KYBERNETIKA — VOLUME 38 (2002), NUMBER 5, PAGES 541–551

APPROXIMATION OF CONTROL LAWS WITH DISTRIBUTED DELAYS: A NECESSARY CONDITION FOR STABILITY

SABINE MONDIÉ, MICHEL DAMBRINE AND OMAR SANTOS

The implementation of control laws with distributed delays that assign the spectrum of unstable linear multivariable systems with delay in the input requires an approximation of the integral. A necessary condition for stability of the closed-loop system is shown to be the stability of the controller itself. An illustrative multivariable example is given.

1. INTRODUCTION

A wide class of systems meet in practice have delays in their inputs [7]. The control of such systems can be achieved through the use of control laws that include distributed delays and having the appealing feature, when no uncertainty is present, to lead to a closed-loop system whose assigned dynamic is described by a polynomial as in the non-delayed case [1, 8].

As explained in the work of Manitius and Olbrot [8], when the system is unstable the realization of the distributed delay by a differential difference equation must be discarded since it involves an unstable pole-zero cancelation as for the Smith Predictor. It is then suggested to realize directly the control law by a numerical computation of the integral terms at each time instant.

However, experimental results on a simple example presented by Van Assche et al [15], further examined in Engelsborghs et al [4] and the analysis of the resulting closed-loop quasipolynomial [13] show that, when the integral term is approximated by a quadrature method such as the trapezoidal rule, the closed-loop system is unstable when the precision of the approximation is sufficiently high. It was shown by Mondié and Santos [10] that, in the monovariable case, closed-loop stability can be achieved only if the ideal controller is stable.

The problem is stated in a multivariable framework in Section 2, and the stability of the ideal control law is shown to be a necessary condition for the stability of the scheme when the controller is approximated in Section 3. An illustrative multivariable example is presented in Section 4 and Section 5 is devoted to some comments and concluding remarks. Technical results are collected in the Appendix.

2. PROBLEM STATEMENT

Consider a linear multivariable system with delay in the input described by

$$\dot{x}(t) = Ax(t) + Bu(t-h) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ and $h \geq 0$ is the delay. According to the proposal of Manitius and Olbrot [8], consider the control law described by the following Volterra integral equation of the second kind

$$u(t) = K\left[x(t) + e^{-hA}\int_{-h}^{0} e^{-\theta A}Bu(t+\theta)\mathrm{d}\theta\right]. \tag{2}$$

Notice that

$$Bu(t-h) = BK\left[x(t-h) + e^{-hA}\int_{-h}^{0} e^{-\theta A}Bu(t-h+\theta)\mathrm{d}\theta\right]$$

and that for $t > h$, $\theta \in [-h, 0]$

$$Bu(t-h+\theta) = \dot{x}(t+\theta) - Ax(t+\theta).$$

Therefore,

$$\dot{x}(t) - Ax(t) = BK\left[x(t-h) + e^{-hA}\int_{-h}^{0} e^{-\theta A}(\dot{x}(t+\theta) - Ax(t+\theta))\mathrm{d}\theta\right]$$

and the closed-loop system is described by the functional differential equation of neutral type

$$\dot{x}(t) - BKe^{-hA}\mathcal{L}(\dot{x}_t) - Ax(t) - BKx(t-h) + BKAe^{-hA}\mathcal{L}(x_t) = 0,$$

where $\mathcal{L}$ is the operator defined as $\mathcal{L} : x_t \mapsto \int_{-h}^{0} e^{-\theta A}x(t+\theta)\,\mathrm{d}\theta$.

The characteristic equation of the closed-loop system is then

$$\det\Big(s\Big(I_n - BKe^{-hA}\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta\Big) - A - BKe^{-hs} + BKAe^{-hA}\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta\Big) = 0. \tag{3}$$

Using the equality

$$\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta = (sI-A)^{-1}\left[I - e^{-h(sI-A)}\right], \tag{4}$$

the expression (3) simplifies to

$$\det\left(sI_n - A - BKe^{-hA}\right) = 0.$$

Now, the control law requires the realization of the integral term in (2). As explained in Manitius and Olbrot's paper [8], if matrix A is Hurwitz, this term can be realized as the solution of the differential difference equation

$$\dot{z}\,(t) = Az(t) + e^{-hA}Bu(t) - Bu(t-h).$$

However, if the matrix A is not Hurwitz, such a realization involves an unstable pole-zero cancellation. The authors suggest then to realize the control law using some numerical approximation of the integral.

3. NECESSARY CONDITIONS FOR SPECTRAL ASSIGNMENT

In this paper, the use of fixed-step approximation methods such as the trapezoidal or Newton–Cote methods [9] is considered for the realization of the integral term in the control law. In the general case, the approximation can be written as

$$\int_{-h}^{0} e^{-\theta A}Bu(t+\theta)\,\mathrm{d}\theta \approx \sum_{p=0}^{q} \eta_p e^{\frac{p}{q}hA}Bu\left(t-\frac{p}{q}h\right), \tag{5}$$

where q determines the precision of the method, and where the parameters η_p depend on the chosen numerical scheme.

The closed-loop system is now described by the differential-difference equation of neutral type

$$\dot{x}(t) - BKe^{-hA}\mathcal{N}(\dot{x}_t) - Ax(t) - BKx(t-h) + BKAe^{-hA}\mathcal{N}(x_t) = 0, \tag{6}$$

where $\mathcal{N}$ is the operator defined as

$$\mathcal{N} : x_t \mapsto \sum_{p=0}^{q} \eta_p e^{\frac{p}{q}hA}x\left(t-\frac{p}{q}h\right).$$

Next, our main result is proved. It is shown that the stability of the ideal controller itself is a necessary condition for the stability of the closed loop when a fixed-step approximation is used for the implementation of the control law.

Theorem 1. Consider a spectrally controllable linear system with delay in the input described by

$$\dot{x}\,(t) = Ax(t) + Bu(t-h)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $h \geq 0$ is the delay, and a control law given by

$$u(t) = K\left[x(t) + e^{-hA}\int_{-h}^{0} e^{-\theta A}Bu(t+\theta)\mathrm{d}\theta\right]$$

that assigns theoretically a prescribed stable closed-loop structure with characteristic equation

$$\det(sI_n - A - e^{-hA}BK) = 0.$$

The implementation of this control law through a constant step approximation method (5) for a sufficiently great q results in an unstable closed-loop system if the characteristic function of the control law

$$\det\left\{I_m - Ke^{-hA}(sI-A)^{-1}(I-e^{-h(sI-A)})B\right\} \tag{7}$$

has at least one zero with positive real part.

Proof. We observe first that

$$\begin{aligned} &\det\left\{I_m - Ke^{-hA}(sI-A)^{-1}(I-e^{-h(sI-A)})B\right\} \\ = \ &\det\left\{I_n - BKe^{-hA}(sI-A)^{-1}(I-e^{-h(sI-A)})\right\} \end{aligned}$$

which, according to (4), is equal to

$$p(s) := \det\left(I - BKe^{-hA}\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta\right).$$

Similarly, we denote $p_q(s)$ the characteristic quasipolynomial of the approximated control law:

$$p_q(s) := \det\left(I - BKe^{-hA}\sum_{p=0}^{q}\eta_p e^{-\frac{p}{q}h(sI-A)}\right).$$

Assume now that $p(s)$ has a zero in the complex right-half plane, namely s_0 with $\mathrm{Re}(s_0) > 0$.

We first show that the sequence of functions $p_q(s)$ converges uniformly to $p(s)$ in a given neighborhood of s_0.

For this, let us recall that for a smooth real function f the error between the integral value $I(f) = \int_{-h}^{0} f(\zeta)\mathrm{d}\zeta$ and its approximation $I_q(f)$ obtained with a fixed-step method depends on the value of q and is given by an expression of the form

$$I(f) - I_q(f) = -\frac{h^{\beta+1}}{\alpha q^{\beta}} f^{(\gamma)}(\zeta_0),$$

where the point ζ_0 belongs to the interval $[-h,\ 0]$ and where α, β and γ are positive integers that depend on the chosen method (for instance, in the trapezoidal method, $\beta = 2$, $\gamma = 2$ and $\alpha = 12$). This result may be easily extended to the case of a smooth function $g : \mathbb{R} \to \mathbb{C}$:

$$|I(g) - I_q(g)| \leq \frac{\kappa}{q^{\beta}} \max_{\theta\in[-h,0]} \left|g^{(\gamma)}(\theta)\right|. \tag{8}$$

Let λ_i $(i = 1,\ldots,k)$ denote the eigenvalues of matrix A and η_i the order of multiplicity of λ_i with respect to the characteristic polynomial of A, and let Z_{ij} $(i = 1,\ldots,k,\ j = 1,\ldots,\eta_i)$ be the components of matrix A (see Chapter 5 in

Gantmacher [5]). We then have

$$\begin{aligned} &\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta - \sum_{p=0}^{q} \eta_p e^{-\frac{p}{q}h(sI-A)} \\ = \ &\sum_{i=1}^{k}\sum_{j=1}^{\eta_i} Z_{ij}\left(I(\theta^{j-1}e^{\theta(s-\lambda_i)}) - I_q(\theta^{j-1}e^{\theta(s-\lambda_i)})\right). \end{aligned}$$

Using inequality (8) with $g(\theta) = \theta^{j-1}e^{\theta(s-\lambda_i)}$, it is straightforward to prove that there is a positive real number M such that

$$||\Delta(s) - \Delta_q(s)|| \leq \frac{M}{q^\beta}$$

for all $s \in C = \{s \in \mathbb{C} : |s - s_0| \leq \varepsilon\}$, where $\varepsilon > 0$, $||\,.\,||$ denotes some matrix norm, $\Delta(s) = I_n - BKe^{-hA}\int_{-h}^{0} e^{\theta(sI-A)}\mathrm{d}\theta$, and $\Delta_q(s) = I_n - BKe^{-hA}\sum_{p=0}^{q}\eta_p e^{-\frac{p}{q}h(sI-A)}$.

This proves that the sequence $\{\Delta_q(s)\}$ converges uniformly to the matrix $\Delta(s)$ on C. Because of the continuity of the determinant application and C is a compact set of $\mathbb{C}$, we conclude that $\{p_q(s)\}$ converges uniformly to $p(s)$ on C as well. Finally, it follows from Lemma 2 of the Appendix that for q sufficiently large, each quasipolynomial $p_q(s)$ has a root s_0^q such that the sequence $\{s_0^q\}$ tends to s_0.

Now, the characteristic quasipolynomial of the approximated closed-loop system is given by the Laplace transform of (6):

$$\det(s\Delta_q(s) - A - BKe^{-hs} + BKAe^{-hA}\sum_{p=0}^{q}\eta_p e^{-\frac{p}{q}h(sI-A)}). \tag{9}$$

This determinant can be written as

$$a_0\left(e^{-\frac{h}{q}s}\right)s^n + a_1\left(e^{-\frac{h}{q}s}\right)s^{n-1} + \ldots + a_n\left(e^{-\frac{h}{q}s}\right)$$

where $a_i(e^{-\frac{h}{q}s}), i = 0, \ldots, n$ are quasipolynomials in $e^{-\frac{h}{q}s}$ and where the coefficient of highest degree in s, $a_0(e^{-\frac{h}{q}s})$, coincides with the characteristic function $p_q(s)$ of the approximated control law.

As shown above, $a_0(e^{-\frac{h}{q}s})$ has a root located at s_0^q with $\mathrm{Re}(s_0^q) > 0$, for q sufficiently large. According to Lemma 1 of the Appendix, this particular neutral-type quasipolynomial has an infinite number of roots located with regularity on a vertical line with this same positive real part.

Clearly, such roots of $p_q(s)$ are roots of the analytic function $f(s)$ defined as

$$\begin{aligned} f(s) \ &:= \ a_0\left(e^{-\frac{h}{q}s}\right)s^n \\ &= \ \det\left(I_n - BK\sum_{p=0}^{q}\eta_p e^{-\frac{p}{q}h(sI-A)}\right)s^n. \end{aligned}$$

Consider now the analytic function $g(s)$

$$g(s) := \left\{ a_1 \left(e^{-s\frac{h}{q}} \right) s^{n-1} + \ldots + a_n \left(e^{-s\frac{h}{q}} \right) \right\}.$$

Notice that there exists an isolated zero, say s_0', of magnitude large enough on the above mentioned vertical line with positive real part $\mathrm{Re}(s_0^q)$ so that on the contour $\mathcal{R}$ centered at s_0' of radius r_0, chosen so that $\mathcal{R}$ belongs to the right-half complex plane, we have that $|f(s)| > |g(s)|$ for each point on $\mathcal{R}$ and $f(s)$ and $g(s)$ do not vanish on $\mathcal{R}$. Therefore it follows from Rouché's Theorem (see Chapter 12 in Bellman and Cooke [3]) that $f(s)$ and $f(s) + g(s)$, i. e. the closed-loop characteristic function (9), have the same number of zeros inside $\mathcal{R}$ and we can conclude that (9) has a zero inside this contour. This zero has indeed positive real part and the result follows. □

Remark 1. A consequence of the above result is that the parameter K in the control law (2) must meet not only the design requirements of the assigned closed-loop, but it also must be such that the control law itself is stable.

Remark 2. In the general multivariable case, for a choice of K leading to a desired closed-loop assignment, one can verify, using a graphical test (such as a Mikhailov diagram), if the necessary condition is satisfied or not. It is also possible to perform a time domain analysis of the stability of the control law with respect to the size of the delay h, using the Lyapunov–Krasovskii approach.

4. ILLUSTRATIVE EXAMPLE

In simple cases, it is possible to parametrize the closed-loop and the control law characteristic equation in order to perform a stability analysis with respect to the parameters of K. Consider the example introduced by Manitius and Olbrot [8]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t-h).$$

where $h = 1$. According to the design procedure recalled in Section 2 the control law (2) with $[k_1, k_2] = [-4e^h + 1, -4e^h]$ that assigns the closed-loop $(s+1)^2$ is:

$$u(t) = (1 - 4e^h)\, x_1(t) - 4e^h x_2(t) + \int_{-h}^{0} u(t+\theta)\, \mathrm{d}\theta - 4 \int_{-h}^{0} e^{\theta} u(t+\theta)\, \mathrm{d}\theta. \quad (10)$$

It is described in the frequency domain as:

$$\left(1 - \frac{1 - e^{-hs}}{s} + 4 \frac{1 - e^{-h(s-1)}}{s-1} \right) u(s) = (1 - 4e^h)\, x_1(s) - 4e^h x_2(s).$$

The simulation results when the integrals in (10) are approximated using a trapezoidal quadrature method [9] show the instability of the closed-loop system (Figure 1).

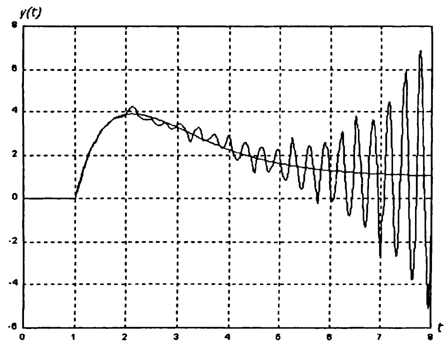

Fig. 1. Closed-loop response.

This behavior is explained, according to the arguments presented in the previous section by the instability of the ideal controller, whose characteristic equation is:

$$1 - \frac{1 - e^{-hs}}{s} + 4\frac{1 - e^{-h(s-1)}}{s-1} = 0. \tag{11}$$

The Mikhailov diagram of (11) shows indeed two encirclements of zero indicating two unstable roots (Figure 2).

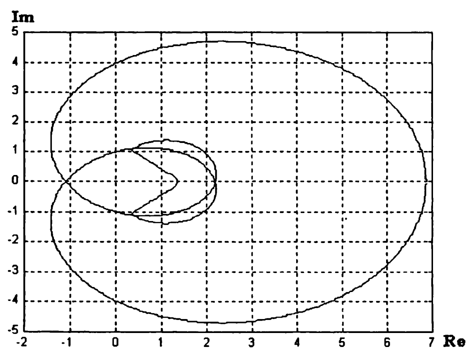

Fig. 2. Controller Mikhailov diagram.

In this example, it is possible to parametrize the control law in order to find if other spectrum assignments fullfil the necessary condition of stability of the controller. A $\mathcal{D}$-partition analysis [7] of the characteristic equation of the controller and of the closed-loop assigned spectrum is performed in order to do so. This analysis is based on the fact that the roots move continuously as the parameters vary continuously, and that they necessarily cross the imaginary axis when they go from a region of stability to one of instability. Moreover, according to the argument principle, the number of roots in a given region is constant. For convenience, the parameters are chosen as $\alpha_1 = k_1 - k_2$ and $\alpha_2 = e^{-h}k_2$. The resulting parametrized ideal controller characteristic equation is

$$\left(1 - \alpha_1 \frac{1 - e^{-hs}}{s} - \alpha_2 \frac{1 - e^{-h(s-1)}}{s - 1}\right) = 0$$

and the parametrized closed-loop spectrum is given by

$$s^2 - (1 + \alpha_1 + \alpha_2)\, s + \alpha_1 = 0.$$

The boundaries of the regions are determined by substituting $s = 0$ and $s = j\omega$. The stability regions of the closed-loop and of the ideal controller shown in Figure 3 are named Regions I and II, respectively. Their intersection is called Region III. It is the region where there is hope to have closed-loop stability.

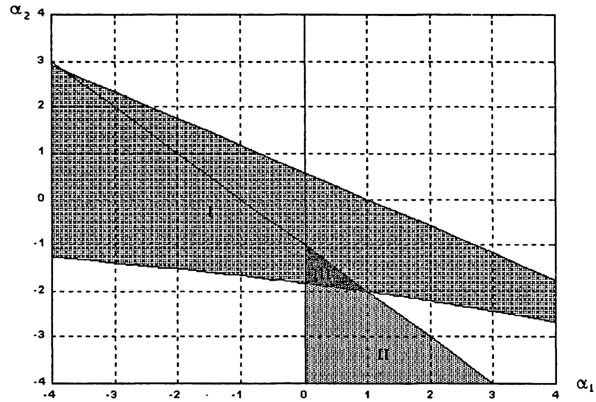

Fig. 3. Stability regions of the controller (I) and of the closed-loop (II).

It follows from Figure 3 that a choice of $\alpha_1 = 0.25$ and $\alpha_2 = -1.3$, insures the stability of the ideal closed-loop spectrum and of the ideal control law. One can verify with the help of Mikhailov diagrams that for a numerical approximation of $q = 16$ the approximated control law is stable but that the resulting closed-loop is unstable. Stability of the closed-loop is achieved for higher precision of the approximation of the control law, namely $q \geq 150$.

This fact raises the question wether or not, for precise enough approximations, the stability of the ideal closed-loop, combined with that of the controller are sufficient conditions for the stability of the closed-loop approximated scheme. However, the analysis of the previous section is based on the study of the roots of large magnitude, and although we know from Rouché's Theorem that there are roots located in the vicinity of the ideal assigned roots, we cannot say much about the location of roots of medium magnitude.

Notice that in this example, the region where these conditions are fulfilled (Region III in Figure 3) is quite reduced, hence the freedom of the designer in assigning the spectrum is very restricted.

5. COMMENTS AND CONCLUSIONS

The origin of instability arising when an approximation of the control law of Manitius and Olbrot [8] is used to stabilize multivariable input delay systems is explained with the help of a complete frequency domain analysis of the closed-loop quasipolynomial. Roughly speaking, one can say that if the control law is unstable, a precise enough approximation has an unstable root in the vicinity of this root. Due to its particular nature, the approximated control law, has an infinite number of roots of arbitrarily large magnitude associated to each root, in particular the unstable ones. The fact that the characteristic equation of the control is the principal term of the closed-loop characteristic equation, implies that its unstable roots of large magnitude are transmitted to the closed-loop.

Control laws that include distributed delay terms are common in control laws for delayed systems. This is the case of well known control schemes such as those based on a prediction (Smith controller [12, 14], Process Model Control [17]) or those derived from design procedures in appropriate rings [6, 11] or Bezout domain [2]. The study performed in this paper indicates that caution is in order when approximating these distributed delays.

In the case of linear systems with commensurate delays in the framework of a Bezout domain [2], a parametrization of stabilizing controllers in the spirit of Youla–Bongiorno can be achieved. This raises the question of finding conditions for the existence of a stable stabilizing controller that would generalize the concept of strong stabilization for linear systems [16].

APPENDIX

The following key lemmas for the proof of our main result are recalled or proved here.

Lemma 1. (Bellman and Cooke [3]) Consider a quasipolynomial such that

$$f(s,e^{hs}) = f(e^{hs}) = \sum_{j=1}^{l} p_j e^{jhs}.$$

Then the zeros of $f(e^{hs})$ are given by

$$\begin{aligned} s &= h^{-1}\log(z_j) = h^{-1}\log|z_j| + ih^{-1}\{\arg(z_j) + 2k\pi\}, \\ j &= 1,\dots,l; \quad k = 0, \pm 1, \pm 2, \dots \end{aligned}$$

where $z_j, j = 1, \dots, l$ are the l zeros of $f(z)$, the polynomial in z with constant coefficients of degree l obtained by letting $z = e^{-hs}$.

Remark 3. (Bellman and Cooke [3]) The zeros of $f(e^{hs})$ lie on a finite number of chains. Each chain consists of a countable infinity of zeros spaced $2\pi h^{-1}$ units apart on a vertical line defined by $Re(s) = h^{-1} \log |z_j|$.

Lemma 2. Consider a function f with an isolated zero at $s_0 \in \mathbb{C}$, which is analytic in a neighborhood V of s_0, and a sequence of analytic functions $\{f_n\}$ in V that uniformly converge to f in V. Then, $\forall \varepsilon > 0$, there exist $N > 0$ such that, for $n > N$, f_n has a zero in an ε-neighborhood s_0.

Proof. Consider the circle $\mathcal{C}$ centered at s_0 of radius $\varepsilon > 0$. For small enough ε we have that $\eta := \min\{|f(s)|\,; s \in \mathcal{C}\}$ is strictly positive (because f is continuous and has an isolated zero at s_0). Now, it follows from the uniform convergence of f_n to f, that there exist $N > 0$ such that $\forall n > N$, $\forall s \in \mathcal{C}$, $|f_n - f| < \eta$. $\forall s \in \mathcal{C}$, $|f_n - f| < \eta \leq |f(s)|$, therefore Rouché's Theorem allow us to conclude that, as f, $f_n = f + (f_n - f)$ has a zero inside $\mathcal{C}$. □

ACKNOWLEDGEMENTS

The authors thank Vladimir Kharitonov for fruitful comments.

(Received March 8, 2002.)

REFERENCES

[1] Z. Artstein: Linear systems with delayed controls: a reduction. IEEE Trans. Automat. Control *AC-27* (1982), 4, 869–879.

[2] D. Brethé and J. J. Loiseau: An effective algorithm for finite spectrum assignment of single-input systems with delays. Math. Comput. Simulation *45* (1998), 339–348.

[3] R. Bellman and K. L. Cooke: Differential Difference Equations. Academic Press, London 1963.

[4] K. Engelborghs, M. Dambrine, and D. Roose: Limitations of a class of stabilization methods for delay systems. IEEE Trans. Automat. Control *AC-46* (2001), 2, 336–339.

[5] F. R. Gantmacher: The Theory of Matrices. Vol. 1. AMS Chelsea Publishing, New York 1959.

[6] E. W. Kamen, P. P. Khargonekar, and A. Tannenbaum: Proper stable bezout factorizations and feedback control of linear time delay systems. Internat. J. Control *43* (1986), 3, 837–857.

[7] V. B. Kolmanovski and V. R. Nosov: Stability of Functional Differential Equations. Academic Press, New York 1986.

[8] A. Z. Manitius and A. W. Olbrot: Finite spectrum assignment problem for systems with delays. IEEE Trans. Automat. Control *AC-24* (1979), 4, 541–553.

[9] J. H. Mathews: Numerical Methods for Mathematics, Science, and Engineering. Prentice–Hall, Englewood Cliffs, N.J. 1992.

[10] S. Mondié and O. Santos: Une condition nécessaire pour l'implantation de lois de commande à retards distribués. In: Conférence Internationale Francophone d'Automatique, Lille 2000, pp. 201–206.

[11] A. S. Morse: Ring models for delay-differential systems. Automatica *12* (1976), 529–531.
[12] Z. J. Palmor: Modified predictors. In: The Control Handbook (W. Levine, ed.), CRC Press, Boca Raton 1996, Section 10.9.
[13] O. Santos and S. Mondié: Control laws involving distributed time delays: robustness of the implementation. In: American Control Conference, Chicago 2000, pp. 2479–2480.
[14] O. J. M. Smith: Closer control of loops with dead time. Chem. Engrg. Prog. *53* (1959), 217–219.
[15] V. Van Assche, M. Dambrine, J. F. Lafay, and J. P. Richard: Some problems arising in the implementation of distributed-delay control laws. In: Proc. 38th IEEE Conference on Decision and Control, Phoenix 1999.
[16] M. Vidyasagar: Control System Synthesis. MIT Press, Cambridge, MA 1985.
[17] K. Watanabe and M. Ito: A process model control for linear systems with delay. IEEE Trans. Automat. Control *AC-26* (1981), 6, 1261–1268.

Dr. Sabine Mondié, CINVESTAV-IPN, Departamento de Control Automático, Av. I.P.N. No. 2508, Col. Zacatenco, A.P. 14-740, 07300 México, D.F. México.
e-mail: smondie@ctrl.cinvestav.mx

Dr. Michel Dambrine, LAIL, UPRESA CNRS 8021, Ecole Centrale de Lille, Cité Scientifique, B.P. 48 Villeneuve d'Ascq-cedex. France.
e-mail: Michel.Dambrine@ec-lille.fr

Omar Santos, Centro de Investigación en Tecnologías y Sistemas, ICBI, Universidad Autónoma del Estado de Hidalgo, Pachuca. México.
e-mail: omarj@uaeh.reduaeh.mx