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LINEARIZATION BY COMPLETELY GENERALIZED INPUT-OUTPUT INJECTION[1]

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The problem addressed in this paper is the linearization of nonlinear systems by generalized input-output (I/O) injection. The I/O injection (called *completely generalized I/O injection*) depends on a finite number of time derivatives of input and output functions. The practical goal is the observer synthesis with linear error dynamics. The method is based on the I/O differential equation structure. Thus, the problem is solved as a realization one. A necessary and sufficient condition is proposed through a constructive algorithm and is based on the exterior differentiation.

1. INTRODUCTION

The problem addressed in this paper is the linearization of a nonlinear system by a generalized state coordinate transformation (cf. [5]), and *completely generalized I/O injection* (i. e. function of a finite number of input and output time derivatives, cf. [6, 15]). Its solution plays a key role in the synthesis of nonlinear observers [1, 2, 8, 16, 17]. The final goal is to build an observer, which has exact linear error dynamics, converges and is stable.

The linearization by I/O injection has been mainly tackled with geometric tools [9, 11, 12, 18] and algebraic tools [6, 7, 10, 13], and used in also some practical applications [3, 14, 17]. Since about ten years ago, and specially in [17], time derivatives are used in the observer synthesis for bilinear systems with an application to biological systems. In [8], it is stated as a problem of resolution of partial differential equations and solved only for 2 and 3 dimensional systems. In [16], only first order time derivatives are dealt with an algebraic method while [15] considers only input time derivatives. This paper is motivated by some recent results, where it is used numerical differentiation for observer synthesis (cf. [4]). Since the observability property assumption asked in [4], numerical differentiation is used to compute the necessary time derivatives of inputs and outputs, the state being derived with a static map. The main shortcoming is the high sensitivity to measurement noise (especially whether the derivatives are computed within a short sampling period).

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In this paper, necessary and sufficient conditions (NSC) are given for the linearization of MIMO nonlinear systems by a generalized state coordinate transformation and *completely generalized I/O injection*. The fully constructive conditions of the existence of a solution are stated in terms of exterior differential systems. The method is based on the study of the structure of the I/O differential equations, and then the problem stated as a realization problem. Our practical goal is to build a Luenberger-like observer, which has stable linear error dynamics.

This frame has been already used [6, 15]. In [6], a NSC is given for linearization by state coordinate transformation and I/O injection. In [15], linearization by a generalized I/O injection with only input time derivatives for MIMO systems is studied. This paper is a generalization of these results. The main problem for the generalization to MIMO case is that I/O differential equations associated to the output functions could be linearly dependent. The characterization of these output functions plays a key role in the solution of MIMO case.

2. PROBLEM STATEMENT

Let us consider the nonlinear system

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in I\!R^n$ is the state, $u \in I\!R^m$ is the input, $y \in I\!R^p$ is the output; f and h are meromorphic functions of their arguments.

In the sequel, nonlinear systems considered here are supposed to be *generically observable* [15] and will be called *observable*.

Example. The following nonlinear system

$$\begin{aligned} &\dot{x}_1 = x_2^2 u, \\ &\dot{x}_2 = f(x,u), \quad y = x_1, \end{aligned} \tag{2}$$

is observable (generically) with a *singular* set in $(x_2 = 0,\ u = 0)$.

Moreover, the k order time derivative of y (resp. u) is denoted $y^{(k)}$ (resp. $u^{(k)}$). The system (1) is supposed to be under its I/O representation. Denoting k_i the observability index of the output y_i (cf. [9]). One gets a system of p I/O differential equations given by $(1 \le i \le p)$

$$y_i^{(k_i)} = P_i(y_1, \cdots, y_1^{(k_1-1)}, \cdots, y_p, \cdots, y_p^{(k_p-1)}, \bar{u}). \tag{3}$$

where $\bar{u} := \left(u, u^{(1)}, \cdots, u^{(k_1-1)}\right)$ and $\sum_{i=1}^{p} k_i = n$ with $k_1 \ge k_2 \ge \cdots \ge k_p$.

The problem can be stated as a realization one and it consists in testing if the nonlinear system (1) is locally equivalent to a linear system up to a completely generalized I/O injection. The former system is assumed to be composed by p

blocks as follows ($1 \leq i \leq p$):

$$
\begin{array}{rcl}
\dot{\zeta}_{i1} & = & \zeta_{i2} \\
\dot{\zeta}_{i2} & = & \zeta_{i3} \\
& \cdots & \\
\dot{\zeta}_{is_i} & = & \zeta_{is_i+1} \\
\dot{\zeta}_{is_i+1} & = & \zeta_{is_i+2} + \varphi_{is_i+1}(\bar{y}^{(0)}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)}) \\
\dot{\zeta}_{is_i+2} & = & \zeta_{is_i+3} + \varphi_{is_i+2}(\bar{y}^{(0)}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)}) \\
& \cdots & \\
\dot{\zeta}_{ik_i-1} & = & \zeta_{ik_i} + \varphi_{ik_i-1}(\bar{y}^{(0)}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)}) \\
\dot{\zeta}_{ik_i} & = & \varphi_{ik_i}(\bar{y}^{(0)}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)}) \\
y_i & = & \zeta_{i1}
\end{array}
\tag{4}
$$

where:

- s_i is the higher time derivative order of the outputs in the generalized I/O injection terms,
- q_i is the higher time derivative order of the input in the generalized I/O injection terms,
- $\bar{y}^{(r)}$ is composed by the r-order time derivatives of outputs, which have an observability index greater than $r-1$.

Remark 1.

- Obviously $k_i > s_i$.
- The I/O differential equation associated to each block (4) can be written as follows:

$$y_i^{(k_i)} = \sum_{j=s_i+1}^{k_i} \varphi_{ij}^{(k_i-j)}. \tag{5}$$

The synthesis of an observer with linear error dynamics for (4) is then an easy task. For

$$\dot{\zeta} = A\zeta + \varphi\left(\bar{y}, \bar{y}^{(1)}, \cdots, \bar{y}^{(s)}, u, u^{(1)}, \cdots, u^{(q)}\right) \tag{6}$$

where A and C are dual of Brunovsky form, an observer closed to the Luenberger one exists

$$\dot{\tilde{\zeta}} = A\tilde{\zeta} + \varphi\left(\bar{y}, \bar{y}^{(1)}, \cdots, \bar{y}^{(s)}, u, u^{(1)}, \cdots, u^{(q)}\right) + LC(\tilde{\zeta} - \zeta). \tag{7}$$

The choice of the eigenvalues of $(A+LC)$ allows to have an arbitrarily fast estimation error decay.

Preliminaries

The method described in this paper is based on a structural study of the system I/O differential equations. The next Lemma is helpful to verify the integrability and independency of some I/O functions in order to proof the main result. This Lemma is based on the Poincaré's Lemma.

Definition 1. Let us use the variables $a \in I\!R^{\lambda}$ (resp. $b \in I\!R^{\rho}$) where $a_1, \cdots, a_\lambda$ (resp. $b_1, \cdots, b_\rho$) are linearly independent. Moreover, let us define $K(a,b)$ as the set of meromorphic functions.

Lemma 1. (Poincaré) The differential form $\omega \in \mathrm{Span}_{K(a,b)}\,\{\mathrm{d}a_1, \cdots, \mathrm{d}a_\lambda, \mathrm{d}b_1, \cdots, \mathrm{d}b_\rho\}$ ($a \in I\!R^{\lambda}$ and $b \in I\!R^{\rho}$) is locally exact if and only if, $\mathrm{d}\omega = 0$.

A modified version of this Lemma follows

Lemma 2. Let us consider a differential form $\omega \in \mathrm{Span}_{K(a,b)}\,\{\mathrm{d}a_1, \cdots, \mathrm{d}a_\lambda\}$ ($a \in I\!R^{\lambda}$ and $b \in I\!R^{\rho}$). There exists locally a function $\eta(a,b)$ such that

$$\sum_{i=1}^{\lambda} \frac{\partial \eta}{\partial a_i} \cdot \mathrm{d}a_i = \omega,$$

if and only if $\mathrm{d}\omega \wedge \mathrm{d}b_1 \wedge \cdots \wedge \mathrm{d}b_\rho = 0$.

Remark. From now on, take the set of meromorphic functions $K(a,b)$ as $\mathcal{K}(x, (u, \dot{u}, \cdots, u^{(w)}))$.

3. MAIN RESULT

3.1. Preliminary example

Let us consider the system

$$\begin{aligned}
\dot{x}_1 &= x_1 - x_2, \quad y_1 = x_1, \\
\dot{x}_2 &= -x_3 + (x_2 - x_1) \cdot x_4 - (x_1 - x_2)^2, \\
\dot{x}_3 &= -x_1 + x_2 - x_3 + (x_1 - x_2 - 2)\,(x_1 - x_2)^2, \\
\dot{x}_4 &= x_1, \quad y_2 = x_4.
\end{aligned} \tag{8}$$

The output y_1 (resp. y_2) has an observability index k_1 (resp. k_2) equal to 3 (resp. 1). The I/O differential equations are described by

$$\begin{aligned}
y_1^{(3)} &= y_1^{(2)} \cdot \left(y_1 + 2y_1^{(1)}\right) + y_1^{(1)} \cdot \left(y_2 + y_1^{(1)^2}\right), \\
y_2^{(1)} &= y_1.
\end{aligned} \tag{9}$$

By using [15], it is proved that it does not exist a state transformation such that system (8) is locally equivalent to a linear system modulo an output injection (without time derivatives of output). Consider now the following system in the particular form (4)

$$\begin{aligned}
\dot{\zeta}_{11} &= \zeta_{12}, \qquad y_1 = \zeta_{11}, \\
\dot{\zeta}_{12} &= \zeta_{13} + \varphi_{12}\left(y_1, y_1^{(1)}, y_2\right), \\
\dot{\zeta}_{13} &= \varphi_{13}\left(y_1, y_1^{(1)}, y_2\right), \\
\dot{\zeta}_{21} &= \varphi_{21}(y_1, y_2), \qquad y_2 = \zeta_{21},
\end{aligned} \tag{10}$$

with $s_1 = 1(< k_1)$ and $s_2 = 0(< k_2)$. If system (8) is locally equivalent to (10), then equations (9) have to have the form (5)

$$y_1^{(3)} = \varphi_{12}^{(1)} + \varphi_{13}, \qquad y_2^{(1)} = \varphi_{21},$$

Then, the functions φ_{12}, φ_{13}, φ_{21} have to verify

$$\begin{aligned}
\frac{\partial \varphi_{12}}{\partial y_1^{(1)}} y_1^{(2)} + \frac{\partial \varphi_{12}}{\partial y_1} y_1^{(1)} + \frac{\varphi_{12}}{\partial y_2} y_2^{(1)} + \varphi_{13} &= y_1^{(2)}\left(y_1 + 2y_1^{(1)}\right) + y_1^{(1)}(y_2 + y_1^{(1)^2}\right) \\
\varphi_{21} &= y_1.
\end{aligned}$$

Note that these two equations are not independent: the first equation depends on $y_2^{(1)}$. Then, the differential equation $y_1^{(3)}$ is a function of $(y_1, \dot{y}_1, y_1^{(2)}, y_2, \dot{y}_2)$ but $\dot{y}_2$ is a known function given by the second equation of (9). $y_1^{(3)}$ is then a function only of $(y_1, \dot{y}_1, y_1^{(2)}, y_2)$. A solution is

$$\begin{aligned}
\varphi_{12} &= y_1^{(1)} \cdot \left(y_1^{(1)} + y_1\right), \qquad \varphi_{13} = y_1^{(1)} \cdot \left(y_1^{(1)^2} - y_1^{(1)} + y_2\right), \\
\varphi_{21} &= y_1.
\end{aligned}$$

3.2. Necessary and sufficient condition

The main result is obtained using the exterior differential system theory, and gives the linearizing generalized state coordinate transformation, whether it exists. Nonlinear system (1) is supposed to be observable and previously transformed in the p I/O differential equations (after state elimination).

G.I.O.I.d. Algorithm

(A1) For $i := 1$ to p, set $\varphi_{is_i} := 0$ and (from (3)) $P_i^{s_i} := P_i$.

(A2) $s_i := 0$ (to be increased up to $k_i - 1$ if necessary)

(A3) $q_i := 0$ (to be increased up to $k_i - 1$ if necessary)

(A4) For $k := s_i + 1$ to k_i, set

$$P_i^k := P_i^{k-1} - [\varphi_{ik-1}]^{(k_i - k + 1)}. \tag{11}$$

Let d_i^k (resp. p_{s_i}) denote the number of outputs whose the observability index is greater than $(k_i - k + s_i)$ (resp. $s_i - 1$). The differential form ω_i^k is defined as (with $\wedge$ as the exterior product)

$$\omega_i^k := \sum_{j=1}^{d_i^k} \frac{\partial P_i^k}{\partial y_j^{(k_i-k+s_i)}} \mathrm{d}y_j^{(s_i)} + \sum_{j=1}^{m} \frac{\partial P_i^k}{\partial u_j^{(k_i-k+q_i)}} \mathrm{d}u_j^{(q_i)} \tag{12}$$

and

$$\wedge \mathrm{d}y^{[s_i]} := \begin{cases} \wedge \mathrm{d}\bar{y} \wedge \cdots \wedge \mathrm{d}\bar{y}^{(s_i-1)} \wedge \mathrm{d}y_{d_i^k+1}^{(s_i)} \wedge \cdots \wedge \mathrm{d}y_{p_{s_i}}^{(s_i)} \text{ for } d_i^k < p_{s_i} \\ \wedge \mathrm{d}\bar{y} \wedge \mathrm{d}\bar{y}^{(1)} \wedge \cdots \wedge \mathrm{d}\bar{y}^{(s_i-1)} \wedge 1, \text{ for } \ d_i^k = p_{s_i}. \end{cases} \tag{13}$$

$$\wedge \mathrm{d}u^{[q_i]} := \begin{cases} \wedge\ 1 \ \text{ for } \ q_i = 0, \\ \wedge \mathrm{d}u \wedge \mathrm{d}\dot{u} \wedge \cdots \wedge \mathrm{d}u^{(q_i-1)}, \ \text{ otherwise.} \end{cases} \tag{14}$$

– If $\mathrm{d}\omega_i^k \wedge \mathrm{d}y^{[s_i]} \wedge \mathrm{d}u^{[q_i]} = 0$, the function φ_{ik} is a solution of

$$\begin{aligned} &\sum_{j=1}^{d_i^k} \frac{\partial \varphi_{ik}}{\partial y_j^{(s_i)}} \mathrm{d}y_j^{(s_i)} + \sum_{j=1}^{m} \frac{\partial \varphi_{ik}}{\partial u_j^{(q_i)}} \mathrm{d}u_j^{(q_i)} = \omega_i^k, \qquad \text{for} \qquad k < k_i, \\ &\varphi_{ik_i} = P_i^{k_i}, \qquad\qquad\qquad\qquad\qquad\qquad \text{for} \qquad k = k_i. \text{ (last step).} \end{aligned} \tag{15}$$

Return to A4.

– If $\mathrm{d}\omega_i^k \wedge \mathrm{d}y^{[s_i]} \wedge \mathrm{d}u^{[q_i]} \neq 0$, the algorithm stops. System described by the I/O differential equation (3) is not linearizable for both values s_i and q_i. Return (whether $k < k_i$, otherwise algorithm stops) to A3, A2 successively.

A necessary and sufficient condition for the existence of the linearizing transformation $\zeta = \phi(x, u, \dot{u}, \cdots, u^{(q-1)})$ is given by the following Theorem.

Theorem 1. Nonlinear system (1) described by (3) is locally equivalent to (4) if and only if

$$\mathrm{d}\omega_i^k \wedge \mathrm{d}y^{[s_i]} \wedge \mathrm{d}u^{[q_i]} = 0, \tag{16}$$

where $1 \le i \le p$, $s_i + 1 \le k \le k_i$ and $\omega_i^k, \wedge \mathrm{d}y^{[s_i]}$, and $\wedge \mathrm{d}u^{[q_i]}$ as in the former algorithm.

Whether conditions of Theorem 1 hold, the generalized state transformation $\zeta = \phi(x, u, \cdots, u^{(q-1)})$ steers system (1) into system (4). This transformation can be obtained for each block associated to the output y_i, $(1 \le i \le p)$, from system (4) as follows

$$\begin{aligned}
&\zeta_{i1} = h_i(x),\ \zeta_{i2} = h_i^{(1)},\ \cdots \\
&\zeta_{is_i+1} = h_i^{(s_i)} \\
&\zeta_{is_i+2} = h_i^{(s_i+1)} - \varphi_{is_i+1}(\bar{y}^{(0)}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)}) \\
&\zeta_{is_i+3} = h_i^{(s_i+2)} - \varphi_{is_i+1}^{(1)}(\cdot) - \varphi_{is_i+2}(\cdot) \\
&\cdots \\
&\zeta_{ik_i} = h_i^{(k_i-1)} - \varphi_{is_i+1}^{(k_i-(s_i+1))} - \varphi_{is_i+2}^{(k_i-(s_i+2))} - \cdots - \varphi_{ik_i}.
\end{aligned} \tag{17}$$

Remark 3. Theorem 1 generalizes the results of [15]. In order to find this former result, consider $s_i = 0$ in (13) (i.e. no output time derivatives are allowed in the I/O injection). Then $p_{s_i} := p$ (the number of outputs) and equation (13) becomes

$$\wedge \mathrm{d}y := \begin{cases} \wedge \mathrm{d}y_{d_i^k+1} \wedge \cdots \wedge \mathrm{d}y_p, & \text{for } d_i^k < p, \\ \wedge 1, \text{ for } d_i^k = p. \end{cases} \tag{18}$$

as in [15].

Proof of Theorem 1.

Sufficiency. Suppose that condition of Theorem 1 is verified. Then, there exists a function such that

$$\begin{aligned}
&\sum_{j=1}^{d_i^k} \frac{\partial \varphi_{ik}}{\partial y_j^{(s_i)}} \mathrm{d}y_j^{(s_i)} + \sum_{j=1}^{m} \frac{\partial \varphi_{ik}}{\partial u_j^{(q_i)}} \mathrm{d}u_j^{(q_i)} = \omega_i^k, \qquad \text{for} \qquad k < k_i, \\
&\varphi_{ik_i} = P_i^{k_i}, \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for} \qquad k = k_i.
\end{aligned} \tag{19}$$

It is then possible at the end of the algorithm, to derive from (17), the generalized state diffeomorphism, which transforms (1) into (4).

At each step, one gets $\varphi_{ik}(\bar{y}, \cdots, \bar{y}^{(s_i)}, u, \cdots, u^{(q_i)})$ for each block associated to an output variable of system (1) and from (17), dynamics of state variables of system (4) are known. From (5) one has that the $(k_i - s_i)$th dynamic depends on the last $(k_i - s_i - 1)$th's one. Thus, the whole coordinate transformation is well characterized. System (4) is then fully known: system (1) is then locally equivalent to the system (4) by a generalized state coordinate transformation (17). Sufficiency of Theorem 1 is proved.

Necessity. Suppose that the generalized state coordinate transformation (17), which transforms (1) into (4), exists. Then the equation (5) is verified to both systems and for all the y_i functions. Applying the G.I.O.I.d. Algorithm, one gets:

Suppose that $i = 1$ and $k = s_1 + 1$, $P_1^{s_1} := y_1^{(k_1)}$, $\varphi_{1s_1} := 0$

$$P_1^{s_1} = \varphi_{11}^{(k_1-s_1-1)} + \varphi_{12}^{(k_1-s_1-2)} + \cdots + \varphi_{1k_1-s_1}$$

Only the $(k_1 - 1)$th time derivatives of output functions that have an observability index larger to $(k_i - (s_1+1) + s_1) := (k_1 - 1)$ are independent of the lower-order time derivatives, because the other time derivatives of output functions can be written as function on both I/O functions and their time derivatives with a smaller degree (see equation (3)). Note that all the generalized I/O functions of degree $(k_1 - 1)$ are obtained from the $\varphi_{11}^{(k_1-s_1-1)}$ function. In $d_1^{s_1+1}$, one gets then the number of outputs that have an observability index greater than $(k_1 - (s_1 + 1) + s_1) = (k_1 - 1)$. Since the output function time derivative independency, and Lemma 2, there exists locally a function φ_{1s_1+1} such that a differential form (ω) can be written as follows

$$\omega_1^k = \sum_{j=1}^{d_1^{s_1+1}} \frac{\partial\varphi_{1s_1+1}}{\partial y_j^{(s_1)}} \, \mathrm{d}y_j^{(s_1)} + \sum_{j=1}^{m} \frac{\partial\varphi_{1s_1+1}}{\partial u_j^{(q_1)}} \, \mathrm{d}u_j^{(q_1)}.$$

Thus, (16) trivially holds for $k = s_1 + 1$. The next steps follow the same lines for $k = s_1 + 2$ to k_1. Necessity is then proved for the first step, and by the same way for the following steps. □

4. EXAMPLE

Consider system (8) and the output y_1 $(i = 1)$ (with $k_1 = 3$). *Set* $s_1 = 0$, $\varphi_{10} := 0$ and $P_1^0 := P_1$. First one checks if there is a solution without output time derivatives in the output injection in the block associated to y_1.

Step 0. $k = 1$. From (11), $P_1^1 := P_1^0 - [\varphi_{10}]^{(3)} = P_1^0$.
By definition d_1^1 (resp. $p_{s_1} := p_0$) is equal to 1 (resp. 2).
The differential ω_1^1 is derived from (12) as $\omega_1^1 := \left(y_1 + 2y_1^{(1)}\right) \mathrm{d}y_1$.
From (13), $\mathrm{d}\omega_1^1 \wedge \mathrm{d}y_2 \neq 0$. Theorem condition does not hold. Then, a state coordinate transformation steering system (8) into (10) with $s_1 = 0$ does not exist.

Set $s_1 = 1$, $P_1^1 := P_1$ and $\varphi_{11} := 0$.

Step 1. $k = 2$. From (11), $P_1^2 := P_1^1 - [\varphi_{11}]^{(2)} = P_1^1$.
By definition d_1^2 (resp. p_1) is equal to 1 (resp. 2).
From (12), one gets as $\omega_1^2 := \left(y_1 + 2y_1^{(1)}\right) \mathrm{d}y_1^{(1)}$.
From (13), $\mathrm{d}\omega_1^2 \wedge \mathrm{d}y_1 \wedge \mathrm{d}y_2 \wedge \mathrm{d}y_2^{(1)} = 0$, Theorem condition holds and from (15) a solution reads as

$$\varphi_{12} := y_1 \cdot y_1^{(1)} + y_1^{(1)2}.$$

Step 2. $k = 3$. From (11), $P_1^3 := P_1^2 - [\varphi_{12}]^{(1)} = y_1^{(1)} \cdot \left(y_1^{(1)2} - y_1^{(1)} + y_2\right)$.
By definition, one gets $\mathrm{d}_1^3 = 1$, $p_1 = 2$.
From (12), one gets $\omega_1^3 := \left(3y_1^{(1)^2} - 2y_1^{(1)} + y_2\right) \mathrm{d}y_1^{(1)}$.
From (13), $\mathrm{d}\omega_1^3 \wedge \mathrm{d}y_1 \wedge \mathrm{d}y_2 \wedge \mathrm{d}y_2^{(1)} = 0$, and then Theorem condition is satisfied. From (15) a solution reads as

$$\varphi_{13} := y_1^{(1)^3} + y_1^{(1)} \cdot y_2 - y_1^{(1)^2}.$$

The algorithm converges for the output y_1.

Applying the algorithm, in a similar way, a solution reads for the output y_2 as $\varphi_{21} = y_1$.

System (8) is then locally equivalent to:

$$\begin{aligned}
\dot{\zeta}_{11} &= \zeta_{12}, \quad y_1 = \zeta_{11}, \\
\dot{\zeta}_{12} &= \zeta_{13} + y_1^{(1)} \cdot \left(y_1^{(1)} + y_2\right), \\
\dot{\zeta}_{13} &= y_1^{(1)} \cdot \left(y_1^{(1)2} - y_1^{(1)} + y_2\right), \\
\dot{\zeta}_{21} &= y_1, \quad y_2 = \zeta_{21}.
\end{aligned} \tag{20}$$

where the state coordinate transformation defined by (17) follows:

$$\begin{aligned}
\zeta_{11} &= x_1, \\
\zeta_{12} &= x_1 - x_2, \\
\zeta_{13} &= x_1 - x_2 + x_3, \\
\zeta_{21} &= x_4.
\end{aligned} \tag{21}$$

5. CONCLUSIONS

A constructive Necessary and Sufficient Condition was obtained for the problem of linearization of nonlinear systems by generalized state coordinate transformation and generalized I/O injection for MIMO system case. The results are based on the computation of some differential one-forms and integrability conditions and is motivated by some recent results [4]. A practical goal of this result is the nonlinear observer synthesis with linear error dynamics, depending on both time derivatives of I/O functions if necessary.

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