NONLINEAR LEAST-SQUARES ESTIMATION OF THE COLINEARITY CONDITION

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ABSTRACT

In this solution, a procedure for the calibration of non-metric cameras from a single photograph has been established by using a nonlinear least-squares estimation method developed by D. W. Marquardt in 1963 [1]. One of the major advantages of this procedure is that it does not require fiducial marks in the photographs. Although the procedure has been developed for non-metric cameras, it can readily be applied to metric cameras as well.

1. INTRODUCTION

Nonlinear least-squares estimation of the colinearity condition method established an algorithm for least-squares estimation of nonlinear parameters [2,3]. In effect, it performs an optimum interpolation between the Taylor series method and the gradient method [1]. The interpolation is based upon the maximum neighborhood in which the truncated Taylor series gives an adequate representation of the nonlinear model which are condition of colinearity equations. As such, it is a solution of central projection of the object space.

2. BACKGROUND PRINCIPLES

A perspective or central projection is one in which all points are projected onto the reference plane through one point called the perspective center. The position of the perspective center with respect to the image coordinate system represents the geometric elements of interior orientation. The position of reference plane is defined by the object space coordinates of the perspective center X_o, Y_o, Z_o. The orientation, which describes the attitude of the camera at the moment of exposure, refers to the spatial relationship between the object coordinate system and the image coordinate system. The relationship between the image and object coordinate system is expressed by a 3x3 orthogonal matrix [4].

The perspective center, the image point, and the object point lie on a straight line, as shown in Figure 1 and 2. This fundamental relationship is basic to all procedures in photogrammetry. Also we can call the condition of colinearity, as follows [2, 5]

$$\begin{bmatrix} \bar{x} - x_p \\ \bar{z} - z_p \\ c \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X - X_o \\ Z - Z_o \\ Y - Y_o \end{bmatrix} \tag{1}$$

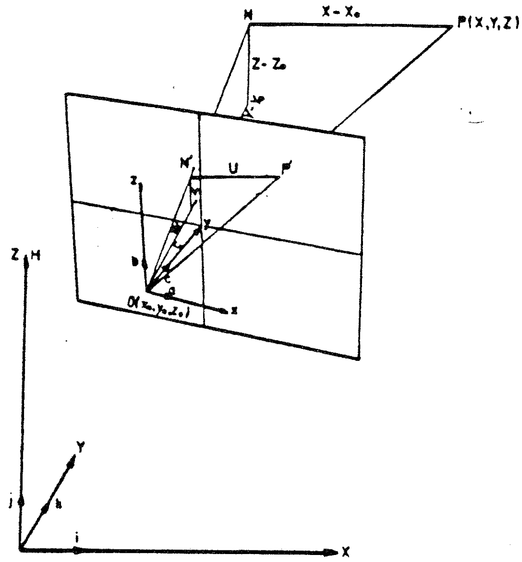

Figure 1- Geometry of space resection of terrestrial photogrammetry

where $\bar{x}$, $\bar{z}$: refined photo coordinates of image points,

z_p, z_p : photo coordinates of the principal point of the photograph,

c : camera principal distance at the moment of exposure,

λ : scale factor,

m_{ij}'s : elements of the rotation matrix in which the rotations ω, ϕ, κ of the photograph are implicit,

X, Y, Z : object space coordinates of point P.

X_o, Y_o, Z_o : object space coordinates of the camera perspective center (more specifically for the photograph under consideration).

The interior and exterior orientation of the photograph were obtained from colinearity condition equations. The left hand side of the colinearity equations may be written in the form [6, 7].

$$\bar{x} - x_p = \lambda_x (x_c - x_o) \quad (2)$$

$$\bar{z} - z_p = \lambda_z (z_c - z_o) \quad (3)$$

where $\bar{x}$, $\bar{z}$, x_p, z_p are as defined previously

x_c, z_c : observed comparator coordinates of an image point,

x_o, z_o : coordinates of the principal point referred to the comparator coordinate system.

also
$$x_c = x + \Delta x + v_x \quad (4)$$
$$z_c = z + \Delta z + v_z \quad (5)$$

where x, z : refined comparator coordinates of an image point.

Δx, Δz: systematic errors in the comparator coordinates,

v_x, v_z: random errors which will contain observed comparator coordinates.

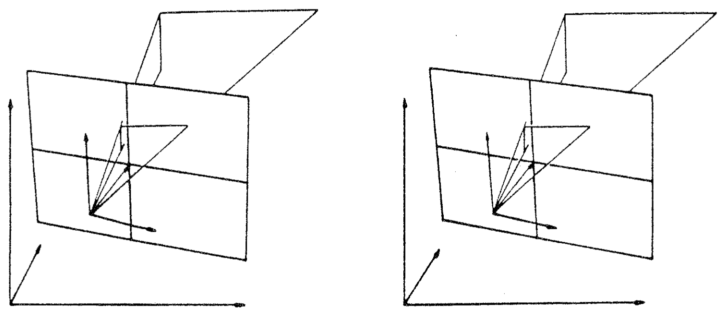

Figure 2- A stereodiagram for the three dimensional viewing of space resection of terrestrial photogrammetry.

In equation (1), dividing the first and second equations by the third,and substituting equations (2), (3) in the resulting relationships, we obtain [2]

$$x_c - x_o = c\,\frac{m_{11}(X - X_o) + m_{12}(Z - Z_o) + m_{13}(Y - Y_o)}{m_{31}(X - X_o) + m_{32}(Z - Z_o) + m_{33}(Y - Y_o)} \qquad (6)$$

$$z_c - z_o = c\,\frac{m_{21}(X - X_o) + m_{22}(Z - Z_o) + m_{23}(Y - Y_o)}{m_{31}(X - X_o) + m_{32}(Z - Z_o) + m_{33}(Y - Y_o)} \qquad (7)$$

When the elements of the transformation matrix are written in terms of the variables ω, ϕ, κ there are nine unknown parameters in equations (6) and (7). These are the exterior orientation elements X_o, Y_o, Z_o, ω, ϕ, κ and the interior orientation elements x_o, z_o and c.

The transformation matrix D is obtained as in the following :

$$D = D_\omega \cdot D_\phi \cdot D_\kappa =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (8)$$

3. STATEMENT OF THE SOLUTION

Let the model to be fitted to the data be

$$YP_i = f(X_{i,1}, X_{i,2}, \ldots , X_{i,n} ; b_1 , b_2 , \ldots , b_k) \qquad (9)$$

where YP_i : dependent variable predicted by the equation,

$X_{i,\ell}$: independent variables of the equation,

b_j : parameters of the equation,

the solution algorithm iteratively adjust the b_j to minimize the functional Φ,

$$\Phi = \sum_{i=1}^{n} (YO_i - YP_i)^2 \tag{10}$$

where YO_i dependent variable observed [1, 8, 9].

In this calibration procedure, fixed target points which are measured according to fixed object space coordinate system with high precision, will be used. The coordinates of the fixed target points can be assumed to be without error [10]. This fact allows one to use these coordinate values as independent variables without error in the solution. The image coordinates of the target points x and z, obtained according to be photograph coordinate system which moves in the above defined fixed object space, constitutes the dependent variables. The observed comparator coordinates of the target points contain systematic and random errors.

In this solution procedure the random errors are minimized for obtaining the best estimated values of the interior and exterior orientation elements of the colinearity equations. The new comparator coordinates of the target points, minimized with respect to random errors and containing only systematic errors, will be calculated in the next and final phase.

4. THEORETICAL BASIS OF ALGORITHM

"An Algorithm for Least-Squares Estimation of Nonlinear Parameters" performed an optimum interpolation between the Taylor series method and the gradient method. The interpolation is based upon the maximum neighborhood in which the truncated Taylor series gives an adequate representation of the nonlinear model. Writing the Taylor Series through the linear terms

$$YP_i(X_i, b_j + \delta_t) = f(X_i, b_j) + \sum_{j=1}^{k} \left(\frac{\partial f_i}{\partial b_j}\right) (\delta_t)_j \tag{11}$$

In (11), the vector δ_t is a small correction to b, YP_i are used to distinguish predictions based upon the linearized model from those based upon the actual nonlinear model. Thus, the value of Φ predicted (11) is

$$\Phi = \sum_{i=1}^{n} (YO_i - YP_i)^2 \tag{12}$$

δ_t appears linearly in (11), and can therefore be found by the standard least-squares method of setting $\partial\Phi/\partial\delta_j = 0$, for all j. Thus δ_t is found by solving

$$A \cdot \delta_t = g \tag{13}$$

where $A^{[kxk]} = P^T \cdot P$ (14)

$$P^{[n\times k]} = \left(\frac{\partial f_i}{\partial b_j}\right), \quad i = 1, 2, \ldots, n \quad ; \quad j = 1, 2, \ldots, k \qquad (15)$$

$$g^{[k\times 1]} = \sum_{i=1}^{n} (YO_i - f_i) \frac{\partial f_i}{\partial b_j}, \quad j = 1, 2, \ldots, k \qquad (16)$$

Theoretical basis of the algorithm is contained several theorems.

THEOREM 1. Let $\lambda \geq 0$ be arbitrary and let δ_o satisfy the equation

$$(A + \lambda I)\, \delta_o = g \qquad (17)$$

Then δ_o minimizes Φ on the sphere whose radius $\| \delta \|$ satifies and λ is a Lagrange multiplier. $\| \delta \|^2 = \| \delta_o \|^2$

THEOREM 2. Let $\delta(\lambda)$ be the solution of (17) for a given value of λ. Then $\| \delta(\lambda) \|^2$ is a continuous decreasing function of λ, such that as $\lambda \to \infty$, $\| \delta(\lambda) \|^2 \to 0$.

THEOREM 3. Let γ be the angle between δ_o and δ_g. Then is a continuous monotone decreasing function of λ such that as $\lambda \to \infty$, $\gamma \to 0$. Since δ_g is independent of λ, it follows that δ_o rotates toward δ_g as $\lambda \to \infty$.

At the rth iteration of this algorithm, the equation

$$(A^{*(r)} + \delta^{(r)} I)\, \delta^{*(r)} = g^{*(r)} \qquad (18)$$

is constructed. A^* and g^* where defined as a scaled matrix, and scaled vector. This equation is then solved for $\delta^{*(r)}$. Then

$$\delta_j = \delta_j^* / \sqrt{a_{jj}} \qquad (19)$$

is used to obtain $\delta^{(r)}$. The new trial vector

$$b^{(r+1)} = b^{(r)} + \delta^{(r)} \qquad (20)$$

will lead to a new sum of squares $\Phi^{(r+1)}$. It is essential to select $\lambda^{(r)}$ such that

$$\Phi^{(r+1)} < \Phi^{(r)} \qquad (21)$$

5. NUMERICAL EXAMPLE

The nonlinear least-squares estimation routine contained in the program is on adaptation of M. W. Morris, formerly with the Department of Environmental and Water Resources at Vanderbilt University. Standard Fortran Language was utilized and the simulations were run on the Cyber Systems 175 of the University of Illinois at Urbana-Champaign. A sample output is provided in the figures below:

```
IDENTIFICATION - NONLINEAR LEAST-SQUARES ESTIMATION OF THE COLLINEARITY CONDITION
IEQ =    1 (EQUATION NO.)
IFORM = FORMAT(I5,4F10.0)
NO. OBSERVATION =  40
NO. PARAMETERS =   9
NO. PARAMETERS SET =   0
NO. INDEPENDANT VARIABLES =   3
IPD = 1 (ANALYTICAL PARTIAL DERIVATIVES USED)
IRPT = 0 (NEW DATA USED)
MAX. NO. CORRECTIONS =  25
YMIN =  -30.00000
YMAX =    30.00000
TSTAT =      2.000
FSTAT =      4.000
GAMMA CRITICAL =        45.0
EPSLON =  0.500E-06
TAU =  0.100E-06
DELPD =  0.1000-04

INITIAL ESTIMATES OF PARAMETERS:
B( 1)=        1.00000000000     X COORDINATE OF THE PRINCIPAL POINT
B( 2)=        100.000000000     CAMERA PRINCIPAL DISTANCE
B( 3)=        0.100000000-03    PHI ROTATION
B( 4)=        0.100000000-03    OMEGA ROTATION
B( 5)=        0.100000000-03    KAPPA ROTATION
B( 6)=        10000.0000000     X COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 7)=        8000.00000000     Y COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 8)=        15000.0000000     Z COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 9)=        1.00000000000     Z COORDINATE OF THE PRINCIPAL POINT
DATA:
  1   0.711000000000    10000.0000000    30000.0000000    10000.0000000
  2    7.62200000000    11500.0000000    30000.0000000    10000.0000000
  3    14.5080000000    13000.0000000    30000.0000000    10000.0000000
  4    21.3670000000    14500.0000000    30000.0000000    10000.0000000
  5    21.2630000000    14500.0000000    30000.0000000    12500.0000000
  6    18.2150000000    13000.0000000    26000.0000000    12500.0000000
  7    9.83700000000    11500.0000000    26000.0000000    12500.0000000
  8   0.700000000000    10000.0000000    30000.0000000    12500.0000000
  9   0.686000000000    10000.0000000    30000.0000000    15000.0000000
 10    9.78800000000    11500.0000000    26000.0000000    15000.0000000
 11    18.1210000000    13000.0000000    26000.0000000    15000.0000000
 12    21.1720000000    14500.0000000    30000.0000000    15000.0000000
 13    21.0840000000    14500.0000000    30000.0000000    17500.0000000
 14    18.0240000000    13000.0000000    26000.0000000    17500.0000000
 15    9.72400000000    11500.0000000    26000.0000000    17500.0000000
 16   0.669000000000    10000.0000000    30000.0000000    17500.0000000
 17   0.655000000000    10000.0000000    30000.0000000    20000.0000000
 18    7.46800000000    11500.0000000    30000.0000000    20000.0000000
 19    14.2410000000    13000.0000000    30000.0000000    20000.0000000
 20    20.9760000000    14500.0000000    30000.0000000    20000.0000000
 21   -25.6440000000    10000.0000000    30000.0000000    10000.0000000
 22   -25.5760000000    11500.0000000    30000.0000000    10000.0000000
 23   -25.5100000000    13000.0000000    30000.0000000    10000.0000000
 24   -25.4580000000    14500.0000000    30000.0000000    10000.0000000
 25   -13.9200000000    14500.0000000    30000.0000000    12500.0000000
 26   -16.5050000000    13000.0000000    26000.0000000    12500.0000000
 27   -16.5660000000    11500.0000000    26000.0000000    12500.0000000
 28   -14.0330000000    10000.0000000    30000.0000000    12500.0000000
 29   -2.51100000000    10000.0000000    30000.0000000    15000.0000000
 30   -2.49700000000    11500.0000000    26000.0000000    15000.0000000
 31   -2.48800000000    13000.0000000    26000.0000000    15000.0000000
 32   -2.48500000000    15000.0000000    30000.0000000    15000.0000000
 33    8.87000000000    14500.0000000    30000.0000000    17500.0000000
 34    11.4020000000    13000.0000000    26000.0000000    17500.0000000
 35    11.4310000000    11500.0000000    26000.0000000    17500.0000000
 36    8.92800000000    10000.0000000    30000.0000000    17500.0000000
 37    20.2710000000    10000.0000000    30000.0000000    20000.0000000
 38    20.2240000000    11500.0000000    30000.0000000    20000.0000000
 39    20.1730000000    13000.0000000    30000.0000000    20000.0000000
 40    20.1230000000    14500.0000000    30000.0000000    20000.0000000
```

```
**********************************************************************************************************
******************************************************* CORRECTIONS **************************************
**********************************************************************************************************
NO.  1:  PHI =  0.16292145D-01: SE =  0.22924944D-01: LENGTH =   702.258930000** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-09: GAMMA =   89.90000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.37300590000
         B( 2) =          100.667540000
         B( 3) =         0.38687511D-01
         B( 4) =        -0.34732696D-01
         B( 5) =        -0.11933418D-02
         B( 6) =          9302.06050000
         B( 7) =          8077.76490000
         B( 8) =          14999.7030000
         B( 9) =         0.964161320000
NO.  2:  PHI =  0.63756891D-03: SE =  0.45350567D-02: LENGTH =   17.7791220000** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-12: GAMMA =   68.70000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.06272090000
         B( 2) =          100.493430000
         B( 3) =         0.35549285D-01
         B( 4) =        -0.34675142D-01
         B( 5) =        -0.19158825D-04
         B( 6) =          9304.23140000
         B( 7) =          8095.35380000
         B( 8) =          15001.0770000
         B( 9) =         0.981084900000
NO.  3:  PHI =  0.63058077D-03: SE =  0.45101347D-02: LENGTH =  0.39484415D-01** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-15: GAMMA =   79.20000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.06425250000
         B( 2) =          100.492920000
         B( 3) =         0.35558862D-01
         B( 4) =        -0.34684256D-01
         B( 5) =        -0.17365967D-04
         B( 6) =          9304.24530000
         B( 7) =          8095.38160000
         B( 8) =          15001.1010000
         B( 9) =         0.982515000000
NO.  4:  PHI =  0.63058077D-03: SE =  0.45101347D-02: LENGTH =  0.11821719D-03** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-15: GAMMA =   88.40000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.06424820000
         B( 2) =          100.492920000
         B( 3) =         0.35558821D-01
         B( 4) =        -0.34684233D-01
         B( 5) =        -0.17368892D-04
         B( 6) =          9304.24530000
         B( 7) =          8095.38150000
         B( 8) =          15001.1010000
         B( 9) =         0.982512310000
NO.  5:  PHI =  0.63058077D-03: SE =  0.45101347D-02: LENGTH =  0.26501161D-06** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-09: GAMMA =   57.70000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.06424820000
         B( 2) =          100.492920000
         B( 3) =         0.35558821D-01
         B( 4) =        -0.34684233D-01
         B( 5) =        -0.17368881D-04
         B( 6) =          9304.24530000
         B( 7) =          8095.38150000
         B( 8) =          15001.1010000
         B( 9) =         0.982512320000
NO.  6:  PHI =  0.63058077D-03: SE =  0.45101347D-02: LENGTH =  0.13745266D-08** ANALYTICAL PARTIAL DERIVATIVES USED
                MARQUARDT VARIABLES:  LAMBDA =  0.100D-06: GAMMA =   73.40000: (K) =   1.000000
         CORRECTED PARAMETERS:
         B( 1) =          1.06424820000
         B( 2) =          100.492920000
         B( 3) =         0.35558821D-01
         B( 4) =        -0.34684233D-01
         B( 5) =        -0.17368881D-04
         B( 6) =          9304.24530000
         B( 7) =          8095.38150000
         B( 8) =          15001.1010000
         B( 9) =         0.982512320000
```

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**************************************************************************************************************
***************************************************** STATISTICS *********************************************
**************************************************************************************************************
FINAL ESTIMATE OF PARAMLTERS:
B( 1)=      1.0642482000U     X COORDINATE OF THE PRINCIPAL POINT
B( 2)=      100.492920000     CAMERA PRINCIPAL UISTANCE
B( 3)=      0.35558821D-01    PHI ROTATIUN
B( 4)=     -0.34684233D-01    OMEGA ROTATION
B( 5)=     -0.17368881D-04    KAPPA ROTATION
B( 6)=      9304.24530000     X COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 7)=      8095.38150000     Y COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 8)=      15001.1010000     Z COORDINATE OF THE CAMERA PERSPECTIVE CENTER
B( 9)=      0.982512320000    Z COORDINATE OF THE PRINCIPAL POINT
STANDART ERROR OF ESTIMATE =  0.45101347D-02
MULTIPLECURRLLA TION COLFFICIENT =  0.999999940000
PARAMLTER CORRELATIONS:
R( 1, 1) =   1.00000000
R( 2, 1) =  0.499610000
R( 3, 1) = -0.999650000
R( 4, 1) = -0.50825D-02
R( 5, 1) = -0.164710000
R( 6, 1) = -0.995480000
R( 7, 1) =  0.628110000
R( 8, 1) =  0.49792D-02
R( 9, 1) =  0.000000000
R( 2, 2) =   1.00000000
R( 3, 2) = -0.518310000
R( 4, 2) = -0.178060000
R( 5, 2) =  0.82953D-03
R( 6, 2) = -0.496960000
R( 7, 2) =  0.972450000
R( 8, 2) =  0.176540000
R( 9, 2) = -0.165550000
R( 3, 3) =   1.00000000
R( 4, 3) =  0.91877D-02
R( 5, 3) =  0.161730000
R( 6, 3) =  0.994990000
R( 7, 3) = -0.644470000
R( 8, 3) = -0.90906D-02
R( 9, 3) =  0.38796D-02
R( 4, 4) =   1.00000000
R( 5, 4) = -0.482420000
R( 6, 4) =  0.38273D-02
R( 7, 4) = -0.19872D-01
R( 8, 4) = -0.994040000
R( 9, 4) =  0.999370000
R( 5, 5) =   1.00000000
R( 6, 5) =  0.168680000
R( 7, 5) = -0.102310000
R( 8, 5) =  0.503880000
R( 9, 5) = -0.502260000
R( 6, 6) =   1.00000000
R( 7, 6) = -0.631430000
R( 8, 6) = -0.36589D-02
R( 9, 6) = -0.13118D-02
R( 7, 7) =   1.00000000
R( 8, 7) =  0.18889D-01
R( 9, 7) = -0.73906D-02
R( 8, 8) =   1.00000000
R( 9, 8) = -0.995400000
R( 9, 9) =   1.00000000
PARAMETER STATISTICS:
```

	STANDARD ERROR	RANGE LOWER	RANGE UPPER	0.05 CONFIDENCE RANGE LOWER	0.05 CONFIDENCE RANGE UPPER
(1)	0.70973575E-01	0.922300990000	1.20619490000	0.638406690000	1.49008940000
(2)	0.75955868E-01	100.341000000	100.644820000	100.037170000	100.948640000
(3)	0.67097670E-03	0.34216866E-01	0.36900774E-01	0.31532962E-01	0.39584678E-01
(4)	0.42457203E-03	-0.35533376E-01	-0.33835087E-01	-0.37231661E-01	-0.32136798E-01
(5)	0.75220392E-04	-0.16780966E-03	0.13307190E-03	-0.46869088E-03	0.43395325E-03
(6)	3.40434270000	9297.43360000	9311.05080000	9283.81640000	9324.66800000
(7)	16.0931240000	8063.19140000	8127.56640000	7998.82030000	8191.93750000
(8)	2.29839610000	14996.5040000	15005.6950000	14987.3090000	15014.8910000
(9)	0.49223848E-01	0.884064610000	1.08095930000	0.687169250000	1.27785490000

IDENT	OBSERVED Y	PREDICTED Y	DIFFERENCE
1	0.711000000000	0.709572530000	0.14274682D-02
2	7.62200000000	7.62631840000	-0.43184478D-02
3	14.5080000000	14.5092800000	-0.12802768D-02
4	21.3670000000	21.3587050000	0.82950623D-02
5	21.2630000000	21.2644020000	-0.14021231D-02
6	18.2150000000	18.2196460000	-0.46458231D-02
7	9.83700000000	9.84268060000	-0.56805662D-02
8	0.700000000000	0.696583810000	0.34161857D-02
9	0.686000000000	0.683697860000	0.23021443D-02
10	9.78800000000	9.78284750000	0.51524890D-02
11	18.1210000000	18.1197010000	0.12993975D-02
12	21.1720000000	21.1708400000	0.11600388D-02
13	21.0840000000	21.0780100000	0.59902395D-02
14	18.0240000000	18.0207120000	0.32878401D-02
15	9.72400000000	9.72358890000	0.41106850D-03
16	0.669000000000	0.670913440000	-0.19134397D-02
17	0.655000000000	0.658229370000	-0.32293723D-02
18	7.46800000000	7.46677810000	0.12218836D-02
19	14.2410000000	14.2425910000	-0.15908039D-02
20	20.9760000000	20.9859030000	-0.99029650D-02
21	-25.6440000000	-25.6457770000	0.17768588D-02
22	-25.5760000000	-25.5804660000	0.44660498D-02
23	-25.5100000000	-25.5154740000	0.54742444D-02
24	-25.4580000000	-25.4507990000	-0.72008891D-02
25	-13.9200000000	-13.9211570000	0.11568926D-02
26	-16.5050000000	-16.5093400000	0.43402602D-02
27	-16.5660000000	-16.5616740000	-0.43262733D-02
28	-14.0330000000	-14.0308100000	-0.21901039D-02
29	-2.51100000000	-2.50773300000	-0.32669931D-02
30	-2.49700000000	-2.49746010000	0.46006157D-03
31	-2.48800000000	-2.48701300000	-0.98704255D-03
32	-2.48500000000	-2.47923960000	-0.57603955D-02
33	8.87000000000	8.86752940000	0.24706459D-02
34	11.4020000000	11.4010780000	0.92173323D-03
35	11.4310000000	11.4317190000	-0.71871178D-03
36	8.92600000000	8.92454000000	0.34600259D-02
37	20.2710000000	20.2670780000	0.39218389D-02
38	20.2240000000	20.2207240000	0.32762022D-02
39	20.1730000000	20.1745920000	-0.15923099D-02
40	20.1230000000	20.1286620000	-0.56820941D-02

Figure 3. Sample Output

6. CONCLUSION

In this paper, a computation procedure for calculation of both the interior and the exterior orientation elements from a single photograph through the use of the colinearity conditions, is presented. The computation time for the algorithm is short, and the values of the parameters converge to their true values in a few iterations,provided that suitable initial estimates are utilized. However, the use of very close estimates is not necessary for the convergence of the algorithm. Another advantage of the procedure is that the fiducial marks are not required in the photographs. Although the procedure has been developed for non-metric cameras, it can readily be applied to metric cameras as well.

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