KYBERNETIKA — VOLUME *56* (2020), NUMBER 4, PAGES 794–809

GLOBAL ROBUST OUTPUT REGULATION OF A CLASS OF NONLINEAR SYSTEMS WITH NONLINEAR EXOSYSTEMS

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An adaptive output regulation design method is proposed for a class of output feedback systems with nonlinear exosystem and unknown parameters. A new nonlinear internal model approach is developed in the present study that successfully converts the global robust output regulation problem into a robust adaptive stabilization problem for the augmented system. Moreover, an output feedback controller is achieved based on a type of state filter which is designed for the transformed augmented system. The adaptive control technique is successfully introduced to the stabilization design to ensure the global stability of the closed-loop system. The result can successfully apply to a tracking control problem associated with the well known Van der Pol oscillator.

Keywords: output regulation, global stability, internal model, nonlinear systems

Classification: 93E12, 62A10, 62F15

1. INTRODUCTION

The goal of output regulation is to reject the disturbances and/or track the desired trajectories. The disturbances as well as exotic signals is usually modeled as unknown deterministic signals with known signal-generating dynamics. The problem of asymptotic rejection of sinusoidal disturbance, which is one kind of common deterministic disturbance, has been widely studied [1, 7, 8, 10, 14]. However, many periodic disturbances are not sinusoidal, such as the Van der Pol oscillator, for which they can't be modeled as an output of a finite-dimensional linear exosystem.

Recently, some progress is reported on output regulation with nonlinear exosystems [2, 3, 17, 19, 26]. Ramos et al. [26] presented a result in terms of sufficient conditions of the state feedback generalized output regulation problem for nonlinear systems with nonautonomous exosystem. Byrnes et al. [2] proposed an algorithm, which uses high gain internal models, to ensure the semi-global output regulation of nonlinear exosystems. Huang et al. [4] for the first time provided a framework to study the robust output regulation problem with nonlinear exosystem. [4] is also the only paper which actually addresses the existence of the steady state generator and internal model when

DOI: 10.14736/kyb-2020-4-0794

the exosystem is nonlinear. Ding [6] proposed an output regulation algorithm for a class of nonlinear systems in the output feedback form. A new nonlinear internal model was constructed based on high gain design and the Hermite–Birkhoff interpolation. The approach was then extended to the application of circle criterion in [31]. Similarly, Chen et al. [3] proposed an asymptotic rejection algorithm, which used the same internal model as in [31], to achieve the asymptotic rejection of nonharmonic disturbances and ensured semi-global stability of the whole systems. Jiang et al. [17, 19, 20] proposed the asymptotic rejection algorithms to achieve the asymptotic rejection of nonharmonic disturbances for some classes of uncertain nonlinear systems and ensured global stability of the whole systems.

Recently, some progress was also reported on output regulation problem. Xu et al. [32] studied the global robust output regulation problem of nonlinear output feedback systems with uncertain exosystems by error output feedback control. Then a novel nonlinear internal model approach was developed which successfully converted the global robust output regulation problem into a robust non-adaptive stabilization problem for the augmented system. By means of a feedback regulator with the same characteristics, Elena Zattoni et al. [32] investigated the problem of achieving output regulation with closed-loop global asymptotic stability in hybrid systems with a continuous-time linear dynamics subject to periodic state jumps. A main contribution of this work is to establish a new necessary and sufficient condition for problem solvability in strict geometric terms. Wang et al. [30] studied the robust output regulation problem for a class of invertible nonlinear MIMO systems. Then a robust regulator was proposed with the combination of nonlinear internal model and a new extended high-gain observer. The novelty of this approach results from the use of an extended observer to estimate, under mild assumptions, all unmeasured terms. Marco et al. [22] dealed with the problem of output regulation for left invariant systems defined on general matrix Lie-Groups. The structure of proposed control law embeds a copy of the exosystem kinematics updated by means of error measurements. A rigorous stability analysis was provided for both the general case and the particular case of systems posed on the special orthogonal group SO(3). Ngoc-Tu Trinh et al. [29] dealed with the PI control/regulation design for a cascaded network of multi systems governed by hyperbolic partial differential equations. Then the PI controllers were designed at junctions and were applied for each subsystem of the network. Subsequently the exponential stability for the closed-loop systems and the output regulation were proven by using the Lyapunov direct method.

But most of the studies in the above were concerned the global output regulation of minimum-phase nonlinear systems. For the study of non-minimum phase nonlinear systems, it has always been a hot issue in the field of control [5, 11, 12, 13, 21, 24, 28]. However, for the non-minimum phase nonlinear systems, the design of stabilizing controllers is challenging. Recently, some progress was also reported on output regulation problem for the non-minimum phase nonlinear systems. A.Isidori et al. [15] proposed an internal model-based output feedback controller for non minimum phase systems and presented a sufficient condition for the existence of an output feedback controller able to semi-globally asymptotically stabilize the augmented system. A.Isidori et al. [16] presented some further results on the semi-global output regulation of non-minimum phase nonlinear systems. Nazrulla and Khalil [25] combined the tool with sliding mode control as well as extended high-gain observer and presentd an output feedback controller for non-minimum phase systems. By using the method of slow integrators as well as high-gain feedback, Huang et al. [9] presented a methodology for the regulation of non-minimum-phase nonlinear systems. And the effectiveness of such methodology was demonstrated on the nontrivial translational oscillator rotating actuator example. Recently, Jiang et al. [18] presented a methodology for the output regulation of output feedback systems with unknown frequency gain.

In this paper, we consider the global output regulation problem with nonlinear exosystems via output feedback. In order to tackle the nonlinearity in the exosystem, we exploit a new nonlinear internal model that can be used to estimate the disturbances. This is a crucial step for solving the output regulation problem. In fact, this internal model design method has been exploited recently for disturbance rejection problem in our paper [17]. Of course there are differences between designing an internal model for disturbance rejection problem and an internal model for output regulation problem, with the later one being more challenging. On the other hand, the exosystems are of special characteristics, which lead to specific conditions to be identified for the internal model design. A general condition will also be specified for nonlinear terms in the dynamic system, which allows more general nonlinear functions than polynomials.

The outline of the paper is as follows. Section 2 describes a class of uncertain nonlinear systems with nonlinear exosystems as a disturbance source. Assumptions are also given. The state transformation is introduced in Section 3, and the design of the internal model is presented in Section 4. Section 5 presents global robust stabilization analysis to determine the controller. Section 6 gives an example to demonstrate the whole design procedure of the proposed method. Finally, the conclusion is given in Section 7.

2. PROBLEM FORMULATION

Considering the following single-input-single-output nonlinear systems which can be transformed into the output feedback form

$$
\begin{aligned}
\dot{x} &= A_c x + \varphi(y)a + D(w) + bu \\
y &= Cx \\
e &= y - q(w)
\end{aligned}
\tag{1}
$$

with

$$
A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \vdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix},
$$

where $x \in R^n$ is the state vector, $u \in R$ is the control input, $y \in R$ is the output, $e \in R$ is the measurement output, $a \in R^q$ and $b \in R^n$ are the vectors of unknown parameters, $b_\rho \neq 0$ indicates that the nonlinear system has a constant relative degree of ρ,

$D : R^m \to R^n, \varphi : R \to R^{n\times q}$ with $\varphi(0) = 0$ and $|\varphi(y_1) - \varphi(y_2)| \le \Delta_1(|y_1|)\sigma_1(|y_1 - y_2|)$ and $\sigma_1(\cdot) \in \Re$ and $\Delta_1(\cdot)$ is nondecreasing and the function $\sigma_1(\cdot)$ is a known smooth function, $q(w)$ is an unknown polynomial of w and $w \in R^m$ is a nonharmonic periodic disturbance vector which is generated from a nonlinear exosystem:

$$\dot{w} = s(w). \tag{2}$$

Remark 1. The assumption about the function φ is satisfied for various types of functions, for example, the polynomial functions.

Assumption 1. The system is of minimum phase, i. e., the polynomial $\Re(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ is Hurwitz, and the high frequency gain b_ρ is known.

Assumption 2. The flows of vector field $s(w)$ are bounded and converged to periodic solutions.

Remark 2. On the basis of Assumption 2, the periodic solutions of the exosystems may include multiple functions, such as harmonic functions and limit cycles of nonlinear dynamic systems.

The output regulation problem that we are going to solve is to find a finite dimensional system

$$\begin{aligned} \dot{\mu} &= v(\mu, e(t)), \\ u &= u(\mu, e(t)) \end{aligned} \qquad \mu \in R^s$$

such that for every $x(0) \in R^n, w(0) \in \Omega \subset R^m, x(t), \mu(t)$ and $u(t)$ are bounded $\forall t \ge 0$, and $\lim_{t\to\infty} e(t) = 0$.

Motivated by the results in [1], the following assumption is proposed in order to solve the output regulation problem.

Assumption 3. There exist $\varpi(w) \in R^n$ and $\iota(w)$ with $\varpi_1(w) = q(w)$ for each a, b such that

$$\frac{\partial \varpi}{\partial w} s(w) = A_c \varpi + \varphi(q(w))a + D(w) + b\iota(w).$$

Assumption 4. There exists a positive integer r and an odd locally Lipschitz function $\gamma : \Omega \to R$ such that

$$\frac{d^r \nu(t)}{dt^r} - \gamma(\nu, \dot{\nu}, \cdots, \nu^{(r-1)}) = 0 \tag{3}$$

where $\Omega \in R^r$ is a compact subset. In addition, there exists a positive number ϱ, such that $|\gamma(\rho_1) - \gamma(\rho_2)| \le \varrho \|\rho_1 - \rho_2\|$, where $\rho_1, \rho_2 \in R^r$.

Remark 4. Assumption 4 is motivated by [8]. We note that the disturbances satisfying Assumption 4 are more general.

3. STATE TRANSFORM

For the system with relative degree $\rho > 1$, we introduce a state transform that is based on the filtered transform in [33] to put the system in the following form

$$\begin{aligned} \dot{z} &= Az + \Xi e + \Omega(y, w, d)a \\ \dot{y} &= z_1 + \tfrac{d_2}{d_1} y + \varphi_1(y)a + D_1(w) + b_\rho \xi_1 \end{aligned} \tag{4}$$

where $z \in R^{n-1}$, $a \in R^q$, ξ_1 is the output of an input filter, with $\lambda_i > 0$

$$\begin{aligned} \dot{\xi}_i &= -\lambda_1 \xi_i + \xi_{i+1}, for \quad i = 1, \cdots, \rho - 2 \\ \dot{\xi}_{\rho-1} &= -\lambda_{\rho-1}\xi_{\rho-1} + u. \end{aligned} \tag{5}$$

Lemma 1. (Xi and Ding [31]) Under Assumption 3 there exists $\pi(w) \in R^{n-1}$ along the trajectories of exosystem satisfying

$$\begin{aligned} \frac{d\pi_i(w(t))}{dt} &= -\frac{d_{i+1}}{d_1}\pi_1(w(t)) + \pi_{i+1}(w(t)) + q(w(t)) \times (\frac{d_{i+2}}{d_1} - \frac{d_{i+1}d_2}{d_1^2}) + D_{i+1}(w(t)) \\ &- D_1(w(t)) \times \frac{d_{i+1}}{d_1} + (\varphi_{i+1}(q(w(t))) - \frac{d_{i+1}}{d_1}\varphi_1(q(w(t))))a, i = 1, \cdots, n-2 \\ \frac{d\pi_{n-1}(w(t))}{dt} &= -\frac{d_n}{d_1}\pi_1(w(t)) - \frac{d_n d_2}{d_1^2} q(w(t)) + (\varphi_n(q(w(t))) - \frac{d_n}{d_1}\varphi_1(q(w(t))))a \\ &+ D_n(w(t)) - \frac{d_n}{d_1} D_1(w(t)). \end{aligned}$$

Based on the above Lemma 1, we have

$$\frac{\partial q(w)}{\partial w} s(w) = \pi_1(w) + \frac{d_2}{d_1} q(w) + \varphi_1(q(w))a + D_1(w) + b_\rho \alpha(w).$$

With ξ_1 viewed as the input, $\alpha(w)$ is the feedback term used for output regulation to tackle the disturbances, and it is given by

$$\alpha(w) = b_\rho^{-1}(\frac{\partial q(w)}{\partial w} s(w) - \pi_1(w) - \frac{d_2}{d_1} q(w) - \varphi_1(q(w))a - D_1(w)).$$

We now introduce the last transformation based on the invariant manifold with

$$\tilde{z} = z - \pi(w(t)),$$

finally, we have the model for the control design

$$\begin{cases} \dot{\tilde{z}} = A\tilde{z} + \Xi e + \Omega(y, w, d)a \\ \dot{e} = \tilde{z}_1 + \frac{d_2}{d_1} e + (\varphi_1(y) - \varphi_1(q(w)))a + b_\rho(\xi_1 - \alpha(w)) \end{cases} \tag{6}$$

where

$$\mathrm{A} = \begin{bmatrix} -\frac{d_2}{d_1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{d_n}{d_1} & 0 & \cdots & 0 \end{bmatrix}$$

$$\Xi = \left(\frac{d_3}{d_1} - \frac{d_2^2}{d_1^2}, \cdots, \frac{d_n}{d_1} - \frac{d_{n-1}d_2}{d_1^2}, -\frac{d_n d_2}{d_1^2} \right)^T$$

$$\Omega(y,w,d) = \begin{pmatrix} \varphi_2(y) - \varphi_2(q(w)) - \frac{d_2}{d_1}(\varphi_1(y) - \varphi_1(q(w))) \\ \vdots \\ \varphi_n(y) - \varphi_n(q(w)) - \frac{d_n}{d_1}(\varphi_1(y) - \varphi_1(q(w))) \end{pmatrix}.$$

Lemma 2. (Xi and Ding [31]) There exist a known function $\zeta(\cdot)$ which is nondecreasing and an unknown constant Δ which is dependent on the initial state w_0 of exosystem, such that

$$\begin{cases} |\Omega(y,w,d)| \le \Delta\,|e|\,\zeta(|e|) \\ |\varphi_1(y) - \varphi_1(q(w))| \le \Delta\,|e|\,\zeta(|e|). \end{cases}$$

Considering a Lyapunov function $V_z = \tilde{z}^T P_0 \tilde{z}$, where

$$P_0\mathrm{A} + \mathrm{A}^T P_0 = -I.$$

Assuming $2xy \le rx^2 + r^{-1}y^2$ or $xy \le rx^2 + (4r^{-1})y^2$ for $x > 0$, $y > 0$, r being any positive real constant and $\zeta^2(|e|) \le \zeta^2(1+e^2)$, there exist unknown positive real constants δ_1, δ_2 such that

$$\begin{aligned} \dot{V}_z &= -\tilde{z}^T\tilde{z} + 2\tilde{z}^T P_0(\Xi e + \Omega(y,w,d)a) \\ &\le -\frac{3}{4}\tilde{z}^T\tilde{z} + \delta_1 e^2 + \delta_2 e^2 \zeta^2(1+e^2) \end{aligned} \tag{7}$$

note that

$$2\tilde{z}^T P_0 \Xi e \le \frac{1}{8}\tilde{z}^T\tilde{z} + 8e^T\Xi^T P_0^2 \Xi e \le \frac{1}{8}\tilde{z}^T\tilde{z} + \delta_1 e^2$$

and

$$\begin{aligned} 2\tilde{z}^T P_0 \Omega(y,w,d)a &\le \frac{1}{8}\tilde{z}^T\tilde{z} + 8a^T\Omega^T P_0^2 \Omega a \le \frac{1}{8}\tilde{z}^T\tilde{z} + \delta_2\,|\Omega|^2 \\ &\le \frac{1}{8}\tilde{z}^T\tilde{z} + \delta_2\Delta^2 e^2\zeta^2(|e|) \le \frac{1}{8}\tilde{z}^T\tilde{z} + \delta_2 e^2\zeta^2(1+e^2). \end{aligned}$$

4. INTERNAL MODEL DESIGN

To solve the problem, we need an assumption on the structure of the exosystem. Firstly, we define

$$\theta = col(\nu, \dot{\nu}, \cdots, \nu^{(r-1)}). \tag{8}$$

Then, there exists an immersion of the exosystem

$$\begin{aligned} \dot{\theta} &= F\theta + G\gamma(\theta) \\ \alpha &= \psi\theta \end{aligned} \tag{9}$$

where $\theta \in R^r$,

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \psi^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

We select any controllable pair (M, N) with $M \in R^{r\times r}, N \in R^r$, where M is Hurwitzian and has disjoint spectra with F. Because the pair (ψ, F) is observable, there exists a unique and nonsingular matrix T satisfying the Sylvester equation

$$TF - MT = N\psi. \tag{10}$$

Assuming $\vartheta = T\theta$, we have

$$\dot{\vartheta} = TFT^{-1}\vartheta + TG\gamma(T^{-1}\vartheta), \quad \alpha = \psi T^{-1}\vartheta. \tag{11}$$

We design the internal model as

$$\dot{\eta} = M(\eta - b_\rho^{-1}Ne) + TG\gamma(T^{-1}(\eta - b_\rho^{-1}Ne)) + N\xi_1, \tag{12}$$

where $N \in R^r$, because M is Hurwitzian, there exists a positive matrix P and Q satisfying

$$PM + M^TP = -Q,$$

$$\eta^TQ\eta \geq \gamma_0\|\eta\|^2, \quad \gamma_0 > 0.$$

We consider the mismatch between the states of (11) and (12), and define an auxiliary error as

$$\tilde{\eta} = \vartheta - \eta + b_\rho^{-1}Ne.$$

It follows that

$$\begin{aligned} \dot{\tilde{\eta}} = M\tilde{\eta} &+ TG\gamma(T^{-1}\vartheta) - TG\gamma(T^{-1}(\eta - b_\rho^{-1}Ne)) \\ &+ b_\rho^{-1}N[\tilde{z}_1 + \frac{d_2}{d_1}e + (\varphi_1(y) - \varphi_1(q(w)))a]. \end{aligned}$$

Let $V_\eta = \tilde{\eta}^TP\tilde{\eta}$, where ϱ and γ_0 are two positive reals. Then there exists unknown positive real constants δ_3 and δ_4 such that

$$\begin{aligned} \dot{V}_\eta &= -\tilde{\eta}^TQ\tilde{\eta} + 2\tilde{\eta}^TPTG(\gamma(T^{-1}\vartheta) - \gamma(T^{-1}(\eta - b_\rho^{-1}Ne))) \\ &\quad + 2\tilde{\eta}^TPb_\rho^{-1}N(\varphi_1(y) - \varphi_1(q(w)))a + 2\tilde{\eta}^TPb_\rho^{-1}N(\tilde{z}_1 + \frac{d_2}{d_1}e) \\ &\leq -\frac{3}{4}\gamma_0\|\tilde{\eta}\|^2 + \varrho\|\tilde{\eta}\|^2 + \frac{12}{\gamma_0}b_\rho^{-2}\tilde{z}_1^2 + \delta_3e^2 + \delta_4e^2\zeta^2(1 + e^2). \end{aligned} \tag{13}$$

5. CONTROL DESIGN

From (11) we have

$$\alpha = \lambda_1\vartheta_1 + \lambda_2\vartheta_2 = \lambda_1(\tilde{\eta}_1 + \eta_1 - b_\rho^{-1}Ne) + \lambda_2(\tilde{\eta}_2 + \eta_2 - b_\rho^{-1}Ne), \quad \lambda_1, \lambda_2 \in R.$$

Then from (6) we have

$$\dot{e} = \tilde{z}_1 + \frac{d_2}{d_1}e + (\varphi_1(y) - \varphi_1(q(w)))a + \bar{\xi}_1 + b_\rho(\tilde{\xi}_1 - \lambda_1(\tilde{\eta}_1 + \eta_1) - \lambda_2(\tilde{\eta}_2 + \eta_2) + (\lambda_1 + \lambda_2)b_\rho^{-1}Ne)$$

where $\tilde{\xi}_1 = \xi_1 - \hat{\xi}_1$ and $\hat{\xi}_1 = b_\rho^{-1}\bar{\xi}_1$.

For the virtual control $\hat{\xi}_1$, we design $\bar{\xi}_1$ as, with $c > 0$

$$\bar{\xi}_1 = -ce + b_\rho(\lambda_1\eta_1 + \lambda_2\eta_2) - (\lambda_1 + \lambda_2)Ne - \hat{l}e(1 + \zeta^2(1 + e^2)) \tag{14}$$

where $\hat{l}$ is an adaptive coefficient with $\hat{l}(0) = 0$. Then we have the resultant error dynamics

$$\dot{e} = \tilde{z}_1 - ce + \frac{d_2}{d_1}e - \hat{l}e(1 + \zeta^2(1 + e^2)) + (\varphi_1(y) - \varphi_2(q(w)))a + b_\rho(\tilde{\xi}_1 - \lambda_1\tilde{\eta}_1 - \lambda_2\tilde{\eta}_2).$$

Then for $V_e = \frac{1}{2}e^2$, there exists unknown positive real constants δ_5 and δ_6 and a sufficiently large unknown positive constant β such that

$$\begin{aligned}\dot{V}_e &= -ce^2 + e\tilde{z}_1 + \frac{d_2}{d_1}e^2 + eb_\rho(\tilde{\xi}_1 - \lambda_1\tilde{\eta}_1 - \lambda_2\tilde{\eta}_2)\\ &\quad + e(\varphi_1(y) - \varphi_1(q(w)))a - \hat{l}e^2(1 + \zeta^2(1 + e^2))\\ &\le -ce^2 + \frac{1}{8}\beta\tilde{z}_1^2 + \frac{1}{4}\gamma_0\,|\tilde{\eta}|^2 + \delta_5 e^2 + \delta_6 e^2\zeta(1 + e^2)\\ &\quad - \hat{l}e^2(1 + \zeta^2(1 + e^2)) + b_\rho e\tilde{\xi}_1.\end{aligned} \tag{15}$$

If the relative degree $\rho = 1$, we set $u = \hat{\xi}_1$. For $\rho > 1$, the adaptive backstepping method can be used to obtain the following results:

$$\begin{aligned}\hat{\xi}_2 &= -b_\rho e - c_1\tilde{\xi}_1 - k_1(\frac{\partial\hat{\xi}_1}{\partial e})^2\tilde{\xi}_1 + \frac{\partial\hat{\xi}_1}{\partial e}[b_\rho(\xi_1 - \lambda_1\eta_1 - \lambda_2\eta_2) + (\lambda_1 + \lambda_2)Ne]\\ &\quad + \frac{\partial\hat{\xi}_1}{\partial\hat{\eta}}\dot{\hat{\eta}} + \frac{\partial\hat{\xi}_1}{\partial\hat{l}}\dot{\hat{l}}\\ \hat{\xi}_i &= -\tilde{\xi}_{i-2} - c_{i-1}\tilde{\xi}_{i-1} - k_{i-1}(\frac{\partial\hat{\xi}_{i-1}}{\partial e})^2\tilde{\xi}_{i-1} + \frac{\partial\hat{\xi}_{i-1}}{\partial e}[b_\rho(\xi_1 - \lambda_1\eta_1 - \lambda_2\eta_2)\\ &\quad + (\lambda_1 + \lambda_2)Ne] + \frac{\partial\hat{\xi}_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}} + \frac{\partial\hat{\xi}_{i-1}}{\partial\hat{l}}\dot{\hat{l}}, \text{ for } i = 3, \dots, \rho,\end{aligned} \tag{16}$$

where $\tilde{\xi}_\rho = u - \hat{\xi}_\rho$, $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$ for $i = 2, \dots, \rho-2$, c_i and k_i, $i = 2, \dots, \rho-1$, are positive real design parameters. When $i = \rho$, the control input appears in the dynamics of $\tilde{\xi}_i$ through the term $\tilde{\xi}_\rho$. Finally, we design the control input by setting $\tilde{\xi}_\rho = 0$, which gives

$$u = \hat{\xi}_\rho. \tag{17}$$

In the following sections, we will establish the boundedness of all the variables and the convergence to zero of the measurement output.

Define a Lyapunov function candidate

$$V = \beta V_z + V_\eta + V_e + \frac{1}{2}\gamma^{-1}(\hat{l} - l)^2 + \frac{1}{2}\sum_{i=1}^{\rho-1} \tilde{\xi}_i^2 \tag{18}$$

where the positive real β is chosen, with the design of $\hat{\xi}_i$, for $i = 1, \dots, \rho$, the dynamics of $\tilde{\xi}_i$ can be evaluated easily. According to the dynamics of V_z in (7), the dynamics of V_η in (13), the dynamics of V_e in (15), and the virtual controls which designed in the above, we have the derivative of V as

$$\begin{aligned}\dot{V} \le& -\beta\tilde{z}^T\tilde{z} - \gamma_0\|\tilde{\eta}\|^2 - ce^2 + \beta(\delta_1 e^2 + \delta_2 e^2\zeta^2(1+e^2)) + \delta_3 e^2 \\ &+ \delta_4 e^2\zeta^2(1+e^2) + \delta_5 e^2 + \delta_6 e^2\zeta^2(1+e^2) - \hat{l}e^2(1+\zeta^2(1+e^2)) + b_\rho e\tilde{\xi}_1 \\ &+ \gamma^{-1}(\hat{l} - l)\dot{\hat{l}} - b_\rho e\tilde{\xi}_1 + \sum_{i=1}^{\rho-1}[-c_i\tilde{\xi}_i^2 - g_i(\frac{\partial\hat{\xi}_i}{\partial e})^2\tilde{\xi}_i^2 \\ &- \tilde{\xi}_i(\frac{\partial\hat{\xi}_i}{\partial e})(\tilde{z}_1 + \frac{d_2}{d_1}e + (\varphi_1(y) - \varphi_1(q(w)))a) + \tilde{\xi}_i(\frac{\partial\hat{\xi}_i}{\partial e})b_\rho(\lambda_1\tilde{\eta}_1 + \lambda_2\tilde{\eta}_2)].\end{aligned} \tag{19}$$

By using the inequalities $2xy \le rx^2 + r^{-1}y^2$ or $xy \le rx^2 + (4r^{-1})y^2$ for $x > 0$, $y > 0$ and r being any positive real constant, the stability analysis could be proceeded to deal with the cross terms between the variables $\tilde{\eta}, \tilde{z}, e, \tilde{\xi}$ for $i = 1, \dots, \rho-1$. Then we let $l = \beta(\delta_1 + \delta_2) + \delta_3 + \delta_4 + \delta_5 + \delta_6$, which is an unknown positive real constant. And define

$$\dot{\hat{l}} = \gamma e^2(1 + \zeta^2(1+e^2)). \tag{20}$$

It can be shown that there exist sufficiently big positive real constants $\beta \ge (\frac{96}{5\gamma_0})b_\rho^{-2}$ and $\varrho \le \frac{1}{4}\gamma_0$ such that the following result holds

$$\dot{V} \le -ce^2 - \beta\|\tilde{z}\|^2 - \frac{1}{4}\gamma_0\|\tilde{\eta}\|^2 - \sum_{i=1}^{\rho-1} c_i\tilde{\xi}_i^2. \tag{21}$$

The boundedness of V further implies $\tilde{\eta}, \tilde{z}, e, \tilde{\xi} \in L_2 \cap L_\infty$ for $i = 1, \dots, \rho-1$, and the boundedness of l. Since the disturbance w is bounded, $\tilde{\eta}, \tilde{z}, e \in L_\infty$ implies the boundedness of y, z, η, which further implies the boundedness of $\hat{\xi}_1$ and the boundedness of ξ_1. The boundedness of $\hat{\xi}_1$ and ξ_1, together with the boundedness of e, η, l, implies the boundedness of $\hat{\xi}_2$ and then the boundedness of ξ_2 follows the boundedness of $\tilde{\xi}_2$. Applying the above reasoning recursively, we can establish the boundedness of $\hat{\xi}_i$ for $i = 2, \ldots, \rho - 1$. Then we can conclude that all the variables are bounded. Furthermore, together with the derivatives of $\dot{\tilde{\eta}}, \dot{\tilde{z}}, \dot{e}, \dot{\tilde{\xi}}$ are bounded, by invoking Barbalat's lemma, we have $\lim_{t\to\infty} \tilde{z} = 0$, $\lim_{t\to\infty} \tilde{\eta} = 0$, $\lim_{t\to\infty} e = 0$ and $\lim_{t\to\infty} \tilde{\xi} = 0$. The result of this section is summarized in the following.

Theorem 1. For the uncertain nonlinear systems (1) with nonlinear exosystem (2), satisfying Assumption 1-4, reveals that the feedback control system that consists of the ξ filters (5), the nonlinear internal model (12), the control input (17), and the adaptive law (20) can solve the global robust output regulation problem.

6. ILLUSTRATIVE EXAMPLE

We use a simple example to illustrate the proposed control design, concentrating on the design of nonlinear internal model. Considering a first order system

$$\dot{y} = y + \ell \sin y - \ell \sin w_1 + w_2 + u$$

$$e = y - w_1$$

where ℓ is an unknown parameter, the nonharmonic periodic disturbance $\alpha = -w_1$ is the output of the following Van der Pol oscillator:

$$\dot{w}_1 = w_2$$

$$\dot{w}_2 = -w_1 + 0.5(1 - w_1^2)w_2.$$

Then, it is easy to show that ν satisfies the following equation:

$$\ddot{\nu} + \nu - 0.5\dot{\nu}(1 - \nu^2) = 0.$$

Let

$$\gamma(\nu, \dot{\nu}) = -\nu + 0.5\dot{\nu}(1 - \nu^2), \quad \theta = col(\nu, \dot{\nu})$$

and

$$\begin{cases} q(w) = w_1 \\ \pi = w_1. \end{cases}$$

From the exosystem and the desired feedforward input α, it can be seen that θ is in the format of (9) with

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \psi = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad T = \begin{bmatrix} -0.5 & 2.75 \\ -1 & 1 \end{bmatrix}.$$

Let

$$M = \begin{bmatrix} -2 & 5 \\ 0 & -1 \end{bmatrix}, \quad N = \begin{bmatrix} 4 & -1 \end{bmatrix}^T.$$

We can easily verify that (M, N) is a pair of controllable matrices with M being Hurwitz. Then based on the proposed control method, the internal model is designed as the following

$$\dot{\eta}_1 = -2(\eta_1 - 4e) + 5(\eta_2 + e) + 2.75\gamma(0.4444(\eta_1 - 4e) - 1.2222(\eta_2 + e)) + 4u$$

$$\dot{\eta}_2 = -(\eta_2 + e) + \gamma(0.4444(\eta_1 - 4e) - 0.2222(\eta_2 + e)) - u.$$

The control input is given by

$$u = -c_0 e + 0.4444\eta_1 - 1.2222\eta_2 - \hat{l}e(1 + (1 + e^2)^2)$$

$$\dot{\hat{l}} = \gamma e^2(1 + (1 + e^2)^2).$$

In the simulation, the initial condition is set to be $c_0 = 10, \ell = 10, \gamma = 10$, and the initial state is $w_1(0) = 2, w_2(0) = 2, y(0) = 1$. The phase portrait of Van der Pol oscillator, the system error output, the control input, the feedforward term and its estimation are shown in Figs.1-4 respectively. As illustrated in the figures, the diturbance is well reproduced by the designed nonlinear internal model, and the system error is regulated to zero.

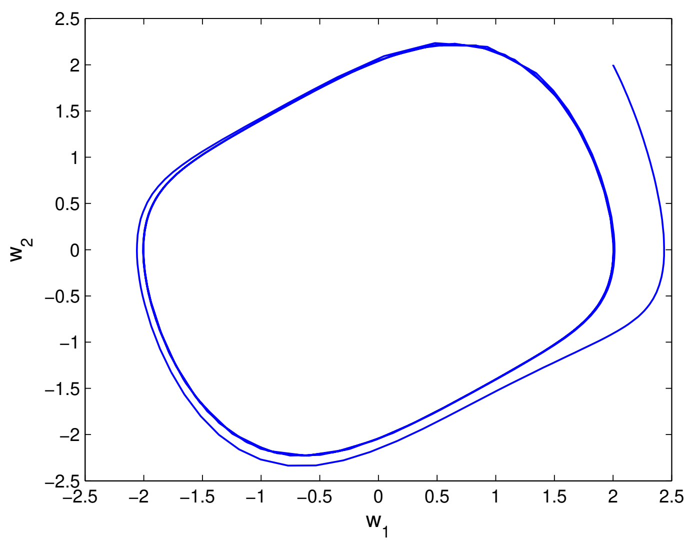

Fig. 1. Phase portrait of the exosystem.

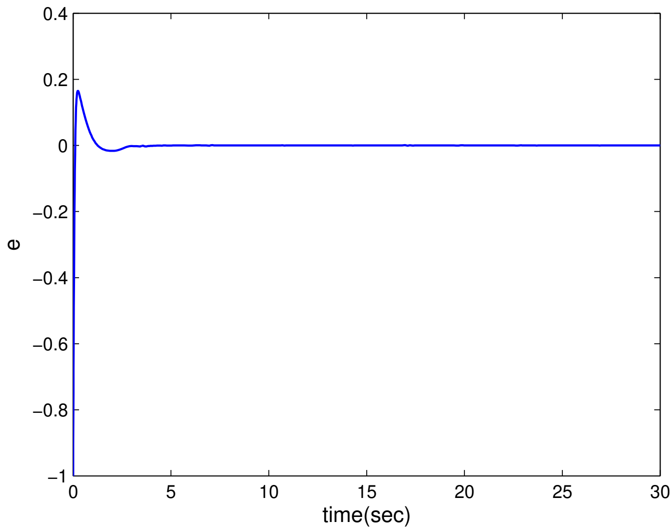

Fig. 2. The systems output e.

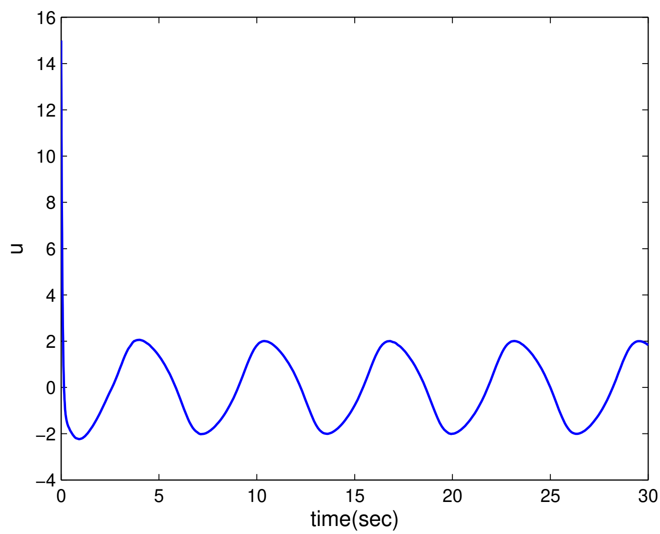

Fig. 3. The system input u.

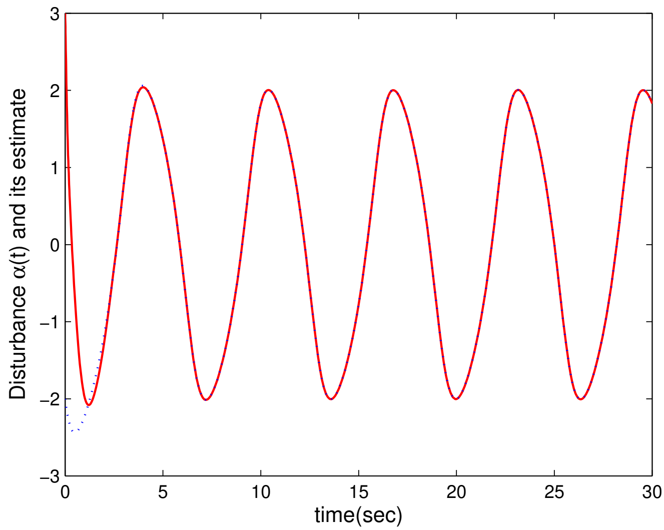

Fig. 4. Disturbance $\alpha(t)$ and its estimate.

7. CONCLUSIONS

We have proposed a new control design method for output regulation with nonlinear exosystems, and a new nonlinear internal model is proposed. With the proposed internal model design, the global robust output regulation problem has been solved for a class of nonlinear output feedback systems. The proposed controller ensures that all the signals in the closed-loop system are bounded and the measurement output converges to zero. Simulation results demonstrates the effectiveness of our algorithm.

ACKNOWLEDGEMENTS

This work was partially supported by National Natural Science Foundation of China (61663030, 61663032), Natural Science Foundation of Jiangxi Province (20142BAB207021), the Foundation of Jiangxi Educational Committee (GJJ150753), Key Laboratory of Nondestructive Testing (Nanchang Hangkong University), Ministry of Education (ZD29529005) and the reform project of degree and postgraduate education in Jiangxi (JXYJG-2017-131).

(Received December 3, 2018)

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