

# LEAST SQUARE MATCHING MODEL AMMGC-LSM FOR MULTI-LINE-ARRAY DIGITAL IMAGES

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## ABSTRACT:

Based on the investigation and comparing of available multi-image matching algorithms, a fresh multi-image matching algorithm model, AMMGC-LSM, which is appropriate for multi-line-array digital images, is proposed in this paper. For the AMMGC algorithm, its purpose is providing good initial values for the multi-line-array least square image matching, which is more precise and can reach sub-pixel level. Experiments prove that AMMGC-LSM model is quite efficient and can be used for the multi-line-array images matching of feature points or grid points.

## 1. INTRODUCTION

### 1.1 General Instructions

Generally, Least Square Image Matching is recognized as a highly precise image matching method. Since the concepts of least square image matching method were firstly proposed by professor Ackerman, rapidly, the method has been extended to the matching of multiple images. Currently, most of the researches relating to least square image matching techniques are focused on traditional frame-based aerial photos. Rare time and effort is spent on the investigation and application of the least square image matching theories for multi-line-array digital images. Therefore, in order to optimize the image matching techniques, least square image matching theories for multi-line-array digital images have to be efficiently and effectively applied. And, the way to apply these theories is the researching focus of this paper. Based on the investigation and comparing of available multi-image matching algorithms, a high accurate multi-image matching algorithm model, AMMGC – LSM, which is appropriate for multi-line-array digital images, is proposed in this paper. In the AMMGC – LSM model, AMMGC is an adaptive multi-Image matching algorithm with geometric constraints, which is used to provide good initial values for the least square image matching, i.e. LSM, which is more precise and can reach sub-pixel level.

## 2. AMMGC MODEL

As is shown in Figure 1, consider three ADS40 digital images, which are obtained on the same flight line.  $I_0$ ,  $I_1$  and  $I_2$  are nadir, forward and backward image respectively, where  $I_0$  is selected as reference image, and  $I_i$  ( $i=1, 2$ ) are selected as searching images. The basic working process of AMMGC model can be described as follows.

- (1) Define or extract points  $p_i$  ( $i=0, 1, 2, 3, \dots$ ) to be matched on the reference image.
- (2) For each point  $p_i$ , determine its approximate height  $Z_i$  and height error  $\Delta Z_i$ .  $Z_i$  and  $\Delta Z_i$  can be predefined by users, or obtained by rough matching of high layer image pyramids or initial DSM. The quasi-epipolar lines of  $p_i$  on the searching images are determined by known exterior parameters and line fitting methods. Define an image window  $W$  around  $p_i$ , and  $W$  is named as correlation window.
- (3) On one searching image's quasi-epipolar line, select a searching window (the same size as the correlation window) around each point of the line, compute the correlation

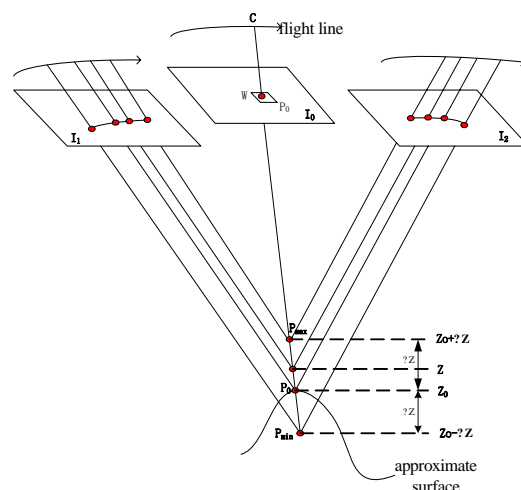


Figure 1. Illustration of AMMGC model

coefficient between the searching window and correlation window, and find the local max correlation coefficients.

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(4) Check the local max correlation coefficients by another searching image to obtain one or zero candidate matching result (matching pixels).

(5) Compute the ground coordinates (Xi, Yi, Zi) of ground point Pi by forward intersection of the matching results.

### 3. PRECISE MODEL OF MULTIPLE IMAGES MATCHING

If we have obtained the image point in the reference image and its corresponding image point in searching image by AMMGC method, now how to use the least square matching of multiple images method to improve the results of initial mathching should be discussed in detail.

#### 3.1 Gray observation equation

Suppose that is an image patch (a rectangle or a square generally) whose center is in the reference image, is an image patch whose center is in the searching image and is the pixel coordinate. The reference image patch can be considered an observation of the searching image patch in the least squares adjustment. So the following observation equation can be established.

$$v_i(x, y) = T_r(g_i(T_g(x, y))) - g_0(x, y) \quad (i=1, 2, \dots, n) \quad (1)$$

where  $g_0$  = gray values of the reference image patch  
 $g_i$  = gray values of the corresponding serching image patch  
 $(x, y)$  = pixe coordinates of image point  
 $vi(x, y)$  = true error function  
 $T_r$  = radiation distortion of image  
 $T_g$  = geometic distortion of image

The radiation distortion can be solved when calculating the correlative coefficient, therefore the gray distortion between the reference image and the serching images do not be considered here. The equation (1) can be given by

$$v_i(x, y) = g_i(T_g(x, y)) - g_0(x, y) \quad (2)$$

The above equation is the least squares gray observation equation. Linearize the equation with regard to the pixel coordinate (x, y) and ignore the high-order terms, according to

$$v_i(x, y) = g_i(T_g(x_0, y_0)) + \frac{\partial g_i}{\partial x_i} \Delta x_i + \frac{\partial g_i}{\partial y_i} \Delta y_i - g_0(x, y) \quad (3)$$

Because of the very small field-of-view angle of an image patch, an affine transformation is considered to be satisfactory to model the geometic transformation between the grey values, with a reasonable assumption of a locally planar object surface.

$$T_g = \begin{cases} x_i = u_i + u_{xxi}x_0 + u_{xyi}y_0 \\ y_i = v_i + v_{yxi}x_0 + v_{yyi}y_0 \end{cases} \quad (i = 1, 2, \dots, n) \quad (4)$$

The first order differentials of equation (4) can be expressed as

$$\begin{cases} \Delta x_i = \Delta u_i + x_0 \Delta u_{xxi} + y_0 \Delta u_{xyi} \\ \Delta y_i = \Delta v_i + x_0 \Delta u_{yxi} + y_0 \Delta u_{yyi} \end{cases} \quad (5)$$

with the simplified notation:

$$g_x^i = \frac{\partial g_i}{\partial x_i}; g_y^i = \frac{\partial g_i}{\partial y_i} \quad (6)$$

Combining equation (3) and equation (5) we obtain

$$v_i(x, y) = g_i(T_g(x_0, y_0)) + g_x^i \Delta u_i + g_x^i x_0 \Delta u_{xxi} + g_x^i y_0 \Delta u_{xyi} + g_y^i \Delta v_i + g_y^i x_0 \Delta v_{yxi} + g_y^i y_0 \Delta u_{yyi} - g_0(x, y) \quad (7)$$

With the notations

$$\begin{cases} X_i^T = [\Delta u_i, \Delta u_{xxi}, \Delta u_{xyi}, \Delta v_i, \Delta v_{yxi}, \Delta v_{yyi}] \\ l_i = g_0(x, y) - g_i(T_g(x_0, y_0)) \\ A_i = [g_x^i, g_x^i x_0, g_x^i y_0, g_y^i, g_y^i x_0, g_y^i y_0] \end{cases} \quad (i = 1, 2, \dots, n)$$

$$X^T = [X_1^T, X_2^T, \dots, X_n^T]; \quad l = [l_1, l_2, \dots, l_n]; \quad A = \begin{bmatrix} A_1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & A_n \end{bmatrix} \quad (8)$$

So the gray observation equation (1) can be expressed as

$$v(x, y) = AX - l; P \quad (\text{the weight coefficient matrix of } l) \quad (9)$$

#### 3.2 The geometrical observation equations

The geometrical constraints have been proved that it can improve the accuracy and reliability of the matchings. For the multiple images matching system composed by frame photographs, the collinearity condition equations is usually used as the geometrical constraints. But for multi-image least square matching of multi-line-array images, the collinear equation constraints involves huge computation amount, which mainly results by the imaging modles of sensor. Many experiments have proved that the difference of two constraints on the orientation accuracy is little, but the computation that use the

quasi-epipolar line as the combined adjustment system of the geometrical observation equations is less than that use the collinear equation, so that quasi-epipolar line is used as constraint factor and maked up of the geometrical observation equations in this paper.

As is shown in Figure 1, the corresponding point  $p_i, i = 1, 2, \dots, n (n \geq 2)$  in searching image must locate at their corresponding quasi-epipolar line, therefore these quasi-epipolar lines can be used as geometrical constraints and form corresponding geometrical observation equations to join the adjustment. Generally, the quasi-epipolar line can be expressed by the formation of polynomials

$$y_i = f(x_i) = a_{im}x_i^m + \dots + a_{i1}x_i + a_{i0} \quad (10)$$

For the L0 level images of the linear array sensor, the quasi-epipolar lines are not the straight lines generally. But for the L1 level images, the quasi-epipolar line can be simulated by a straight line. Then Equation (10) becomes

$$y_i = f(x_i) = a_{i1}x_i + a_{i0} \quad (11)$$

If we have get the approximate pixel coordinate  $(x_i^0, y_i^0)$  of the image point by AMMGC, the geometrical observation equations can be expressed by

$$v(x, y) = \Delta y_i - a_{i1}\Delta x_i + (y_i^0 - f(x_i^0)) \quad (12)$$

With the notations

$$\begin{cases} X_i^T = [\Delta x_i, \Delta y_i] \\ l_{gi} = -y_i^0 + f(x_i^0) \quad (i = 1, 2, \dots, n) \\ B_i = [-a_{i1}, 1] \end{cases}$$

$$X = [X_1^T, X_2^T, \dots, X_n^T]^T; \quad l_g = [l_{g1}, l_{g2}, \dots, l_{gn}]^T; \quad B = \begin{bmatrix} B_1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & B_n \end{bmatrix} \quad (13)$$

Here  $(\Delta x_i, \Delta y_i)$  is corresponding to  $(\Delta u_i, \Delta v_i)$  gotten in the gray observation equations, so the geometrical observation equations can be given by

$$v(x, y) = BX - l_g; P_g \text{ (the weight coefficient matrix of } l_g \text{)} \quad (14)$$

Gray observation equations (9) and geometrical observation equations (14) compose a combined adjustment system, which are associated with each other via the common conversion parameter  $(u_i, v_i)$ . The least square solution of combined adjustment is

$$\hat{X} = (A^T P A + B^T P_g B)^{-1} (A^T P l + B^T P_g l_g) \quad (15)$$

The answer of equation (15) is the final matching points coordinates, which can reach the 1/10 pixel matching accuracy in theory.

#### 4. EXPERIMENTS AND CONCLUSION

Three ADS40 three-line-array digital images were used in this paper. These images were obtained form the same flight strip. The main attributes of the images are shown in Table 1.

Camera	Level	Location	Focus	GSD	Flight height
ADS40	1	Lintong, China	62.7cm	0.48m	1000m

Table 1. Main attributes of experiment data

In the matching process, we use AMMGC—LSM model to match three images, where nadir image is used as reference image and backward and forward images are used as searching images. Some experiments result are shown as below Figure 2 and Figure 3.

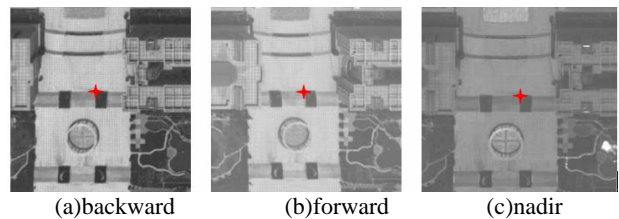


Figure2-1. Initial matching points provided by AMMGC

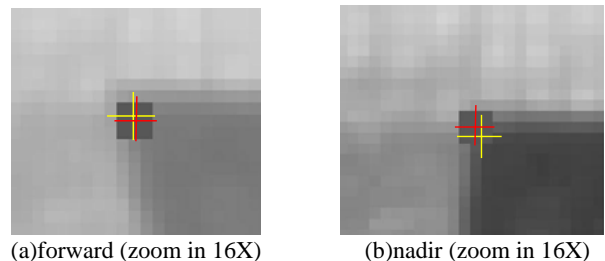


Figure2-2. Final Matching results provided by LSM (small red crosses are initial points and big yellow ones are final matching points)

Figure2. Matching results by AMMGC-LSM model

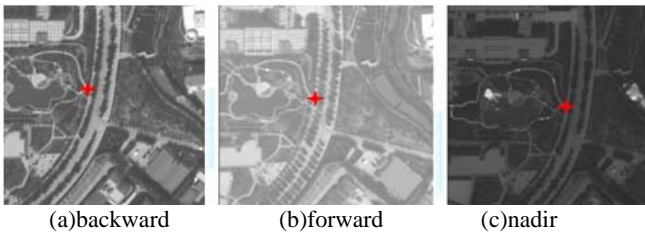


Figure3-1. Initial matching points provided by AMMGC

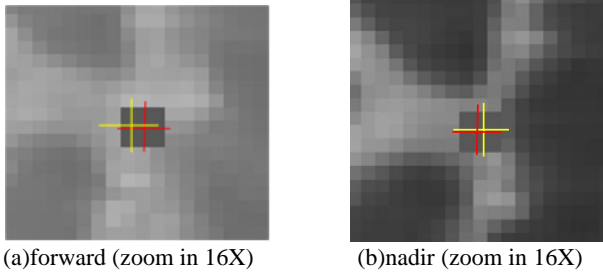


Figure3-2. Final Matching results provided by LSM (small red crosses are initial points and big yellow ones are final matching points)

Figure3. Matching results by AMMGC-LSM model

From many experiments we found that the AMMGC-LSM matching model can reach 1/10 pixel matching level for multi-line-array digital images when good initial values can be provided by AMMGC model. The weight of quasi-epipolar line constraints has to be changed reasonably. High weight has to be applied in the first a few AMMGC-LSM iterations to insure convergence, then the weight is gradually decreased to enable that, out of the quasi-epipolar lines, correct and sub-pixel-level matching results can be effectively found. Experiments prove that the model is quite efficient and can be efficiently used for the multiple-image matching of feature points and image grid points.

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