed as follows:

1. Measurement of frame points and image points in the comparator system

2. Determination of the measurement frame using image frame points

3. Transformation of the measured frame to a reference frame

4. Using the functions A, B and S for the distortions, the image coordinates are completed.

## 4. Object coordinates

As we have already mentioned, the determination of object coordinates should only be carried out using analytical methods. Fisheye photographs were calculated with the help of Fortran-programs, which were carried out using the two-step solution method and the bundle method.

When using the two-step solution method, the first step is to calculate model coordinates X, Y, Z according to the S c h u t - m e t h o d from the image coordinates:

$$K = \begin{vmatrix} bx & by & bz \\ X_i' & Y_i' & Z_i' \\ X_i'' & Y_i'' & Z_i'' \end{vmatrix} = 0$$

K ... condition of coplanarity

bx ... basic components

$X_i'$ ... model coordinates in the photo coordinate system (left image)

The second step is to transform the model coordinates through spatial similarity transformation into object coordinates U, V and W.

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix}_i = \begin{bmatrix} U0 \\ V0 \\ W0 \end{bmatrix} + \lambda \cdot R \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i$$

U0 ... translations

$\lambda$ ... scale

R ... rotation matrix

In contrast, when using the bundle method the object coordinates are calculated from the image coordinates in one stage. The bundle method also has the extra advantage that additional parameters, additional observations and characteristics of the object in question as well as further photographs can be included in the adjustment computation.

The image coordinates are a function of the camera focal length c, the exterior orientation elements $\omega$, $\varphi$, $\varkappa$, U0, V0, W0 and the object point coordinates U, V and W

$$(x, y)_i' = f(c, \omega, \varphi, \varkappa, U0, V0, W0, U_i, V_i, W_i)$$

The basic formulae of central perspective underlie the coordinates as a functional model. In the stochastic model the image coordinates do not carry the same weight, because the level of accuracy decreases towards the outer regions of the image. Weight estimation was arrived at by using the linear function

$$p_i = const. \, /(a + b \cdot \tan \alpha)^2$$

As additonal observations (including control points) and object characteristics, one could introduce the following with the corresponding weights:

- complete and incomplete control points
- differences in height
- horizontal and vertical straight lines
- spatial straight lines

Moreover a third image can improve the conditions of intersection.

## 5. Examples

All the measurements were carried out on an analytical plotting system Zeiss Planicomp C 100. The computations were made on a HP-1000 computer and a ICL 1906-Computer (Computing centre of the University Braunschweig).

a) Domsingschule Braunschweig

The Domsingschule is situated on the southern side of the cathedral in Braunschweig (see fig. 5 and 6). It is a dome ca. 9 m high and has a cross-section of ca. 9 m. The shape of the dome begins half way up. Corresponding control points were measured geodetically.
The standard errors depending on the numbers of control points have been set down in table 1. It soon becomes evident that 4 control points are adequate enough to arrive at a good result. If one uses all of the control points, the deviations illustrated in figure 4 occur. The mean errors in the coordinates were
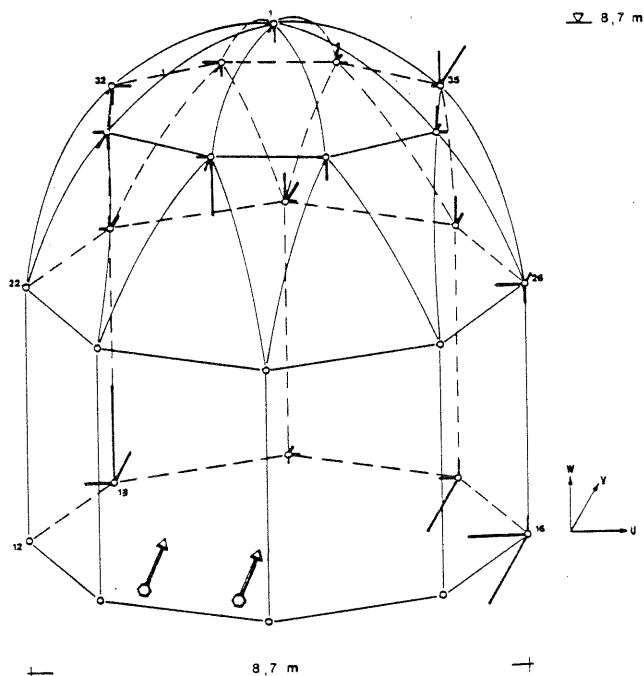
mU = ± 4,3 mm

mV = ± 6,6 mm

mW = ± 7,6 mm

Fig. 4

Vectors of errors
Example Domsingschule
Braunschweig

( ———— 10 mm )

Fig. 5 Domsingschule Braunschweig
Fisheye-photograph FE-66



Fig. 6 Photograph taken with a
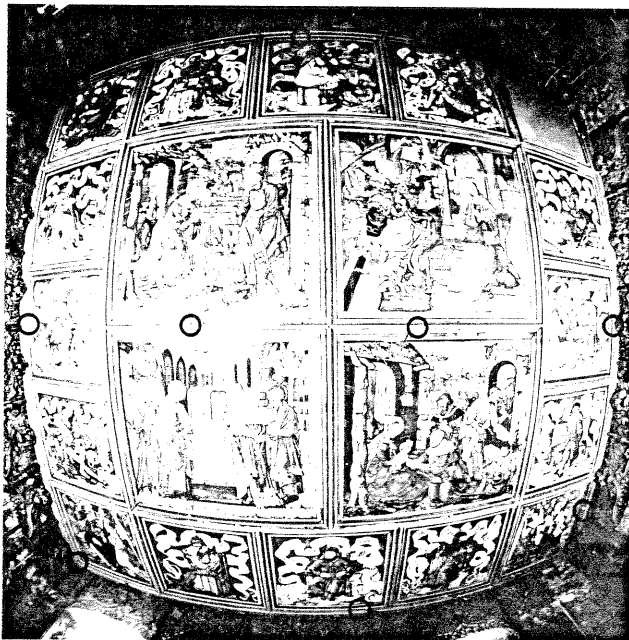wide angled metric-camera
TMK 6



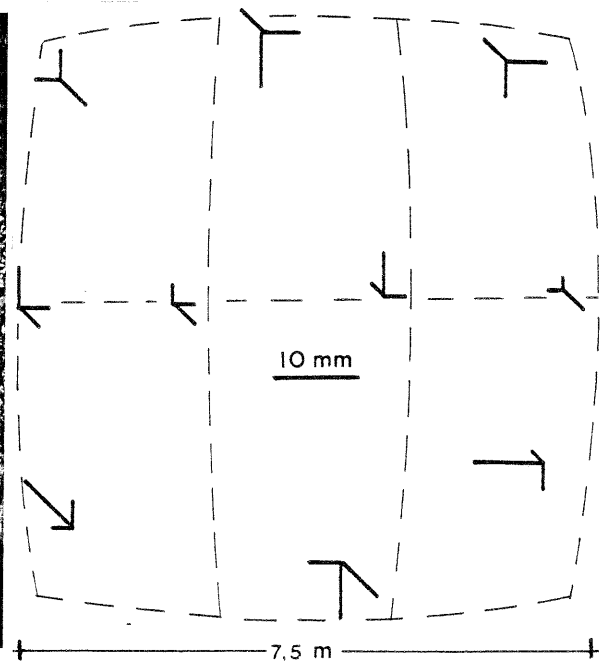Fig. 7 Huldigungssaal Goslar
Fisheye-photograph FE-66



Fig. 8 Vectors of errors to Fig. 7
(---UMK model photographs)

If object spatial information and a third image in addition to this is used, then
the level of accuracy shown in table 2 are attained ( in % ). The best results
are obtained from the third image (version B3). A third picture is taken
quickly and the additional effort involved in evaluating it is little.
In this particular case the bundle method (without object spatial information
and a third image) has a slight advantage over the two-step method. However,

one could obtain better results with the two-step method in another example, since the basis-distance ratio was infavorable.

| Version | Number of control points | ± (mm) | | |
|---------|--------------------------|--------|--------|--------|
|         |                          | $m_U$  | $m_V$  | $m_W$  |
| PP 10   | 10                       | 2      | 5      | 3      |
| PP 8    | 8                        | 2      | 5      | 5      |
| PP 6    | 6                        | 4      | 5      | 5      |
| PP 4    | 4                        | 4      | 4      | 5      |

Tab. 1  Version of control points

| Version | (%) | | | |
|---------|-----|-----|-----|---|
|         | U   | V   | W   |   |
| S4      | 15  | 15  | 8   | 4 lengths |
| DH5     | 15  | 16  | 18  | 5 differences in height |
| SDH     | 25  | 25  | 18  | 4 lengths and 5 differences in height |
| HV      | 4   | 5   | 4   | 2 horizontal and 3 vertical stright lines |
| B3      | 25  | 17  | 17  | third image |

Tab. 2  Increase in accuracy through object spatial information and a third image

b) Huldigungssaal Goslar

The Huldigungssaal in Goslar was measured photogrammetrically during restoration work. In addition to UMK 10/1318 photographs, the ceiling was photographed with the FE-66 camera (fig. 7). The coordinates attained show a level of accuracy similar to that in example a) (fig. 8). Figure 8 taken with an UMK 10/1318 camera illustrates moreover the models necessary for the ceiling.

## 6. Conclusion

Fisheye-objectives are of especial value in architecture, when carring out measurements of the interior of buildings, where the given accuracy is adequate. An increase in the level of accuracy can be aimed at by using object spatial information and above all a third image.
The given examples also show that in contrast to conventional cameras the number of images or rather models can be reduced greatly.

## 7. References

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