LINEARIZATION OF THE COLLINEARITY EQUATIONS WITHOUT SERIAL EXPANSION AND ITS APPLICATION TO THE SPACE INTERSECTION

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Commission Nr: V

ABSTRACT

The space intersection method which bases on the collinearity condition is generally used to compute the object space coordinates of a point which is imaged in two common photographs, in the analytical photogrammetry. Since the collinearity condition is expressed by non -linear equations, these equations are commonly linearized by the help of serial expansion, and generally Taylor's method is preferred. In this case, this method requires iteration.

In this study, the collinearity equatins were linearized without using the conventional serial expansion, and the object space coordinates of the photographic points were computed by means of this method. In addition the developed model is tested by means of the simulation method.

INTRODUCTION

The first of two fundamental steps of the aerial and terrestrial photogrammetry is the space resection. By carrying out this process, the object space coordinates of the camera perspective center, exposure station, and the rotation of the photographic coordinate system according to the object (geodesic) coordinate system are determined. The second step is the space intersection. By performing this step, the object space coordinates of the points, which correspond to the same object point imaged in two or more photographs, are computed.

In the analogue restitution (instrumental) both of the processes are carried out on the analogue plotting instruments via some optical and mechanical treatments, namely the results of relative and absolute orientation. However, in the analytical photogrammetry the all of the restitution is performed numerically.

In both space resection and intersiction, the analytical process can be carried out by employing various mathematical methods which are based on the principles of projective or solid analytical geometry. The optimum solution (Veress and Hatzopoulos, 1979) is the collinearity condition. The basic principle in the collinearity condition is that a certain object point, its image on the photographs and the perspective center of the camera (the frontal point of the camera lens) lie along the same straight line (Finsterwalder and Hofmann,1968; Wolf 1974). The non-linear equations systems are faced when the collinearity condition is expressed by means of the analytical equations.

Up to the recent studies, these condition equations have been commonly linearized using the serial expansion, and often Taylor's therom has been preferred. Although there are some available direct linear applications (Schut, 1960; Oswal and Balasubramanian, 1968) in performing space resection, for computing the object space coordinates of any ground point the collinearity equaitions have been employed after serial expansion in space intersection, (Veress's the vector method is excluded, Veress and Sun,1978; Veress and Hatzopoulos,1979). DLT (Abdel-Aziz and Karara, 1971, 1974; Marzan and Karara, 1975) and the 11 Parameter solution (Bopp and Krauss, 1978) are linear solutions for a non-linear problem, but the specific aspect of their studies is the coverage of the calibration of non-metric cameras. Furthermore, they depend on the 11 transformation parameters in determining the object space coordinates of the points.

This study is completely and directly based on the collinearity equations and these equations are linearized without using the traditional serial expansion, and the assumption of using a metric camera. The object space coordinates of the camera perspective center and the elements m_{ij} of the orthogonal transformation matrix can be obtained by means of using the computation of one of the linear or non-linear method. Another way of obtaining the above mentioned parameters is the direct measurement of all of them on the field (see, Erlandson and Veress, 1974,1975) or the measurement of some of them on the field and obtaining some of them by calculation (see, Erez, 1971; Brandenberger and Erez, 1972; Veress and Sun, 1978). Above mentioned cases may change due to the properties of the work, researcher and the available conditions.

LINEARIZATION OF THE COLLINEARITY EQUATIONS BY A SIMPLE METHOD

The object point, its image on photographs and perspective center all lie on the same straight line. This case is expressed by the collinearity equations which are the basis for the computation of object space coordinates of points in photogrammetry. These condition equations are formulated according to the shown coordinate system in Figure 1, as follows:

$$x_{ij} = f_j \frac{m_{11}(X-X_L)+m_{12}(Y-Y_L)+m_{13}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)} \quad (1.a)$$

$$y_{ij} = f_j \frac{m_{21}(X-X_L)+m_{22}(Y-Y_L)+m_{23}(Z-Z_L)}{m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)} \quad (1.b)$$

where the index i refers to any ground points which are imaged on two overlapped photographs, j refers to any exposure station.

- x,y = refined photo coordinates of a point
- f = camera principle distance (calibrated focal length)
- m's = elements of the orthogonal transformation matrix in which the rotations omega,phi,kappa of the photographs are implicit.
- X,Y,Z = object space coordinates of any point

X_L, Y_L, Z_L = object space coordinates of the camera perspective center (exposure station).

Besides refined systematic errors, measured quantities will contain random errors v_x, v_y respectively; then equations (1.a) and (1.b) become as follows:

$$x + v_x = f\frac{M_1X}{M_3X} \tag{2.a}$$

$$y + v_y = f\frac{M_2X}{M_3X} \tag{2.b}$$

where $M_1 = (m_{11}\ m_{12}\ m_{13})$

$M_2 = (m_{21}\ m_{22}\ m_{23})$

$M_3 = (m_{31}\ m_{32}\ m_{33})$

$X^T = (X-X_L\quad Y-Y_L\quad Z-Z_L)$

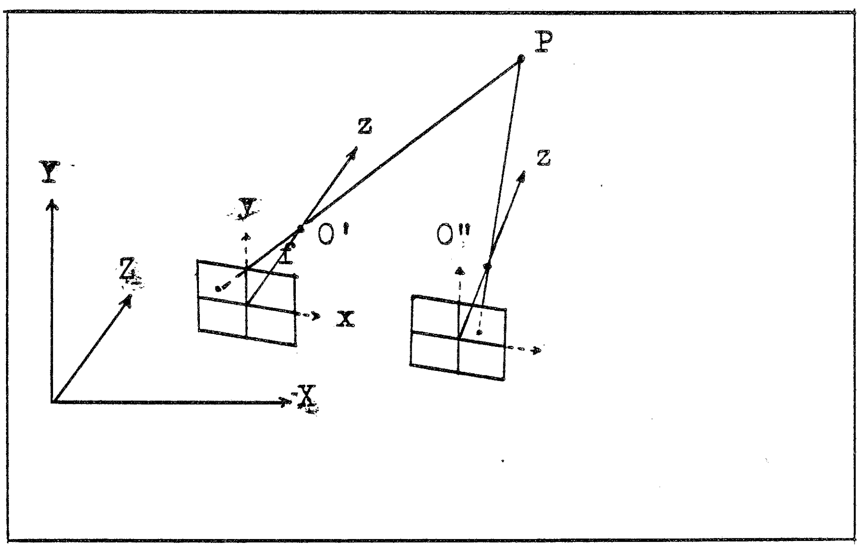

Figure: 1. Geodesic and photographic coordinate systems

Then the equations can be written as

$$v_x = f\frac{M_1X}{M_3X} - x \tag{3.a}$$

$$v_y = f\frac{M_2X}{M_3X} - y \tag{3.b}$$

If x and y are multiplied by M_3X in above equations, we obtain

$$v_x = \frac{fM_1X - xM_3X}{M_3X} \tag{4.a}$$

$$v_y = \frac{f\ M_2X - yM_3X}{M_3X} \tag{4.b}$$

If the notations M_1, M_2, M_3, X are substituted in equations (4.a) and (4.b) one gets the

$$v_x = \frac{fm_{11}X-fm_{11}X_L +fm_{12}Y-fm_{12}Y_L +fm_{13}Z-fm_{13}Z_L -xm_{31}X+xm_{31}X_L-xm_{32}Y+xm_{32}Y_L -xm_{33}Z+xm_{33}Z_L}{m_{31}(X-X_L) +m_{32}(Y-Y_L) +m_{33}(Z-Z_L)} \tag{5.a}$$

$$v_y = \frac{fm_{21}X-fm_{21}X_L +fm_{22}Y-fm_{22}Y_L +fm_{23}Z-fm_{23}Z_L -ym_{31}X+ym_{31}X_L-ym_{32}Y+ym_{32}Y_L -ym_{33}Z+ym_{33}Z_L}{m_{31}(X-X_L) +m_{32}(Y-Y_L) +m_{33}(Z-Z_L)} \tag{5.b}$$

equations,

If the equations (5.a) and (5.b) are arranged according to the constants and variables, we can write as follows

$$v_x = X\left(\frac{fm_{11}-xm_{31}}{\bar{M}_3}\right) + Y\left(\frac{fm_{12}-xm_{32}}{\bar{M}_3}\right) + Z\left(\frac{fm_{13}-xm_{33}}{\bar{M}_3}\right) - X_L\left(\frac{fm_{11}-xm_{31}}{\bar{M}_3}\right) - Y_L\left(\frac{fm_{12}-xm_{32}}{\bar{M}_3}\right) - Z_L\left(\frac{fm_{13}-xm_{33}}{\bar{M}_3}\right) \tag{6.a}$$

$$v_y = X\left(\frac{fm_{21}-ym_{31}}{\bar{M}_3}\right) + Y\left(\frac{fm_{22}-ym_{32}}{\bar{M}_3}\right) + Z\left(\frac{fm_{23}-ym_{33}}{\bar{M}_3}\right) - X_L\left(\frac{fm_{21}-ym_{31}}{\bar{M}_3}\right) - Y_L\left(\frac{fm_{22}-ym_{32}}{\bar{M}_3}\right) - Z_L\left(\frac{fm_{23}-ym_{33}}{\bar{M}_3}\right) \tag{6.b}$$

where $\bar{M}_3=m_{31}(X-X_L)+m_{32}(Y-Y_L)+m_{33}(Z-Z_L)$

For each point i in photo j, if the equations (6a) and (6.b) are written in matrix form, we obtain

$$\begin{bmatrix} v_{x_{ij}} \\ v_{y_{ij}} \end{bmatrix} = \begin{bmatrix} a_{1_{ij}} & a_{2_{ij}} & a_{3_{ij}} \\ b_{1_{ij}} & b_{2_{ij}} & b_{3_{ij}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} a_{1_{ij}} & a_{2_{ij}} & a_{3_{ij}} \\ b_{1_{ij}} & b_{2_{ij}} & b_{3_{ij}} \end{bmatrix} \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \tag{7}$$

where

$$a_{1_{ij}} = \frac{fm_{11}-xm_{31}}{\bar{M}_3} \qquad b_{1_{ij}} = \frac{fm_{21}-ym_{31}}{\bar{M}_3}$$

$$a_{2_{ij}} = \frac{fm_{12}-xm_{32}}{\bar{M}_3} \qquad b_{2_{ij}} = \frac{fm_{22}-ym_{32}}{\bar{M}_3} \tag{8}$$

$$a_{3_{ij}} = \frac{fm_{13}-xm_{33}}{\bar{M}_3} \qquad b_{3_{ij}} = \frac{fm_{23}-ym_{33}}{\bar{M}_3}$$

Consequently the matrix equation (7) is the linear form of the non-linear collinearity condition equations, and the linearization has been achieved without using the tredational serial expansion.

COMPUTATION OF THE OBJECT SPACE COORDINATES

The equation (7) contains only three unknowns which are the geodesic coordinates X,Y and Z of a ground point. Since these three unknowns can be computed by a least three equations, there have to be at least two overlapped photos in which the point i is imaged. If there are n photographs used in the solution, we will have 2n number of equations (7) to compute the unknowns X,Y and Z. The number of the redundant observation equations will be R=2n-3. For point i in photo j the pairs of condition equations (7) can be written as

$$V_{ij} = A_{ij}\, X_i - L_j \tag{9}$$

where

$$V=\begin{bmatrix} v_x \\ v_y \end{bmatrix} ; \quad A=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} ; \quad X=\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ; \quad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = -L$$

V_{ij} = 2x1 correction vector of the image coordinates

A_{ij} = 2x3 matrix of the coefficients of the unknowns

L_j = 2x1 vector of constant

In generally speaking, we can write the following equation:

$$V = A\, X_i - L \tag{10}$$

In this case, for a least squares solution, it requires

$$V^TWV = X^TA^TWAX - X^TA^TWL - L^TWAX + L^TWL \tag{11}$$

where W = weight matrix for the condition equations.

$$\frac{\partial V^TWV}{\partial X} = A^TWAX - A^TWL = 0 \qquad (12)$$

Therefore least squares method will result in

$$X = (A^TWA)^{-1}A^TWL$$
$$= N^{-1}.\ n \qquad (13)$$

where

$$N = A^TWA$$
$$n = A^TWL$$

To obtain the sum of the residuals squared, we premultiply equation (12) by X^T and substitute the results into (11), then we get

$$V^TWV = L^TWL - L^TWAX \qquad (14)$$

The standart error of the unit weight is computed as follows

$$m_o^2 = \frac{V^TWV}{2n - u}$$

$$m_o = \left(\frac{V^TWV}{R}\right)^{1/2} \qquad (15)$$

where n = number of photos used in the solution
u = number of unknowns
R = degrees of freedom

The standard error of the unknowns can be also computed as follows

$$m_X = m_o(N^{-1})^{1/2} \qquad (16)$$

COMPUTATION OF $\overline{M}_3$

When the coefficients which are represented in equations (8) are examined, it will be seen that the quantity $\overline{M}_3$, which forms the denominator of the coefficients, contains the unknowns X,Y and Z, Therefore, the approximate valuesof them must be computed. For this computation the equations (1.a) and (1.b) can be used directly. If we rewrite the equations (1.a) and (1.b) clearly, remove the parenthesises and arrange them according to the unknowns, we will get

$$X(fm_{11}-xm_{31})+Y(fm_{12}-xm_{32})+Z(fm_{13}-xm_{33})-X_L(fm_{11}-xm_{31})-Y_L(fm_{12}-xm_{32})-$$
$$Z_L(fm_{13}-xm_{33}) = 0 \qquad (17.a)$$

$$X(fm_{21}-ym_{31})+Y(fm_{22}-ym_{32})+Z(fm_{23}-ym_{33})-X_L(fm_{21}-ym_{31})-Y_L(fm_{22}-ym_{32})-$$
$$Z_L(fm_{23}-ym_{33}) = 0 \qquad (17.b)$$

Then if the long terms are expressed by a_i' and b_i' notations, the equations (17.a) and (17.b) become as follows:

$$a_1'X + a_2'Y + a_3'Z - (a_1'X_L + a_2'Y_L + a_3'Z_L) = 0$$
$$b_1'X + b_2'Y + b_3'Z - (b_1'X_L + b_2'Y_L + b_3'Z_L) = 0 \qquad (18)$$

where

$$a_1' = fm_{11} - xm_{31} \qquad b_1' = fm_{21} - ym_{31}$$
$$a_2' = fm_{12} - xm_{32} \qquad b_2' = fm_{22} - ym_{32}$$
$$a_3' = fm_{13} - xm_{33} \qquad b_3' = fm_{23} - ym_{33}$$

For each point which is imaged at least in two common photos, two pairs of equations can be written from each photos similar to equations (17.a) and (17.b). We can obtain the presolution of unknowns as using any three of these equations. Then these values are substituted into M_3 and the coefficients a_i and b_i are determined.

DETERMINATION OF THE WEIGHT COEFFICIENTS

If the adjustment will be performed with respect to the weighted measurement, various methods are followed in determination of the weight coefficients. Some researchers (Erez, 1971) accept the inverse of distance, which is distance between the perspective center and the respective object point lying on the same ray, as the weight coefficient. Some other researchers (Marzan and Karara, 1975) use the law of propagation of variance which requires the long computations, to determine the weight coefficients. In some studies (Erlandson and Veress, 1975) the weight coefficients have been considered as unity. Marzan (1975) proved that the accuracy of the object space coordinates of points is a function of configuration, according to him configuration is defined by three items as follows: (1) base, (2) object distance, (3) the convergence of the two camera axes. As it will be mentioned later, the results of our simulation test have indicated that the parallactic angle and the object distance have great effect on the accuracy of the object space coordinates. In this case, the weight coefficient can be computed as follows: if it applies to the equations (7), the law of propagation of variance seperately,

$$F_x = a_1m_X + a_2m_Y + a_3m_Z - a_1m_{X_L} - a_2m_{Y_L} - a_3m_{Z_L}$$
$$F_y = b_1m_X + b_2m_Y + b_3m_Z - b_1m_{X_L} - b_2m_{Y_L} - b_3m_{Z_L} \qquad (19)$$

$$m^2_{F_x} = a_1^2m_X^2 + a_2^2m_Y^2 + a_3^2m_Z^2 + a_1^2m_{X_L}^2 + a_2^2m_{Y_L}^2 + a_3^2m_{Z_L}^2 \qquad (20)$$

$$m^2_{F_y} = b_1^2m_X^2 + b_2^2m_Y^2 + b_3^2m_Z^2 + b_1^2m_{X_L}^2 + b_2^2m_{Y_L}^2 + b_3^2m_{Z_L}^2$$

If the equations (20) are arranged , one gets

$$m_{F_x}^2 = a_1^2(m_X^2 + m_{X_L}^2) + a_2^2(m_Y^2 + m_{Y_L}^2) + a_3^2(m_Z^2 + m_{Z_L}^2)$$

$$m_{F_y}^2 = b_1^2(m_X^2 + m_{X_L}^2) + b_2^2(m_Y^2 + m_{Y_L}^2) + b_3^2(m_Z^2 + m_{Z_L}^2) \qquad (21)$$

where m_X^2, m_Y^2, m_Z^2 the variances of the object space coordinates which are determined by the space intersection, and they are computed via Marzan's equations (12), (1975) as follows:

$$m_X^2 = \frac{1}{\sqrt{2}}\,\frac{D}{c}\,\frac{m}{(1-1/2\tan\Theta\sin\phi)\cos\phi}$$

$$m_Y^2 = \frac{1}{\sqrt{2}}\,\frac{D}{c}\,m \qquad (22)$$

$$m_Z^2 = \sqrt{2}\,\frac{D}{c}\,\frac{m}{\tan\Theta\cos^2\phi + 2\sin\phi}$$

In the above equations:

D= object distance
c= principle distance of the camera
m= the plate coordinate error
ϕ= the convergence angle
Θ= the overlapped angle (from $\tan\Theta = \frac{B}{D}$)

Thus, the factors (object distance and convergency) which have the greatest effect on the accuracy of the object space coordinates of the points are introduced into the computations of the weight coefficients.In this case the weight coefficients become

$$W_i = \begin{bmatrix} w_{x_i} & 0 \\ 0 & w_{y_i} \end{bmatrix} \qquad W_i = \begin{bmatrix} 1/m_{F_x}^2 & 0 \\ 0 & 1/m_{F_y}^2 \end{bmatrix} \qquad (23)$$

SIMULATION STUDY

In order to assess the accuracy and appropriate of the developed model, it was tested via the simulation method. Several simulation experiments were performed to find the optimum geometric configuration and the number of plates which effect the accuracy.

The simulation experiments start by generating fictitious photographic coordinates which are obtained in a mathmaticalmanner by using directly equations (1a) and (1.b) . Assumed model is based on a mathematical test area which has dimensions of 90x90 meters. It includes 100 ground points which have the geodesic coordinates. The locations of the ground points are according to a grid pattern in order to cover the whole test area. The simulated cameras have been selected as Wild P32 which has an image format 65x90 mm (picture format 60x80 mm) and focal length of 64 mm. The mathematical test area is photographed from two and four different camera stations. The convergency is preferred to preservation of ratio of the base -distance in taking the photographs,since the convergency has great effect on the accuracy of the intersected coordinates of an object point (Kenefick, 1971; Hottier, 1976). Consequently the used parallactic angles and the exterior orientation elements are given in Table 1.

Table 1. Parameters used for simulation study

Comb. no	Station no	The Exterior Orientation Elements						Parallac. Angle
		X_m	Y_m	Z_m	Omega	phi	Kappa	
1	1	980	145	150	0^g	-20^g	0^g	40^g
	2	1130	145.5	152	0	+20	0	
2	1	975	151	175	0	-30	0	60
	2	1150	152	176	0	+30	0	
3	1	970	145	225	0	-40	0	80
	2	1130	145	226	0	+40	0	
4	1	825	183	150	0	-50	0	100
	2	1250	164	152	0	+50	0	
5	1	970	145	225	0	-40	0	80
	2	1130	145	226	0	+40	0	
	3	975	151	175	0	-30	0	60
	4	1150	152	176	0	+30	0	
6	1	980	145	150	0	-20	0	40
	2	1130	145.5	152	0	+20	0	
	3	900	151	200	0	-40	0	80
	4	1180	152	201	0	+40	0	
7	1	980	145	150	0	-30	0	60
	2	1130	145.5	152	0	+30	0	
	3	900	151	200	0	-40	0	80
	4	1180	152	201	0	+40	0	
8	1	850	175	200	0	-50	0	100
	2	1132	176	201	0	+50	0	
	3	825	183	150	0	-50	0	100
	4	1250	164	152	0	+50	0	
9		perturbed 2. combination						
10		perturbed 4. combination						
11		perturbed 8. combination						

After computation of the fictitious image coordinates, the lens distortion error was introduced to these coordinates. The average lens distortion coefficients are listed in Table 2. For this purpose, for some certain points, the lens distortion values were taken from the calibration data of a Wild P32 camera;, then the lens distortion curve was obtained as expressed by the following equation:

$$\Delta r = K_1 r + K_2 r^3 + K_3 r^5 + K_4 r^7$$

where Δr is the value of radial displacament of a point in a photograph due to the lens distortion. K_i are distortion coefficients. r is the radial distance from the principal point to the photographic image point. As no significant improvement in the accuracy was gained by adding the term K_4 (Karara, 1980) it was omitted in our computing the lens distortion coefficient. As the most significant distortion caused by the camera lens is generally symmetrical radial lens distortion (Wolf, 1974; Veress and Hatsopoulos, 1979), it was taken into account for computations only.

For the simulation study, first the exterior orientation elements of each camera namely the object space coordinates of the camera and the rotation elements omega, phi, kappa were assumed fixed. The object spaca coordinates of the points which are included in the test field, were then obtained by using the developed model. In this run, first two camera stations were used in order to intersect the points. The parallactic angles were chosen as 40, 60, 80 and 100 grads, corresponding to the combinations No. 1, 2, 3 and 4 respectively. Then four camera stations were employed to compute the geodesic coordinates of the points, given as combinations No. 5, 6, 7 and 8. The parallactic angles were also preferred as 80-60, 40-80, 60-80, 100-100 grads respectively.

In the second step, the object space coordinates of the ground points were obtained using perturbed exterior orientation elements of each camera, by employing equations (7), given with combinations No. 9, 10 and 11. The magnitudes of the perturbations were determined greater than Veress's specifications (Erlandson and Veress, 1974, 1975). In this case, the object space coordinates of the camera perspective centers were perturbed as $\pm$ 5 cm. and the orientation angles omega, phi and kappa were perturbed as $\pm$ 15^{cc} for the combinations 9, 10, 11 respectively. Still there was no decrease in the accuracy. The final results of all combinations are given in Table 3.

Table 2. Average lens distortion coefficients

K_1 = 0.31908724x10^{-3}
K_2 = -0.63047551x10^{-6}
K_3 = 0,24266095x10^{-9}

EVALUATION OF THE SIMULATION RESULTS

There are different ways of analysing the accuracy of the photogrammetric system. Since the geodesic coordinates of all points in the test area have been predetermined, these coordinates can be assumed free of errors. In this case, the quantities S_X, S_Y, S_Z which are computed by means of differences between the geodesic coordinates and the computed photogrammetric coordinates, represent the obtained precision much better. The mean square value of the differences S_X, S_Y, S_Z are expressed as follows:

$$S_X = \sqrt{\frac{V_{Xi} V_{Xi}}{n}} \qquad S_Y = \sqrt{\frac{V_{Yi} V_{Yi}}{n}} \qquad S_Z = \sqrt{\frac{V_{Zi} V_{Zi}}{n}}$$

where n is the number of the groundpoints used in analysing; V_{Xi}, V_{Yi}, V_{Zi} are differences as follows:

$$V_{Xi} = X_s - X_p$$

$$V_{Yi} = Y_s - Y_p$$

$$V_{Zi} = Z_s - Z_p$$

where s refers to the simulated geodesic coordinates and p refers to computed photogrammetric coordinates. The position error is also as follows:

$$S_p = \sqrt{S_X^2 + S_Y^2 + S_Z^2}$$

These quantities are given in Table 3, according to the obtained final results of the simulatinn study. The above values S_X, S_Y, S_Z and S_p are same the quantities RX, RY, RZ and RXYZ respectively, devised by Hottier (1976).

Table 3. The final results of all combinations

Combi. no	S_X mm	S_Y mm	S_Z mm	S_p mm
1	9.6	3.7	124.0	124.4
2	11.7	3.2	95.2	96.0
3	10.1	3.1	55.6	56.6
4	9.3	0.6	12.4	15.5
5	8.8	3.7	40.6	41.7
6	6.9	1.0	51.2	51.7
7	6.1	0.4	15.6	16.8
8	5.4	1.2	54.0	54.3
9	52.4	8.1	50.7	73.2
10	10.8	2.3	18.9	21.9
11	10.3	2.7	58.9	59.9

As it can be seen from the Table 3, according to the obtained final results the mimimum position error is proveded by the combinatien No. 4 and 7, due to the maximum parallactic angle in these combinations. This is a normal results since for the special convergent case where the camera axes are directed towards the mean plane containing the targets, the optimum configuration is attained when the convergence is equal to 50 grads (Marzan, 1975). The combination No. 5 and 10 also provide good results, as we have used four stations and greater parallactic angle relatively in order to intersect the points. Although the exterior orientation elements of combination No. 10 were perturbed, still the results are very good. However, all combinations provide very good results for the X and Y coordinates while only some combinations provide slightly greater differences in the Z direction. This is due to two reasons: in the combinations where differences in Z coordinates are great either the parallactic angle is small or the exposure distance is somewhat greater. The increasing the number of station does not change appreciably the accuracy in Z coordinates, namely the incraesing the number of plates from two to 3, or more.

Although the object distance influences the accuracy of the results, the convergence has greater effect than the object distance on the accuracy, especially on the Z direction. This case is seen from the combinations No. 1 and No. 4 in Table 3. Both of the combinations have the same object distance but different parallactic angle.

A conclusion from the above analysis that the chosen geometric configuration, namely parallactic angle and the object distance affect the accuracy of the results. According to the results of the simulation study, the parallactic angle has the maximum effect on the accuracy. The taking distance and the number of station have less effect than the parallactic angle.

In order to compare the other methods with the developed model in this study, the object space coordinates of the test points were computed by means of the vector method by developed Veress (1978, 1979). Data of the combinations of 1, 2, 3 and 4 were used in these computations without changing. The obtained results are given in Table 4.

Table 4. The results of the Vector Method

Combi. no.	S_X mm	S_Y mm	S_Z mm	S_p mm	Developed Model S_p mm
1	9.0	3.8	126.7	127.1	124.4
2	13.4	4.6	88.6	89.6	96.0
3	9.0	3.1	58.2	59.0	56.6
4	5.0	0.6	16.0	16,8	15.5

CONCLUSIONS

Two basic principles on which the analytical photogrammetry depends, are the collinearity and the coplanarity conditions. The optimum condition (Veress and Hatzopoulos, 1979) is also the collinearity condition. Generally, this mentioned condition is preferred for computation of the object space coordinates by means of the space intersection. Since these condition equations are non-linear, they have been commonly linearized via traditional serial expansion. Because of the large numbers of intermediate treatment the computation time is long as well (see, Bopp and Krauss, 1978). Since the computation is done applying approximate values to the unknowns quantities in serial expansion, iteration is required. In this case the number of iterations vary according to the appopriacy of the applied approximate values (Wolf, 1974).

In this study, the collinearity equations were linearized without using the conventional serial expansion. This developed model has the following properties:

1. The number of the intermediate steps are minimised during the performance.

2. Applicabilty in the programming of the mathematical expressions is straightforward and the solution time is short.

3. This method can be used in space intersection and resection.

4. This model does not require any iteration.

5. Asimple way was used for the calculation of the presolutions of the object space coordinates of the points.

6. The precision of the calculated approximate coordinates of the points does not effect the accuracy of the results.

7. Under all these conditions, the accuracy of the results does not decrease.

ACKNOWLEGMENT

Sincere gratitude is expressed to Professor M.Aytaç, Technical University of Istanbul, help and encouragement throughout this work.

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