

# OUTPUT FEEDBACK REGULATION FOR LARGE-SCALE UNCERTAIN NONLINEAR SYSTEMS WITH TIME DELAYS

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This paper is concerned with the problem of global state regulation by output feedback for large-scale uncertain nonlinear systems with time delays in the states and inputs. The systems are assumed to be bounded by a more general form than a class of feedforward systems satisfying a linear growth condition in the unmeasurable states multiplying by unknown growth rates and continuous functions of the inputs or delayed inputs. Using the dynamic gain scaling technique and choosing the appropriate Lyapunov–Krasovskii functionals, we explicitly construct the universal output feedback controllers such that all the states of the closed-loop system are globally bounded and the states of large-scale uncertain systems converge to zero.

*Keywords:* global regulation, large-scale systems, output feedback, time-delay systems, uncertain nonlinear systems

*Classification:* 34K35, 62F35, 93A15, 93B52, 93C10, 93C23

## 1. INTRODUCTION

Large-scale systems, which are composed of a set of interconnected subsystems, can be found in many practical systems of the real world, such as economic systems, urban traffic networks, power systems, multi-agent systems and digital communication networks. In the control of large-scale systems, decentralized control schemes present a practical and effective means for designing control algorithms that just utilize the local state without the need for information exchange amongst subsystems. On the other hand, it is widely known that time-delay phenomenon is frequently encountered in the real control systems, such as nuclear reactors, chemical process, turbojet engines. All these systems have the characteristics of time delay. The existence of time delay usually leads to poor performances and often causes instability (see e. g., [2, 3, 4, 5, 6]). Therefore the problem of decentralised state feedback or output feedback stabilization of large-scale time-delay systems has received considerable attention (see e. g., [14, 19, 20, 21, 24, 25]).

In the last decade, the problem of global output feedback control of nonlinear systems with linear unmeasurable states multiplying by the various growth functions has received considerable attention and still remains as an active research topic (see e. g., [1, 2, 9, 10, 11, 12, 13, 15, 16, 18, 22, 23, 24, 25, 26]). For example, a time-varying output feedback controller has been proposed for the global regulation of nonlinear uncertain systems

with an unbounded time-varying delay in the input in [9]. Specifically, based on the Lyapunov–Krasovskii theorem, the output feedback controllers have been constructed to stabilize a class of large-scale nonlinear time-delay systems that are dominated by the upper or lower triangular time-delay systems in [24] and [25], respectively.

Motivated by [9, 12, 24, 25], in this paper, we consider the problem of global adaptive regulation via output feedback for large-scale uncertain nonlinear systems with time delays in the states and inputs. To the best of our knowledge, there is no work dealing with such a class of large-scale systems satisfying Assumption 2.1 in the literature at present, see Remark 2.3.

## 2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a large-scale uncertain nonlinear time-delay system composed of  $N$  interconnected subsystems

$$\left\{ \begin{array}{l} \dot{x}_{i,1}(t) = x_{i,2}(t) + \phi_{i,1}(t, x(t), u(t), x(t - \tau_{i1}), u(t - \tau_{i1})), \\ \vdots \\ \dot{x}_{i,n_i-1}(t) = x_{i,n_i}(t) + \phi_{i,n_i-1}(t, x(t), u(t), x(t - \tau_{i1}), u(t - \tau_{i1})), \\ \dot{x}_{i,n_i}(t) = u_i(t - \tau_{i2}), \\ y_i(t) = x_{i,1}(t), \end{array} \right. \quad i = 1, 2, \dots, N, \tag{1}$$

where  $x_i(t) = [x_{i,1}(t), x_{i,2}(t), \dots, x_{i,n_i}(t)]^T \in \mathbb{R}^{n_i}$ ,  $u_i(t) \in \mathbb{R}$  and  $y_i(t) \in \mathbb{R}$  are the states, control input, and output of the  $i$ th subsystem, respectively; constants  $\tau_{im}$  satisfying  $0 \leq \tau_{im} \leq \tau, m = 1, 2$  are known time delays of the  $i$ th subsystem; in this paper, we always denote  $x_i(t), \varepsilon_i(t), z_i(t)$  by  $x_i, \varepsilon_i, z_i$ ;  $x = [x_1^T, x_2^T, \dots, x_N^T]^T$ ,  $u = [u_1, \dots, u_N]^T$  and  $y = [y_1, y_2, \dots, y_N]^T$ . The continuously differentiable uncertain functions  $\phi_{i,j} : \mathbb{R}^+ \times \mathbb{R}^{2(n_1 + \dots + n_N + N)} \rightarrow \mathbb{R}, j = 1, \dots, n_i - 1$ , represent the nonlinearities within the  $i$ th subsystem and the nonlinear interconnection effects between the  $i$ th subsystem and other subsystems, and satisfy the following growth condition.

**Assumption 2.1.** For the unknown functions  $\phi_{i,j}(\cdot)$ , there exist an unknown constant  $\theta > 0$  and known nonnegative continuous functions  $f_i(u, u(t - \tau_{i1}))$  such that for any  $s \in (0, 1]$ , the following inequality holds

$$\sum_{j=1}^{n_i-1} s^{n_i-j+1} |\phi_{i,j}(\cdot)| \leq \theta s^2 f_i(u, u(t - \tau_{i1})) \left[ \sum_{p=1}^N \left( \left( \sum_{q=1}^{n_p} s^{n_p-q+1} (|x_{p,q}| + |x_{p,q}(t - \tau_{i1})|) \right) + |u_p| + |u_p(t - \tau_{i1})| \right) \right], \quad i = 1, \dots, N.$$

**Remark 2.2.** It is not difficult to prove that if the following condition for some unknown

constant  $\theta' > 0$

$$|\phi_{i,j}(\cdot)| \leq \theta' f_i(u, u(t - \tau_{i1})) \sum_{p=1}^N \sum_{q=\max\{2+n_p+j-n_i, 1\}}^{n_p} (|x_{p,q}| + |x_{p,q}(t - \tau_{i1})| + |u_p| + |u_p(t - \tau_{i1})|), \quad j = 1, \dots, n_i - 1, \quad i = 1, \dots, N \tag{2}$$

is satisfied, then Assumption 2.1 is always satisfied, but not vice versa. Then system (1) is a more general form than a class of large-scale feedforward systems satisfying (2).

**Remark 2.3.** For system (1) satisfying (2) with  $N = f_i(\cdot) = 1, \tau_{i1} = \tau_{i2} = 0$ , the output feedback stabilization or regulation problem has been investigated in [1, 26]. For system (1) satisfying (2) with  $\tau_{i2} = 0$  and  $\theta' = 1$ , the output feedback stabilization problem has been considered in [23, 24]. For system (1) satisfying (2), where  $N = f_i(\cdot) = 1, \tau_{11} = 0$  and  $\tau_{12}$  is a time-varying functions, the output feedback regulation problem has been studied in [9]. However, since  $\theta'$  is an unknown positive constant and  $f_i(\cdot)$  are the inputs or delay inputs functions, system (1) satisfying Assumption 2.1 do not belong to the systems considered in the existing related literature. Therefore, for system (1) satisfying Assumption 2.1, the problem of output feedback regulation is unsolvable by any existing design method, and then is worth of investigation.

We introduce two technical lemmas that will be crucial in establishing our main result.

**Lemma 2.4.** (Krishnamurthy and Khorrami [11], Zhang et al. [24]) For  $i = 1, \dots, N$ , there exist a constant  $\alpha > 0$ , symmetric matrices  $P_i > 0, Q_i > 0$ , and vectors  $a_i = (a_{i,1}, \dots, a_{i,n_i})^T, b_i = (b_{i,1}, \dots, b_{i,n_i})^T$  such that

$$\begin{aligned} A_i^T P_i + P_i A_i &\leq -I, \text{ and } D_i P_i + P_i D_i \geq \alpha I, \\ B_i^T Q_i + Q_i B_i &\leq -2I, \text{ and } D_i Q_i + Q_i D_i \geq \alpha I, \end{aligned} \tag{3}$$

where

$$A_i = \begin{bmatrix} -a_{i,1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{i,n_i-1} & 0 & \cdots & 1 \\ -a_{i,n_i} & 0 & \cdots & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_{i,1} & -b_{i,2} & \cdots & -b_{i,n_i} \end{bmatrix},$$

$$D_i = \text{diag}\{n_i, n_i - 1, \dots, 1\}.$$

**Lemma 2.5.** (Zhang et all. [22]) For any constant  $\tau > 0$  and continuous vector  $\eta(t) \in \mathbb{R}^n$ , the following inequality holds

$$\int_{t-\tau}^t \eta^T(s) ds \int_{t-\tau}^t \eta(s) ds \leq \tau \int_{t-\tau}^t \|\eta(s)\|^2 ds.$$

### 3. GLOBAL REGULATION BY OUTPUT FEEDBACK

In this section, we will show that under Assumption 2.1, system (1) can be globally regulated by the output feedback controller. The main results are given below.

**Theorem 3.1.** Suppose that Assumption 2.1 holds, the states of system (1) achieve global adaptive regulation by the following output feedback controller

$$\begin{aligned} \dot{\hat{x}}_{i,1} &= \hat{x}_{i,2} + \frac{a_{i,1}}{LM}(y_i - \hat{x}_{i,1}), & i = 1, \dots, N, \\ &\vdots \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{\hat{x}}_{i,n_i} &= u_i + \frac{a_{i,n_i}}{(LM)^{n_i}}(y_i - \hat{x}_{i,1}), \\ u_i &= -\left(b_{i,1} \frac{\hat{x}_{i,1}}{(LM)^{n_i}} + b_{i,2} \frac{\hat{x}_{i,2}}{(LM)^{n_i-1}} + \dots + b_{i,n_i} \frac{\hat{x}_{i,n_i}}{LM}\right), \end{aligned} \tag{5}$$

$$\dot{M} = \frac{1}{\alpha M} \max \left\{ \varpi(u, u(t - \tau_{11}), \dots, u(t - \tau_{N1})) - \frac{M}{2}, 0 \right\}, \tag{6}$$

$$\dot{L} = \sum_{i=1}^N \frac{M}{(LM)^2} \left( \frac{y_i - \hat{x}_{i,1}}{(LM)^{n_i}} \right)^2, \text{ with } M(t) = L(t) = 1, \text{ for } t \in [-\tau, 0], \tag{7}$$

where  $\alpha, a_{i,j}$  and  $b_{i,j}$ ,  $j = 1, \dots, n_i - 1$ ,  $i = 1, \dots, N$  are the appropriately chosen parameters such that Lemma 2.4 holds,  $\varpi(\cdot) \geq 0$  is a continuously differentiable function to be designed later.

*Proof.* For the convenience of the readers, we break up the proof into four parts.

**Part I:** The changes of coordinates and the closed-loop system.

For  $i = 1, \dots, N$ , let

$$[\tilde{x}_{i,1}, \dots, \tilde{x}_{i,n_i-1}, \tilde{x}_{i,n_i}]^T = \left[ x_{i,1}, \dots, x_{i,n_i-1}, x_{i,n_i} + \int_{t-\tau_{i2}}^t u_i(s) ds \right]^T, \tag{8}$$

and

$$\varepsilon_{i,j} = \frac{\tilde{x}_{i,j} - \hat{x}_{i,j}}{(LM)^{n_i-j+1}}, \quad z_{i,j} = \frac{\hat{x}_{i,j}}{(LM)^{n_i-j+1}}, \quad j = 1, \dots, n_i. \tag{9}$$

Then, for  $i = 1, \dots, N$ , based on (1), (4)–(5) and (8)–(9), the dynamics of  $\varepsilon_i$  and  $z_i$  can be given by the following compact form

$$\dot{\varepsilon}_i = \frac{1}{LM} A_i \varepsilon_i - \frac{1}{(LM)^2} E_i \int_{t-\tau_{i2}}^t u_i(s) ds + \Phi_i(\cdot) - \left( \frac{\dot{L}}{L} + \frac{\dot{M}}{M} \right) D_i \varepsilon_i, \tag{10}$$

$$\dot{z}_i = \frac{1}{LM} B_i z_i + \frac{1}{LM} a_i \varepsilon_{i,1} - \left( \frac{\dot{L}}{L} + \frac{\dot{M}}{M} \right) D_i z_i, \tag{11}$$

where  $a_i, A_i, B_i$  and  $D_i$  are defined by Lemma 2.4,  $\varepsilon_i = (\varepsilon_{i,1}, \dots, \varepsilon_{i,n_i})^T$ ,  $z_i = (z_{i,1}, \dots, z_{i,n_i})^T$ ,  $\Phi_i(\cdot) = \left[ \frac{\phi_{i,1}(\cdot)}{(LM)^{n_i}}, \frac{\phi_{i,2}(\cdot)}{(LM)^{n_i-1}}, \dots, \frac{\phi_{i,n_i-1}(\cdot)}{(LM)^2}, 0 \right]^T$ ,  $E_i = [0, \dots, 0, 1, 0]^T \in \mathbb{R}^{n_i}$ , and we have  $u_i = -b_i^T z_i$ .

It is easy to see that the closed-loop system consisting of (5)–(7) and (10)–(11) has a unique solution  $(\varepsilon, z, L, M)$  on a small time interval  $[0, T)$ , where  $\varepsilon = [\varepsilon_1^T, \dots, \varepsilon_N^T]^T$ ,  $z = [z_1^T, \dots, z_N^T]^T$ . Without loss of generality, we suppose that this solution can be extended to the maximal interval  $[0, T_f)$  for some  $T_f$ , with  $0 < t_f \leq +\infty$ .

From (6)–(7), it can be seen that for  $\forall t \in [0, T_f)$

$$\begin{cases} \dot{M} \geq 0, M(t) \geq M(t - \tau_{i1}) \geq 1, \frac{M}{2} + \alpha M \dot{M} \geq \varpi(\cdot), & (12a) \\ \dot{L} = \sum_{i=1}^N \frac{M}{(LM)^2} \varepsilon_{i,1}^2 \geq 0, L(t) \geq L(t - \tau_{i1}) \geq 1. & (12b) \end{cases}$$

**Part II:** The choice of Lyapunov–Krasovskii functional and continuously differentiable function  $\varpi(\cdot)$ .

Consider the Lyapunov functions  $V_\varepsilon = \sum_{i=1}^N (\mu + 1) \varepsilon_i^T P_i \varepsilon_i$  and  $V_z = \sum_{i=1}^N z_i^T Q_i z_i$ , where  $P_i$  and  $Q_i$  are given by Lemma 2.4,  $\mu = \max_{1 \leq i \leq N} \{\|Q_i a_i\|^2\}$ . A simple calculation gives

$$\begin{aligned} \dot{V}_\varepsilon &\leq \sum_{i=1}^N \left( -\frac{\mu + 1}{LM} \|\varepsilon_i\|^2 - \alpha(\mu + 1) \frac{\dot{M}}{M} \|\varepsilon_i\|^2 \right) \\ &\quad + \sum_{i=1}^N \left( -2 \frac{\mu + 1}{(LM)^2} \varepsilon_i^T P_i E_i \int_{t-\tau_{i2}}^t u_i(s) ds + 2(\mu + 1) \varepsilon_i^T P_i \Phi_i(\cdot) \right), \end{aligned} \tag{13}$$

$$\dot{V}_z \leq \sum_{i=1}^N \left( -\frac{2}{LM} \|z_i\|^2 + 2 \frac{1}{LM} z_i^T Q_i a_i \varepsilon_{i,1} - \alpha \frac{\dot{M}}{M} \|z_i\|^2 \right). \tag{14}$$

Note that  $u_i = -b_i^T z_i$  and Lemma 2.5 holds, then

$$\begin{aligned} -\sum_{i=1}^N 2 \frac{\mu + 1}{(LM)^2} \varepsilon_i^T P_i E_i \int_{t-\tau_{i2}}^t u_i(s) ds &\leq \sum_{i=1}^N \frac{1}{(LM)^2} \left\| \int_{t-\tau_{i2}}^t z_i(s) ds \right\|^2 + \sum_{i=1}^N \frac{\mu_1}{(LM)^2} \|\varepsilon_i\|^2 \end{aligned} \tag{15}$$

$$\leq \sum_{i=1}^N \tau_{i2} \left( \int_{t-\tau_{i2}}^t \frac{\|z_i(s)\|^2}{(L(s)M(s))^2} ds \right) + \sum_{i=1}^N \frac{\mu_1}{(LM)^2} \|\varepsilon_i\|^2, \tag{16}$$

where  $\mu_1$  is a known positive constant.

Using Assumption 2.1 and  $2ab \leq a^2 + b^2$ , we obtain

$$\begin{aligned} &\sum_{i=1}^N 2(\mu + 1) \varepsilon_i^T P_i \Phi_i(\cdot) \\ &\leq \sum_{i=1}^N 2\theta_1 \|\varepsilon_i\| \left( \left| \frac{\phi_{i,1}(\cdot)}{(LM)^{n_i}} \right| + \dots + \left| \frac{\phi_{i,n_i-1}(\cdot)}{(LM)^2} \right| \right) \end{aligned}$$

$$\begin{aligned}
 &\leq \sum_{i=1}^N 2\theta_2 \|\varepsilon_i\| \frac{f_i(\cdot)}{(LM)^2} \left[ \sum_{p=1}^N \left( \left( \sum_{q=1}^{n_p} \left( \frac{|x_{p,q}|}{(LM)^{n_p-q+1}} + \frac{|x_{p,q}(t-\tau_{i1})|}{(LM)^{n_p-q+1}} \right) \right) \right. \right. \\
 &\quad \left. \left. + |u_p| + |u_p(t-\tau_{i1})| \right) \right] \\
 &\leq \sum_{i=1}^N 2\theta_2 \|\varepsilon_i\| \frac{f_i(\cdot)}{(LM)^2} \left[ \sum_{p=1}^N \left( \left( \sum_{q=1}^{n_p} (|\varepsilon_{p,q}| + |z_{p,q}| \right. \right. \right. \\
 &\quad \left. \left. + |\varepsilon_{p,q}(t-\tau_{i1})| + |z_{p,q}(t-\tau_{i1})| \right) \right) + \frac{\int_{t-\tau_{p2}}^t |u_p(s)| ds}{LM} \\
 &\quad \left. + \frac{\int_{t-\tau_{p2}}^t |u_p(s-\tau_{i1})| ds}{L(s-\tau_{i1})M(s-\tau_{i1})} + |u_p| + |u_p(t-\tau_{i1})| \right) \Big] \\
 &\leq \sum_{i=1}^N 2\theta_3 \|\varepsilon_i\| \frac{f_i(\cdot)}{(LM)^2} \left[ \sum_{p=1}^N \left( \|\varepsilon_p\| + \|z_p\| + \|\varepsilon_p(t-\tau_{i1})\| \right. \right. \\
 &\quad \left. \left. + \|z_p(t-\tau_{i1})\| \right) \right] + \sum_{i=1}^N 2\theta_3 \|\varepsilon_i\| \frac{f_i(\cdot)}{LM} \left[ \sum_{p=1}^N \left( \int_{t-\tau_{p2}}^t \left\| \frac{z_p(s)}{L(s)M(s)} \right\| ds \right. \right. \\
 &\quad \left. \left. + \int_{t-\tau_{p2}}^t \left\| \frac{z_p(s-\tau_{i1})}{L(s-\tau_{i1})M(s-\tau_{i1})} \right\| ds \right) \right] \\
 &\leq \sum_{i=1}^N \frac{f_i^2(\cdot)}{(LM)^2} \|\varepsilon_i\|^2 + \sum_{i=1}^N \frac{\theta_4}{(LM)^2} \left( \|\varepsilon_i\|^2 + \|z_i\|^2 \right) \\
 &\quad + \sum_{i=1}^N \sum_{p=1}^N \theta_4 \left( \frac{\|\varepsilon_p(t-\tau_{i1})\|^2 + \|z_p(t-\tau_{i1})\|^2}{L^2(t-\tau_{i1})M^2(t-\tau_{i1})} \right) \\
 &\quad + \sum_{p=1}^N \theta_4 \int_{t-\tau_{p2}}^t \left\| \frac{z_p(s)}{L(s)M(s)} \right\|^2 ds \\
 &\quad + \sum_{i=1}^N \sum_{p=1}^N \theta_4 \left( \int_{t-\tau_{p2}}^t \left\| \frac{z_p(s-\tau_{i1})}{L(s-\tau_{i1})M(s-\tau_{i1})} \right\|^2 ds \right) \tag{17}
 \end{aligned}$$

where  $\theta_i$ ,  $i = 1, 2, 3, 4$  are unknown constants depending on  $\theta$ . In addition, it is obvious that

$$\begin{aligned}
 \sum_{i=1}^N 2 \frac{1}{LM} z_i^T Q_i a_i \varepsilon_{i,1} &\leq \sum_{i=1}^N \frac{1}{LM} \|z_i\|^2 + \sum_{i=1}^N \frac{1}{LM} \|Q_i a_i\|^2 \varepsilon_{i,1}^2 \\
 &\leq \sum_{i=1}^N \frac{1}{LM} \|z_i\|^2 + \sum_{i=1}^N \frac{\mu}{LM} \varepsilon_{i,1}^2 \tag{18}
 \end{aligned}$$

$$\leq \sum_{i=1}^N \frac{1}{LM} \|z_i\|^2 + \sum_{i=1}^N \frac{\mu}{LM} \|\varepsilon_i\|^2. \tag{19}$$

Now, define the Lyapunov–Krasovskii functional

$$\begin{aligned}
 V_1 &= V_\varepsilon + V_z + \sum_{i=1}^N \sum_{p=1}^N \theta_4 (1 + \tau_{p2}) \left( \int_{t-\tau_{i1}}^t \left( \frac{\|\varepsilon_p(s)\|^2 + \|z_p(s)\|^2}{L^2(s)M^2(s)} \right) ds \right) \\
 &\quad + \sum_{p=1}^N (\theta_4 + \tau_{p2}) \left( \int_{-\tau_{p2}}^0 \int_{t+\rho}^t \left\| \frac{z_p(s)}{L(s)M(s)} \right\|^2 ds d\rho \right) \\
 &\quad + \sum_{i=1}^N \sum_{p=1}^N \theta_4 \left( \int_{-\tau_{p2}}^0 \int_{t+\rho}^t \left\| \frac{z_p(s - \tau_{i1})}{L(s - \tau_{i1})M(s - \tau_{i1})} \right\|^2 ds d\rho \right). \tag{20}
 \end{aligned}$$

Using (13)–(20), we get

$$\begin{aligned}
 \dot{V}_1 &\leq \sum_{i=1}^N \left( \left( -\frac{1}{LM} + \frac{\theta_5 M}{(LM)^2} \right) (\|\varepsilon_i\|^2 + \|z_i\|^2) \right) \\
 &\quad - \sum_{i=1}^N \left( \frac{1}{(LM)^2} \left( \frac{M}{2} + \alpha M \dot{M} - f_i^2(\cdot) \right) (\|\varepsilon_i\|^2 + \|z_i\|^2) \right), \tag{21}
 \end{aligned}$$

where  $\theta_5$  is an unknown constant depending on  $\theta$ . Choosing  $\varpi(\cdot) \geq \max_{1 \leq i \leq N} \{f_i^2(\cdot)\}$ , for any  $u$ . Accordingly, we obtain

$$\dot{V}_1 \leq - \sum_{i=1}^N \left( \frac{M}{(LM)^2} (L - \theta_5) (\|\varepsilon_i\|^2 + \|z_i\|^2) \right). \tag{22}$$

**Part III:** Boundedness of the closed-loop system on  $[0, T_f)$ .

Now, we use (22) to prove that the states  $(\varepsilon, z, L, M)$  of the closed-loop system (5)–(7) and (10)–(11) are bounded on  $[0, T_f)$ .

Firstly, we show that the dynamic gain  $L$  is bounded on  $[0, T_f)$ . This can be done by a contradiction argument. Suppose  $\lim_{t \rightarrow T_f} L(t) = +\infty$ . Combining it and (12b) together, we get that there exists a finite time  $t_1 \in (0, T_f)$  such that

$$L(t) \geq \theta_5 + 1, \quad \text{for } \forall t \in [t_1, T_f).$$

Substituting the inequality above into (22), we have

$$\dot{V}_1 \leq - \sum_{i=1}^N \frac{M}{(LM)^2} (\|\varepsilon_i\|^2 + \|z_i\|^2), \quad \text{for } \forall t \in [t_1, T_f). \tag{23}$$

From (12b) and (23), we obtain

$$+\infty = L(T_f) - L(t_1) = \int_{t_1}^{T_f} \dot{L}(t) dt = \int_{t_1}^{T_f} \left( \sum_{i=1}^N \frac{M(t) \varepsilon_{i,1}^2(t)}{[L(t)M(t)]^2} \right) dt \leq V_1(t_1) = \text{constant},$$

which is impossible. Therefore,  $L$  is bounded on  $[0, T_f)$  and  $\lim_{t \rightarrow T_f} L(t) < +\infty$ . Moreover, we obtain that  $\int_0^{T_f} \left( \sum_{i=1}^N \frac{\varepsilon_{i,1}^2(t)}{L^2(t)M(t)} \right) dt < \infty$  and  $\int_0^{T_f} \left( \sum_{i=1}^N \frac{\varepsilon_{i,1}^2(t)}{M(t)} \right) dt < +\infty$ .

Next, we prove that the state  $z$  is bounded on  $[0, T_f)$ . Using (14), (18) and (12a), we have

$$\begin{aligned} \dot{V}_z &\leq -\sum_{i=1}^N \frac{1}{LM} \|z_i\|^2 + \sum_{i=1}^N \frac{\mu}{LM} \varepsilon_{i,1}^2 \\ &= -\sum_{i=1}^N \frac{1}{LM} \|z_i\|^2 + \mu L \dot{L}, \text{ for } \forall t \in [0, T_f). \end{aligned}$$

Consequently, as  $L$  is bounded on  $[0, T_f)$ , for  $\forall t \in [0, T_f)$ , we have

$$\begin{aligned} \sum_{i=1}^N \left( \lambda_{\min}(Q_i) \|z_i(t)\|^2 - z_i^T(0) Q_i z_i(0) \right) &\leq -\int_0^t \left( \sum_{i=1}^N \frac{\|z(s)\|^2}{L(s)M(s)} \right) ds + \frac{\mu}{2} [L^2(t) - 1] \\ &\leq \frac{\mu}{2} [L^2(t) - 1] < +\infty \end{aligned}$$

and

$$\int_0^t \left( \sum_{i=1}^N \frac{\|z(s)\|^2}{L(s)M(s)} \right) ds \leq \sum_{i=1}^N z_i^T(0) Q_i z_i(0) + \frac{\mu}{2} [L^2(t) - 1] < +\infty.$$

Then  $\lim_{t \rightarrow T_f} \|z(t)\| < +\infty$  and  $\int_0^{T_f} \left( \sum_{i=1}^N \frac{\|z_i(t)\|^2}{M(t)} \right) dt < +\infty$ .

Thirdly, we claim that the dynamic gain  $M$  is bounded on  $[0, T_f)$ . This claim can be proven again by a contradiction argument. Suppose  $\lim_{t \rightarrow T_f} M(t) = +\infty$ . Since  $z_i$  are bounded on  $[0, T_f)$ , we obtain that  $u_i = -b_i^T z_i$  are bounded,  $i = 1, \dots, N$ . Furthermore, note that  $\varpi(\cdot) \geq 0$  is a continuous inputs and delay inputs function, then there exists a constant  $K > 0$  such that  $\varpi(\cdot) \leq K$ . From  $\lim_{t \rightarrow T_f} M(t) = +\infty$ , we get that there exists a finite time  $t_2 \in (0, T_f)$  such that  $M(t_2) \geq 2K + 1$ . By (12a), we obtain

$$M(t) \geq M(t_2) \geq 2K + 1 > 2\varpi(\cdot), \text{ for any } t \in [t_2, T_f).$$

From (6), we get  $\dot{M}(t) = 0$ , for any  $t \in [t_2, T_f)$ . Then

$$M(t) \equiv M(t_2) = \text{constant}, \text{ for any } t \in [t_2, T_f),$$

which leads to a contradiction. Thus  $M$  is bounded on  $[0, T_f)$ . Moreover,

$$\lim_{t \rightarrow T_f} \|M(t)\| < +\infty, \int_0^{T_f} \left( \sum_{i=1}^N \varepsilon_{i,1}^2(t) \right) dt < +\infty \text{ and } \int_0^{T_f} \left( \sum_{i=1}^N \|z_i(t)\|^2 \right) dt < +\infty.$$

Finally, we verify that the state  $\varepsilon$  is bounded on  $[0, T_f)$ . For this aim, we introduce two suitable unknown positive constants  $\theta_6, \theta_7$  depending on  $\theta$ , then we define the following change of coordinates

$$\eta_{i,j} = \frac{\tilde{x}_{i,j} - \hat{x}_{i,j}}{(L^*)^{n_i-j+1}}, \quad j = 1, 2, \dots, n_i, \quad i = 1, \dots, N, \tag{24}$$

where constant  $L^* \geq \max\{L(T_f), M(T_f), \theta_7 + 1\}$ . Then

$$\dot{\eta}_i = \frac{1}{L^*} A_i \eta_i + \frac{1}{L^*} a_i \eta_{i,1} - \frac{1}{L^*} \Gamma_i a_i \eta_{i,1} + \Psi_i^*(\cdot) + \Phi_i^*(\cdot), \tag{25}$$



where  $a_i$  and  $A_i$  are defined by Lemma 2.4,  $\eta_i = (\eta_{i,1}, \dots, \eta_{i,n_i})^T$ ,  $\Psi_i^*(\cdot) = \frac{E_i b_i^T}{(L^*)^2} \int_{t-\tau_{i2}}^t z_i(s) ds$ ,  $\Phi_i^*(\cdot) = \left[ \frac{\phi_{i,1}(\cdot)}{(L^*)^{n_i}}, \frac{\phi_{i,2}(\cdot)}{(L^*)^{n_i-1}}, \dots, \frac{\phi_{i,n_i-1}(\cdot)}{(L^*)^2}, 0 \right]^T$  and  $\Gamma_i = \text{diag} \left\{ \frac{L^*}{LM}, \left(\frac{L^*}{LM}\right)^2, \dots, \left(\frac{L^*}{LM}\right)^{n_i} \right\}$ .

Choose the Lyapunov function  $V_\eta = \sum_{i=1}^N \eta_i^T P_i \eta_i$ , we get

$$\begin{aligned} \dot{V}_\eta \leq & - \sum_{i=1}^N \frac{1}{L^*} \|\eta_i\|^2 + \sum_{i=1}^N 2 \frac{1}{L^*} \eta_i^T P_i a_i \eta_{i,1} - \sum_{i=1}^N 2 \frac{1}{L^*} \eta_i^T P_i \Gamma_i a_i \eta_{i,1} \\ & + \sum_{i=1}^N 2 \eta_i^T P_i \Psi_i^*(\cdot) + \sum_{i=1}^N 2 \eta_i^T P_i \Phi_i^*(\cdot). \end{aligned} \tag{26}$$

By the completion of square, as  $L$  and  $M$  are bounded on  $[0, T_f)$ , the following estimations can be obtained

$$\sum_{i=1}^N 2 \frac{1}{L^*} \eta_i^T P_i a_i \eta_{i,1} \leq \sum_{i=1}^N \frac{1}{(L^*)^2} \|\eta_i\|^2 + \sum_{i=1}^N \frac{\theta_6}{2} \eta_{i,1}^2 \tag{27}$$

and

$$- \sum_{i=1}^N 2 \frac{1}{L^*} \eta_i^T P_i \Gamma_i a_i \eta_{i,1} \leq \sum_{i=1}^N \frac{1}{(L^*)^2} \|\eta_i\|^2 + \sum_{i=1}^N \frac{\theta_6}{2} \eta_{i,1}^2. \tag{28}$$

Moreover, from  $L^* \geq L(T_f) \geq L(t) \geq 1$  and Assumption 2.1, recalling that  $u_i$  are bounded and  $f_i(\cdot)$  are continuous inputs and delay inputs functions,  $i = 1, \dots, N$ , following the procedure of (16) and (17), we have

$$\sum_{i=1}^N 2 \eta_i^T P_i \Psi_i^*(\cdot) \leq \sum_{i=1}^N \tau_{i2} \left( \int_{t-\tau_{i2}}^t \frac{\|z_i(s)\|^2}{(L^*)^2} ds \right) + \sum_{i=1}^N \frac{\theta_6}{(L^*)^2} \|\eta_i\|^2 \tag{29}$$

and

$$\begin{aligned} \sum_{i=1}^N 2 \eta_i^T P_i \Phi_i^*(\cdot) \leq & \sum_{i=1}^N \frac{\theta_6}{(L^*)^2} \|\eta_i\|^2 + \sum_{i=1}^N \frac{\theta_6}{(L^*)^2} \left( \|\eta_i\|^2 + \|z_i\|^2 \right) \\ & + \sum_{i=1}^N \sum_{p=1}^N \theta_6 \left( \frac{\|\eta_p(t - \tau_{i1})\|^2 + \|z_p(t - \tau_{i1})\|^2}{(L^*)^2} \right) \\ & + \sum_{p=1}^N \theta_6 \int_{t-\tau_{p2}}^t \frac{\|z_p(s)\|^2}{(L^*)^2} ds \\ & + \sum_{i=1}^N \sum_{p=1}^N \theta_6 \left( \int_{t-\tau_{p2}}^t \frac{\|z_p(s - \tau_{i1})\|^2}{(L^*)^2} ds \right). \end{aligned} \tag{30}$$

Construct the Lyapunov–Krasovskii functional

$$V_2 = V_\eta + \sum_{i=1}^N \sum_{p=1}^N \int_{t-\tau_{i1}}^t \theta_6 (1 + \tau_{p2}) \left( \frac{\|\eta_p(s)\|^2 + \|z_p(s)\|^2}{(L^*)^2} \right) ds$$

$$\begin{aligned}
 & + \sum_{p=1}^N (\theta_6 + \tau_{p2}) \int_{-\tau_{p2}}^0 \int_{t+\rho}^t \frac{\|z_p(s)\|^2}{(L^*)^2} ds d\rho \\
 & + \sum_{i=1}^N \sum_{p=1}^N \theta_6 \int_{-\tau_{p2}}^0 \int_{t+\rho}^t \frac{\|z_p(s - \tau_{i1})\|^2}{(L^*)^2} ds d\rho.
 \end{aligned} \tag{31}$$

Substituting (26)-(30) into (31), we obtain

$$\begin{aligned}
 \dot{V}_2 & \leq \sum_{i=1}^N \left( -\frac{L^* - \theta_7}{(L^*)^2} \|\eta_i\|^2 + \theta_7 \|z_i\|^2 + \theta_6 \varepsilon_{i,1}^2 \right) \\
 & \leq \sum_{i=1}^N \left( -\frac{1}{(L^*)^2} \|\eta_i\|^2 + \theta_7 \|z_i\|^2 + \theta_6 \varepsilon_{i,1}^2 \right).
 \end{aligned} \tag{32}$$

From (31) and (32), it follows that for any  $t \in [0, T_f]$

$$\sum_{i=1}^N \lambda_{\min}(P_i) \|\eta_i(t)\|^2 \leq V_2(0) + \theta_7 \int_0^t \left( \sum_{i=1}^N \|z_i(s)\|^2 \right) ds + \theta_6 \int_0^t \left( \sum_{i=1}^N \varepsilon_{i,1}^2(s) \right) ds < +\infty \tag{33}$$

and

$$\int_0^t \left( \sum_{i=1}^N \frac{\|\eta_i(s)\|^2}{(L^*)^2} \right) ds \leq V_2(0) + \theta_7 \int_0^t \left( \sum_{i=1}^N \|z_i(s)\|^2 \right) ds + \theta_6 \int_0^t \left( \sum_{i=1}^N \varepsilon_{i,1}^2(s) \right) ds < +\infty. \tag{34}$$

Then, from (33), (34), (24) and (9), we get  $\lim_{t \rightarrow T_f} \|\varepsilon(t)\| < +\infty$  and

$$\int_0^{T_f} \left( \sum_{i=1}^N \|\varepsilon_i(t)\|^2 \right) dt < +\infty.$$

**Part IV:** Convergence of the states.

Up to now, we have proved that  $L, z, M, \varepsilon$  are all bounded on the maximal interval  $[0, T_f]$ . Thus we get  $T_f = +\infty$ . Furthermore, from Part III, we know that  $L, z, M, \varepsilon$  are bounded on  $[0, +\infty)$  and  $\int_0^\infty \|z(t)\|^2 dt < +\infty, \int_0^\infty \|\varepsilon(t)\|^2 dt < +\infty$ . It is easy to obtain the boundedness of  $\dot{z}, \dot{\varepsilon}$  on  $[0, +\infty)$  from the boundedness of  $L, M, z, \varepsilon$  on  $[0, +\infty)$ .

Therefore, we have

$$\varepsilon \in L_2, \dot{\varepsilon} \in L_\infty \quad \text{and} \quad z \in L_2, \dot{z} \in L_\infty.$$

Using the Barbalat's Lemma, we have

$$\lim_{t \rightarrow +\infty} z(t) = \lim_{t \rightarrow +\infty} \varepsilon(t) = 0,$$

which along with (8), (9) and  $u_i = -b_i^T z_i$  leads to

$$\lim_{t \rightarrow +\infty} u(t) = 0 \quad \text{and} \quad \lim_{t \rightarrow +\infty} x(t) = \lim_{t \rightarrow +\infty} \hat{x}(t) = 0.$$

□

**Remark 3.2.** Since  $\dot{L} \geq 0$ ,  $\dot{M} \geq 0$ , and  $L, M$  are bounded on  $[0, +\infty)$ , there exist constants  $\bar{L} > 0$ ,  $\bar{M} > 0$  such that  $\lim_{t \rightarrow \infty} L(t) = \bar{L}$ ,  $\lim_{t \rightarrow \infty} M(t) = \bar{M}$ . That is to say, the dynamic gains  $L$  and  $M$  are time-invariant in nature.

**Remark 3.3.** From the proof procedure of Theorem 1, we see that the dynamic gains  $L$  and  $M$  are introduced to deal with the unknown growth rate  $\theta$  and the function  $f_i(u, u(t - \tau_{i1}))$ , respectively, and both are required.

**Remark 3.4.** It is worth pointing out that Theorem 1 also holds when the time delays  $\tau_{i2}$  are unknown time-varying functions  $\tau_{i2}(t)$  satisfying  $0 \leq \tau_{i2}(t) \leq \tau$  and  $0 < \dot{\tau}_{i2} \leq d < 1$ ,  $i = 1, \dots, N$ .

From (5) and (6), we know that the inputs  $u_i$  are dependent on the time delays  $\tau_{i1}, i = 1, \dots, N$ . As pointed out in [4], it has a crucial fundamental limitation that the designed delay-dependent controller has to use knowledge of the delay explicitly and hence require memory, which is difficult to implement in practice especially for the case of time-varying delay. In what follows, we will introduce another assumption, under which a delay-independent output feedback controller is proposed for system (1), where the unknown time delays  $\tau_{im}$  satisfy  $0 \leq \tau_{im} \leq \tau$ ,  $\tau$  is a known constant,  $i = 1, \dots, N, m = 1, 2$ . The price to be paid for the improvement is that Assumption 3.5 is more stringent than Assumption 2.1.

**Assumption 3.5.** For the uncertain functions  $\phi_{i,j}(\cdot)$ , there exist an unknown constant  $\theta > 0$  and known nonnegative continuous functions  $f_i(u)$  such that for any  $s \in (0, 1]$ , the following inequality holds

$$\begin{aligned} & \sum_{j=1}^{n_i-1} s^{n_i-j+1} |\phi_{i,j}(\cdot)| \\ \leq & \theta s^2 f_i(u(t - \tau_{i1})) \left[ \sum_{p=1}^N \left( \left( \sum_{q=1}^{n_p} s^{n_p-q+1} |x_{p,q}(t - \tau_{i1})| \right) + |u_p(t - \tau_{i1})| \right) \right] \\ & + \theta s^2 f_i(u) \left[ \sum_{p=1}^N \left( \left( \sum_{q=1}^{n_p} s^{n_p-q+1} |x_{p,q}| \right) + |u_p| \right) \right], \quad i = 1, \dots, N. \end{aligned}$$

**Theorem 3.6.** Suppose that Assumption 3.5 holds. Appropriately select constants  $\alpha, a_{i,j}$  and  $b_{i,j}, j = 1, \dots, n_i - 1, i = 1, \dots, N$ , and a continuously differentiable function  $\varpi(u) \geq \max_{1 \leq i \leq N} \{f_i^2(u)\} \geq 0$ , for any  $u$ , then the states of system (1) achieve global adaptive regulation by the output feedback controller consisting of (4), (5), (7) and the following dynamic equation of  $M$

$$\dot{M} = \frac{1}{\alpha M} \max \left\{ \varpi(u) - \frac{M}{2}, 0 \right\}. \tag{35}$$

**Proof.** Since the proof of this theorem is very similar to that of Theorem 3.1, we omitted it for brevity. □

4. SIMULATION EXAMPLE

Consider an interconnected time-delay system

$$\left\{ \begin{array}{l} \dot{x}_{1,1} = x_{1,2} + c_1 u_1(t - \tau_1) \sqrt{\ln(1 + x_{1,1}^6) \ln(1 + x_{2,1}^6(t - \tau_1))} + c_2 u_2^2(t - \tau_1), \\ \dot{x}_{1,2} = x_{1,3} + c_3 u_1(t - \tau_1) u_2 \\ \dot{x}_{1,3} = u_1(t - \tau_2) \\ y_1 = x_{1,1}, \\ \dot{x}_{2,1} = x_{2,2}(t) + c_4 u_1(t - \tau_3) u_2(t - \tau_3), \\ \dot{x}_{2,2} = u_2(t - \tau_4), \\ y_2 = x_{2,1}, \end{array} \right. \tag{36}$$

where  $c_i, i = 1, 2, 3, 4$  are totally unknown parameters, and the unknown delay constants  $\tau_j$  satisfy  $0 \leq \tau_j \leq 1, j = 1, 2, 3, 4$ .

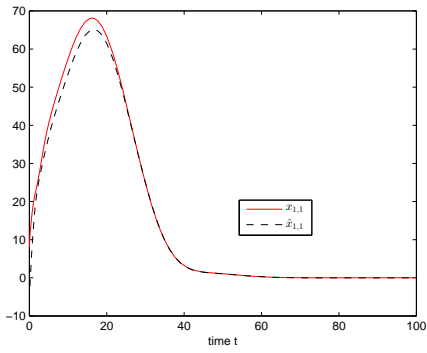
It is not difficult to verify that system (36) satisfies Assumption 2.1 and 3.5, but does not satisfy (2). Using Theorem 3.6, we design the controller for (36)

$$\left\{ \begin{array}{l} \dot{\hat{x}}_{1,1} = \hat{x}_{1,2} + \frac{3}{LM} (y_1 - \hat{x}_{1,1}), \\ \dot{\hat{x}}_{1,2} = \hat{x}_{1,3} + \frac{3}{(LM)^2} (y_1 - \hat{x}_{1,1}) \\ \dot{\hat{x}}_{1,3} = u_1 + \frac{1}{(LM)^3} (y_1 - \hat{x}_{1,1}) \\ \dot{\hat{x}}_{2,1} = \hat{x}_{2,2} + \frac{2}{LM} (y_2 - \hat{x}_{2,1}), \\ \dot{\hat{x}}_{2,2} = u_2 + \frac{1}{(LM)^2} (y_2 - \hat{x}_{2,1}), \\ \dot{M} = \frac{1}{0.4M} \max \left\{ \frac{u_1^2 + u_2^2 + 1}{25} - \frac{M}{2}, 0 \right\}, \\ \dot{L} = \sum_{i=1}^2 \frac{M}{(LM)^2} \left( \frac{y_i - \hat{x}_{i,1}}{(LM)^{4-i}} \right)^2, \text{ with } M(t) = L(t) = 1, \text{ for } t \in [-1, 0], \end{array} \right. \tag{37}$$

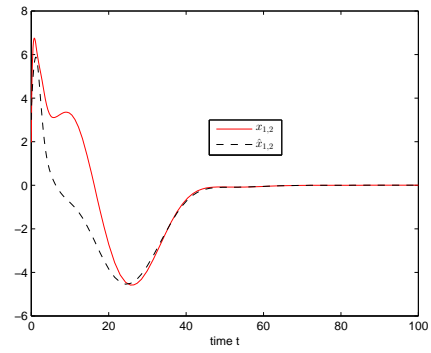
with

$$\left\{ \begin{array}{l} u_1 = - \left( \frac{\hat{x}_{1,1}}{(LM)^3} + 3 \frac{\hat{x}_{1,2}}{(LM)^2} + 3 \frac{\hat{x}_{1,3}}{LM} \right), \\ u_2 = - \left( \frac{\hat{x}_{2,1}}{(LM)^2} + 2 \frac{\hat{x}_{2,2}}{LM} \right). \end{array} \right. \tag{38}$$

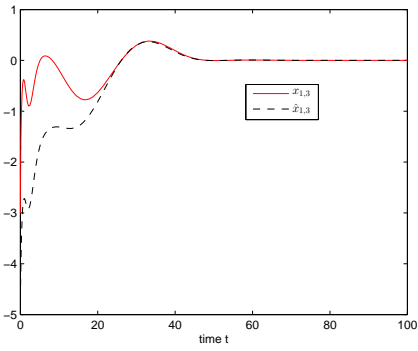
Let  $c_2 = 0.3, c_i = 1, i = 1, 3, 4$ , and  $\tau_j = 0.1, j = 1, 2, 3, 4$ , the simulation results are shown in Figures 1 – 2 for the closed-loop system consisting of (36) – (38). The initial condition is chosen as, for  $t \in [-1, 0], [x_{1,1}(t), x_{1,2}(t), x_{1,3}(t), x_{2,1}(t), x_{2,2}(t), \hat{x}_{1,1}(t), \hat{x}_{1,2}(t), \hat{x}_{1,3}(t), \hat{x}_{2,1}(t), \hat{x}_{2,2}(t), L(t), M(t)] = [8, 2, -3, -1, 3, -6, 3, -5, 9, -4, 1, 1]$ .



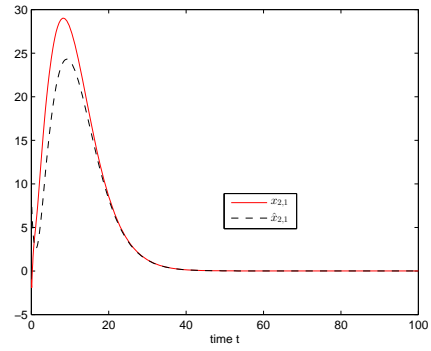
(a) Trajectories of  $x_{1,1}$  and  $\hat{x}_{1,1}$ .



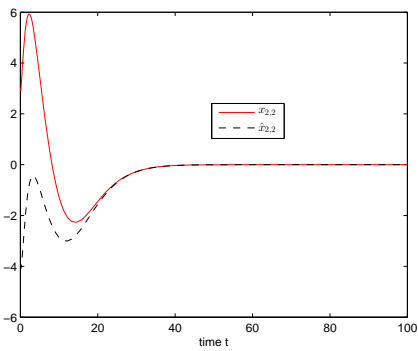
(b) Trajectories of  $x_{1,2}$  and  $\hat{x}_{1,2}$ .



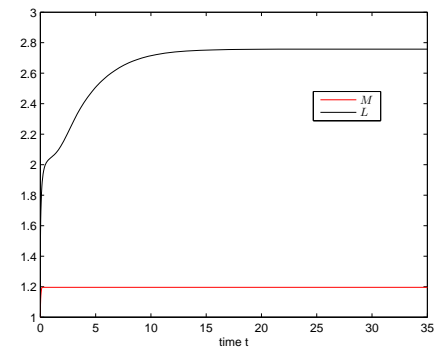
(c) Trajectories of  $x_{1,3}$  and  $\hat{x}_{1,3}$ .



(d) Trajectories of  $x_{2,1}$  and  $\hat{x}_{2,1}$ .

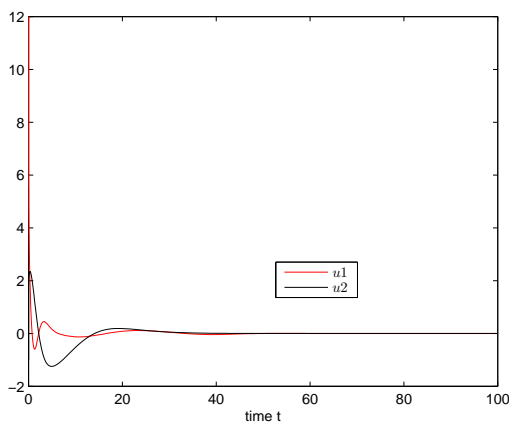


(e) Trajectories of  $x_{2,2}$  and  $\hat{x}_{2,2}$ .



(f) The observer's gain  $L$  and  $M$ .

**Fig. 1.** Transient response of the closed-loop system consisting of (36)–(38).



**Fig. 2.** The control inputs  $u_1$  and  $u_2$

## 5. CONCLUSION

In this paper, we have investigated the problem of global adaptive output feedback regulation for a class of large-scale nonlinear time-delay systems whose nonlinearities satisfy certain growth conditions. By designing the dynamic gain observer and using the rescaling transformation of coordinates, we propose the dynamic output feedback controllers, which have a linear-like structure, to achieve global adaptive regulation of systems. Simulation results have been provided to show the effectiveness of the proposed approach.

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