OUTPUT FEEDBACK H_∞ CONTROL OF NETWORKED CONTROL SYSTEMS BASED ON TWO CHANNEL EVENT-TRIGGERED MECHANISMS

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In this paper, we study dynamical output feedback H_∞ control for networked control systems (NCSs) based on two channel event-triggered mechanisms, which are proposed on both sides of the sensor and the controller. The output feedback H_∞ controller is constructed by taking random network-induced delays into consideration without data buffer units. The controlled plant and the output feedback controller are updated immediately by the sampled input and the sampled output, respectively. By using the approaches of time delay and interval decomposition, linear matrix inequality (LMI) based sufficient conditions are presented to guarantee that the closed-loop system satisfies H_∞ performance. Finally, we provide numerical simulations to illustrate effectiveness of the proposed method.

Keywords: output feedback H_∞ control, event-triggered mechanism, interval decomposition, NCSs, LMI

Classification: 93B52, 93B70

1. INTRODUCTION

A networked control system (NCS) is a type of system, which mainly includes sensor nodes, controller units, actuator units and a controlled plant. With the development of computer technology, network communication and control technology, NCS has been applied in various fields, such as aviation, power system, transportation, factory automation [4, 9, 15, 19, 32, 33]. It has also received widespread attention because of low installation and maintenance costs, easy reconfiguration, high reliability, small wiring volume and high system flexibility. However, the involvement of network (especially wireless network) may bring some problems, for example, transmission delay, packet loss, quantization [20, 21, 28, 35]. In order to solve these issues, the conventional periodic sampling (time-triggered) mechanism has been widely used [8, 12, 22, 25, 29]. In time-triggered mechanism, all sampled data will be transmitted through the network, which may waste the communication bandwidth and communication resources. Therefore, it is necessary to find a new method to solve this problem.

Recently, event-triggered mechanisms that can determine whether the sampled signals are transmitted or not have received extensive attention [3, 5, 6, 7, 13, 14, 16, 18,

23, 24, 26, 27, 30, 31]. In comparison to the traditional periodic sampling method, the network communication burden and the computing resources can be reduced. For instance, in [26], the authors studied event-triggered H_∞ control for NCS with network-induced delay via state feedback. A co-design method of event-triggered mechanism and controller feedback gain was proposed for NCS with network-induced delay and packet loss [5]. In the above literatures, most of the authors proposed controller design by state feedback under the event-triggered mechanism, and less mentioned controller design by output feedback. In [24], a dynamical output feedback controller design method was proposed for NCS based on an event-triggered scheme. The authors in [30] presented a discrete event-triggered mechanism and investigated dynamical output feedback control for NCS. The dynamical output feedback H_∞ controller was proposed for NCS with non-uniform sampling periods and an event-triggered communication scheme [3]. In [18], the authors considered the difference in transmission delays of two transmission channels. An interval decomposition method was proposed to ensure that the output feedback controller and the controlled plant were updated at the same time interval.

In conclusion, the event-triggered mechanism is designed on the sensor-to-controller (S-C) side, which may reduce the network resources on the S-C side. However, due to limitation of the network resources and bandwidth of the controller-to-actuator(C-A) side, design of dual-side event-triggered mechanisms have been proposed by researches. For instance, the authors studied stability and $\mathcal{L}_\infty$ performance of the closed-loop system based on a dynamical output feedback controller with decentralized event-triggered mechanism [7]. In [16], the problem of output feedback control was proposed for sampled-data systems with event detectors. However, transmission delays were not considered both in [7] and [16]. The authors in [13] introduced a novel hybrid event-triggered scheme for NCS with networked delay. In order to facilitate analysis, they put the total transmission delay on the actuator side. In [27], two-channel event-triggered transmission strategies were proposed for NCS with communication delay via state feedback and a novel delay system model was constructed. In [14], the authors studied dual-side event-triggered output feedback H_∞ control for NCS with network delays. However, for the convenience of analysis, the authors only considered the case with a fixed transmission delay.

In this paper, we study output feedback H_∞ control for NCSs with two channel event-triggered schemes. As shown in Figure 1, the event-triggered NCS contains a plant, two samplers, two sensors, a dynamical output feedback controller, two zero-order holds (ZOHs), an actuator and two event generators. The sensor and sampler in the S-C (or C-A) channel are used to measure and sample the output of the plant (or the dynamical output controller), respectively. We assume that the sensor and the sampler are clock-driven, and the dynamical output feedback controller and the actuator are event-driven. The samplers in both channels work at a fixed period h. In the S-C channel, all sampled data will be transmitted from the event generator 1 to the dynamical output feedback controller via a network with transmission delays. In the C-A channel, all sampled data will be sent from the event generator 2 to the actuator in the same way. The dynamical output feedback control and the plant are updated immediately by the sampled output in the S-C channel and the sampled input in the C-A channel, respectively. Our major contributions include:
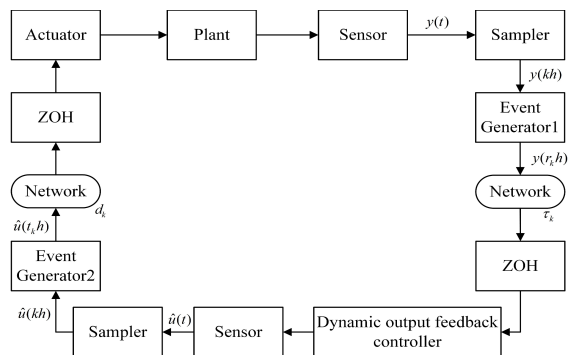
Fig. 1. The structure of NCS with two channel event-triggered communication schemes.

1. Output feedback H_∞ control is addressed for NCS with dual-channel event-triggered mechanisms with random transmission delay. We remove the data buffer units and process the transmitted data whenever they arrive.

2. The approaches of time delay and interval decomposition are introduced to derive the closed-loop system. LMI-based sufficient conditions are presented to guarantee H_∞ performance of the closed-loop system.

This paper is organized as follows. In Section 2, we present some preliminaries and problem description. In Section 3, we give our main results: output feedback H_∞ control design with two side event-triggered schemes. Numerical simulations are provided to illustrate the validity of the proposed design methods in Section 4. This paper concludes in Section 5.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

In this paper, we consider a class of linear time-invariant systems given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \\ z(t) = C_1 x(t), \\ y(t) = C_2 x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $z(t) \in \mathbb{R}^p$, $y(t) \in \mathbb{R}^r$ and $\omega(t)$ are the state vector, the control input, the control output, the measured output and the disturbance input, respectively. A, B, B_ω, C_1, C_2 are known matrices with appropriate dimensions, and $x(t_0) = x_0$ is the initial condition.

For the system (1), we make the following assumptions:

1. The system output is sampled at a fixed period h. $S_1 = \{kh \mid k \in \mathbb{N}\}$ represents the set of sampled instants.

2. In the S-C channel, the set of transmission instants is described by $S_2 = \{r_k h \mid k \in \mathbb{N}\}$. $S_3 = \{t_k h \mid k \in \mathbb{N}\}$ indicates released instants by the event generator 2 in the C-A channel. The logic ZOH before the output feedback controller (or actuator) will keep the input of the output feedback controller (or actuator) unchanged until the next updated data on the S-C (or C-A) side arrive.

3. τ_k and d_k are transmission delays from the sensor to the dynamical output feedback controller and from the dynamical output feedback controller to the plant, respectively. We assume that the upper bounds of τ_k and d_k are $\bar{\tau}$ and $\bar{d}$, respectively, and there exist two positive real numbers n_1 and n_2 such that $\bar{\tau} \leq n_1 h$ and $\bar{d} \leq n_2 h$.

Next, we propose two event-triggered schemes in the S-C channel and in C-A channel. As in [26], the event-triggered mechanism before the output feedback controller is designed as

$$[y(r_k h + lh) - y(r_k h)]^T \Omega_1 [y(r_k h + lh) - y(r_k h)] \leq \sigma_1 y(r_k h + lh)^T \Omega_1 y(r_k h + lh), \quad (2)$$

where Ω_1 is a symmetric positive definite matrix, σ_1 is a given scalar parameter, $y(r_k h + lh)$ $(l = 1, 2, \ldots)$ represents the value of the measurement output $y(t)$ at the current sampling instant, and $y(r_k h)$ represents the value of the measurement output $y(t)$ at the latest triggering instant. The event-triggered mechanism (2) decides whether the current sampled output $y(r_k h + lh)$ is transmitted or not. The sampled output data will be sent to the first ZOH via the network when the event-triggered mechanism (2) is violated; otherwise, the sampled output date will be discarded immediately. From the event-triggered mechanism (2), we know that the next release time is $r_{k+1} h$, where

$$r_{k+1} h = r_k h + \min_l \{lh | [y(r_k h + lh) - y(r_k h)]^T \Omega_1 [y(r_k h + lh) - y(r_k h)]$$
$$\geq \sigma_1 y(r_k h + lh)^T \Omega_1 y(r_k h + lh)\}. \quad (3)$$

The other event-triggered mechanism before the actuator is expressed as

$$e_k{}^T(t) \Omega_2 e_k(t) \leq \sigma_2 \hat{x}(t_k h + jh)^T \Omega_2 \hat{x}(t_k h + jh), \quad (4)$$

where $e_k(t) = \hat{x}(t_k h) - \hat{x}(t_k h + jh)$, Ω_2 is a symmetric positive definite matrix, σ_2 is a given scalar parameter, $u(t_k h) = K\hat{x}(t_k h)$ and $u(t_k h + jh) = K\hat{x}(t_k h + jh)$ $(j = 1, 2, \ldots)$ are the latest transmitted control signal and controller output data at the current sampling time, respectively (K is the control gain and will be determined later). The sampled control data will not be sent to the plant via the network when the event-triggered mechanism (4) is satisfied, which means that it will be discarded immediately. From the event-triggered mechanism (4), we know that the next release time is $t_{k+1} h$, where

$$t_{k+1} h = t_k h + \min_j \{jh | e_k{}^T(t) \Omega_2 e_k(t) \geq \sigma_2 \hat{x}(t_k h + jh)^T \Omega_2 \hat{x}(t_k h + jh)\}. \quad (5)$$

Remark 2.1. The event-triggered mechanism (2) at the sensor side decides whether the plant output data are transmitted or not, and the event-triggered mechanism (4) at the

controller side decides whether the output data are transmitted or not. Thus the data transmission of the event-triggered mechanism (2) and the event-triggered mechanism (4) do not affect each other, which means that the event-triggered mechanisms on both channels can be triggered simultaneously, but not necessarily synchronously.

Remark 2.2. In [14], for the convenience of analysis, the author defined the maximum transmission delay of the dual-side transmission channels as τ, and only considered the case with a fixed time delay. In this paper, we take into account stochastic transmission delays of the sensor side and the actuator side, and remove the data buffer units. That is, the dynamical output feedback controller and the controlled system are updated by the sampled output and the sampled input, respectively, once they arrive.

According to the control strategy shown in Figure 1, we design the following dynamical output feedback controller,

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(r_kh) - C_2\hat{x}(r_kh)), \\ u(t) = K\hat{x}(r_kh), t \in [r_kh + \tau_k, r_{k+1}h + \tau_{k+1}), k \geq 0, \end{cases} \quad (6)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the controller, $K \in \mathbb{R}^{m \times p}$ denotes the control gain and $L \in \mathbb{R}^{m \times m}$ indicates the observer gain. The controlled system is given by

$$\dot{x}(t) = Ax(t) + BK\hat{x}(t_kh) + B_\omega\omega(t), t \in [t_kh + d_k, t_{k+1}h + d_{k+1}). \quad (7)$$

Since τ_k and d_k are different, the event generators are sampled simultaneously at a fixed period. Then the output feedback controller (6) and the system (7) are updated at different time intervals. As in [30], we use the updated interval of (6) to decompose the updated interval of (7) to obtain a closed-loop system. Assume that there exist two positive integers q_1 and q_2 such that

$$t_kh + d_k \in [r_{q_1}h + \tau_{q_1}, r_{q_1+1}h + \tau_{q_1+1}), t_{k+1}h + d_{k+1} \in [r_{q_2}h + \tau_{q_2}, r_{q_2+1}h + \tau_{q_2+1}).$$

By interval decomposition, we can get

$$[t_kh + d_k, t_{k+1}h + d_{k+1}) = \Phi^q = \Phi^{0,q} \cup_{s=1}^{r_{q_2}-r_{q_1}-1} \Phi_s{}^q \cup \Phi^{1,q}. \quad (8)$$

where

$$q = \begin{cases} r_{q_1}, t \in [t_kh + d_k, r_{q_1+1}h + \tau_{q_1+1}), \\ r_{q_1+s}, t \in [r_{q_1+s}h + \tau_{q_1+s}, r_{q_1+s+1}h + \tau_{q_1+s+1}), \\ s = 1, 2, \ldots, r_{q_2} - r_{q_1} - 1, \\ r_{q_2}, t \in [r_{q_2}h + \tau_{q_2}, t_{k+1}h + d_{k+1}), \end{cases} \quad (9)$$

and

$$\Phi^q = \begin{cases} \Phi^{0,q} = [t_kh + d_k, r_{q_1+1}h + \tau_{q_1+1}), q = r_{q_1}, \\ \Phi_s^q = [r_{q_1+s}h + \tau_{q_1+s}, r_{q_1+s+1}h + \tau_{q_1+s+1}), q = r_{q_1+s}, \\ s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ \Phi^{1,q} = [r_{q_2}h + \tau_{q_2}, t_{k+1}h + d_{k+1}), q = r_{q_2}. \end{cases} \quad (10)$$

Further, we decompose the above intervals by the sampled period h. For the interval $\Phi^{0,q}$, note that $t_kh + d_k \in [r_{q_1}h + \tau_{q_1}, r_{q_1+1}h + \tau_{q_1+1})$, then, there exists a positive integer

$n_0 = \min\limits_{j}\{jh | t_k h + d_k < r_{q_1} h + \tau_{q_1} + jh\}$. Since $r_{q_1} h + \tau_{q_1} + n_0 h \leq r_{q_1+1} h + \tau_{q_1+1}$, we consider the following intervals,

$$[t_k h + d_k, r_{q_1} h + \tau_{q_1} + n_0 h), [r_{q_1} h + \tau_{q_1} + s_1 h, r_{q_1} h + \tau_{q_1} + s_1 h + h).$$

Then, there exists a positive integer $N^{0,q}$ satisfying

$$r_{q_1} h + \tau_{q_1} + N^{0,q} h < r_{q_1+1} h + \tau_{q_1+1} < r_{q_1} h + \tau_{q_1} + N^{0,q} h + h.$$

Therefore, the time interval $\Phi^{0,q}$ can be decomposed into the following subintervals,

$$\Phi^{0,q} = I^{0,q}_{n_0-1} \cup^{N^{0,q}}_{s_1=n_0} I^{0,q}_{s_1}, q = r_{q_1},$$

where

$$\begin{cases} I^{0,q}_{n_0-1} = [t_k h + d_k, r_{q_1} h + \tau_{q_1} + n_0 h), \\ I^{0,q}_{s_1} = [r_{q_1} h + \tau_{q_1} + s_1 h, r_{q_1} h + \tau_{q_1} + s_1 h + h), s_1 = n_0, \cdots, N^{0,q} - 1, \\ I^{0,q}_{N^{0,q}} = [r_{q_1} h + \tau_{q_1} + N^{0,q} h, r_{q_1+1} h + \tau_{q_1+1}). \end{cases}$$

Let us define $\eta(t)$ and $e_q(t)$ on the interval $\Phi^{0,q}$ as

$$\eta(t) = \begin{cases} t - r_{q_1} h - (n_0 - 1)h, t \in I^{0,q}_{n_0-1}, \\ t - r_{q_1} h - s_1 h, t \in I^{0,q}_{s_1}, s_1 = n_0, \cdots, N^{0,q} - 1, \\ t - r_{q_1} h - N^{0,q} h, t \in I^{0,q}_{N^{0,q}}, \end{cases} \tag{11}$$

and

$$e_q(t) = \begin{cases} y(r_{q_1} h) - y(r_{q_1} h + (n_0 - 1)h), t \in I^{0,q}_{n_0-1}, \\ y(r_{q_1} h) - y(r_{q_1} h + s_1 h), t \in I^{0,q}_{s_1}, s_1 = n_0, \cdots, N^{0,q} - 1, \\ y(r_{q_1} h) - y(r_{q_1} h + N^{0,q} h), t \in I^{0,q}_{N^{0,q}}. \end{cases} \tag{12}$$

Then, $\eta(t)$ satisfies

$$\begin{cases} 0 < \tau_1 \leq \eta(t) < r_{q_1} h + \tau_{q_1} + n_0 h - r_{q_1} h - (n_0 - 1)h < (1 + n_1)h, t \in I^{0,q}_{n_0-1}, \\ 0 < \tau_1 \leq \tau_{q_1} < \eta(t) < r_{q_1} h + \tau_{q_1} + s_1 h + h - r_{q_1} h - s_1 h < (1 + n_1)h, t \in I^{0,q}_{s_1}, \\ s_1 = n_0, \cdots, N^{0,q} - 1, \\ 0 < \tau_1 \leq \tau_{q_1} < \eta(t) < (1 + n_1)h, t \in I^{0,q}_{N^{0,q}}. \end{cases}$$

where $\tau_1 = \min\{\tau_k\}, \tau_{M_1} = (1 + n_1)h$.

Similarly, Φ^q_s and $\Phi^{1,q}$ can be decomposed by

$$\Phi^q_s = I^q_{s.0} \cup^{N^q_s}_{s_2=1} I^q_{s.s_2}, q = r_{q_1+s}, s = 1, 2, \ldots, r_{q_2} - r_{q_1} - 1,$$

and

$$\Phi^{1,q} = \cup^{N^{1,q}}_{s_3=0} I^{1,q}_{s_3}, q = r_{q_2},$$

where

$$\begin{cases} I^q_{s.0} = [r_{q_1+s} h + \tau_{q_1+s}, r_{q_1+s} h + \tau_{q_1+s} + h), s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ I^q_{s.s_2} = [r_{q_1+s} h + \tau_{q_1+s} + s_2 h, r_{q_1+s} h + \tau_{q_1+s} + s_2 h + h), s_2 = 1, \cdots, N^q_s - 1, \\ I^q_{s.N^q_s} = [r_{q_1+s} h + \tau_{q_1+s} + N^q_s h, r_{q_1+s+1} h + \tau_{q_1+s+1}), \end{cases}$$

and

$$\begin{cases} I_0^{1,q} = [r_{q_2}h + \tau_{q_2}, r_{q_2}h + \tau_{q_2} + h), \\ I_{s_3}^{1,q} = [r_{q_2}h + \tau_{q_2} + s_3h, r_{q_2}h + \tau_{q_2} + s_3h + h), s_3 = 1, \cdots, N^{1,q} - 1, \\ I_{N^{1,q}}^{1,q} = [r_{q_2}h + \tau_{q_2} + N_s{}^q h, t_{k+1}h + d_{k+1}). \end{cases}$$

Further, we define $\eta(t)$ and $e_q(t)$ on the intervals Φ_s^q and $\Phi^{1,q}$, respectively,

$$\eta(t) = \begin{cases} t - r_{q_1+s}h, t \in I_{s.0}^q, s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ t - r_{q_1+s}h - s_2h, t \in I_{s.s_2}^q, s_2 = 1, \cdots, N_s{}^q - 1, \\ t - r_{q_1+s}h - N_s{}^q h, t \in I_{s.N_s^q}^q, \\ t - r_{q_2}h, t \in I_0^{1,q}, \\ t - r_{q_2}h - s_3h, t \in I_{s_3}^{1,q}, s_3 = 1, \cdots, N^{1,q} - 1, \\ t - r_{q_2}h - N^{1,q}h, t \in I_{N^{1,q}}^{1,q}, \end{cases} \tag{13}$$

and

$$e_q(t) = \begin{cases} 0, t \in I_{s.0}^q, s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ y(r_{q_1+s}h) - y(r_{q_1+s}h + s_2h), t \in I_{s.s_2}^q, s_2 = 1, \cdots, N_s{}^q - 1, \\ y(r_{q_1+s}h) - y(r_{q_1+s}h + N_s{}^q h), t \in I_{s.N_s^q}^q, \\ 0, t \in I_0^{1,q}, \\ y(r_{q_2}h) - y(r_{q_2}h + s_3h), t \in I_{s_3}^{1,q}, s_3 = 1, \cdots, N^{1,q} - 1, \\ y(r_{q_2}h) - y(r_{q_2}h + N_s{}^q h), t \in I_{N^{1,q}}^{1,q}. \end{cases} \tag{14}$$

Then,

$$\begin{cases} 0 < \tau_1 \le \tau_{q_1+s} \le \eta(t) < (1+n_1)h, t \in I_{s.0}^q, s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ 0 < \tau_1 \le \tau_{q_1+s} \le \eta(t) < (1+n_1)h, t \in I_{s.s_2}^q, s_2 = 1, \cdots, N_s{}^q - 1, \\ 0 < \tau_1 \le \tau_{q_1+s} \le \eta(t) < (1+n_1)h, t \in I_{s.N_s^q}^q, s = 1, 2, \cdots, r_{q_2} - r_{q_1} - 1, \\ 0 < \tau_1 \le \tau_{q_2} \le \eta(t) < (1+n_1)h, t \in I_0^{1,q}, \\ 0 < \tau_1 \le \tau_{q_2} \le \eta(t) < (1+n_1)h, t \in I_{s_3}^{1,q}, s_3 = 1, \cdots, N^{1,q} - 1, \\ 0 < \tau_1 \le \tau_{q_2} \le \eta(t) < (1+n_1)h, t \in I_{N^{1,q}}^{1,q}. \end{cases}$$

By (11), (12), (13) and (14), we can obtain that

$$y(qh) = y(t - \eta(t)) + e_q(t) = C_2 x(t) - C_2 \int_{t-\eta(t)}^t \dot{x}(s)\,\mathrm{d}s + e_q(t). \tag{15}$$

The event-triggered mechanism (2) is rewritten as

$$e_q{}^T(t)\Omega_1 e_q(t) \le \sigma_1 y^T(t - \eta(t))\Omega_1 y(t - \eta(t))$$
$$= \sigma_1 [C_2 x(t) - C_2 \int_{t-\eta(t)}^t \dot{x}(s)\,\mathrm{d}s]^T \Omega_1 [C_2 x(t) - C_2 \int_{t-\eta(t)}^t \dot{x}(s)\,\mathrm{d}s]. \tag{16}$$

Let $\varsigma(t) = t - qh$. Then, the dynamical output feedback control law (6) can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + BK\hat{x}(t - \varsigma(t)) + L(y(t - \eta(t)) + e_q(t) - C_2\hat{x}(t - \varsigma(t))), t \in \Phi^q. \tag{17}$$

Since $\Phi^q = [t_k h + d_k, t_{k+1} h + d_{k+1})$, for the event generator 2, we can obtain

$$[t_k h + d_k, t_{k+1} h + d_{k+1}) = [t_k h + d_k, t_k h + d_k + h)$$
$$\cup_{i=1}^{i=d_M-1} [t_k h + d_k + ih, t_k h + d_k + ih + h) \cup [t_k h + d_k + d_M h, t_{k+1} h + d_{k+1}).$$

Define

$$\tau(t) = \begin{cases} t - t_k h, t \in [t_k h + d_k, t_k h + d_k + h), \\ t - t_k h - ih, t \in [t_k h + d_k + ih, t_k h + d_k + ih + h), \\ i = 1, 2, \cdots, d_M - 1, \\ t - t_k h - d_M h, t \in [t_k h + d_k + d_M h, t_{k+1} h + d_{k+1}), \end{cases} \tag{18}$$

and

$$e_k(t) = \begin{cases} 0, t \in [t_k h + d_k, t_k h + d_k + h), \\ \hat{x}(t_k h) - \hat{x}(t_k h + ih), t \in [t_k h + d_k + ih, t_k h + d_k + ih + h), \\ i = 1, 2, \cdots, d_M - 1, \\ \hat{x}(t_k h) - \hat{x}(t_k h + d_M h), t \in [t_k h + d_k + d_M h, t_{k+1} h + d_{k+1}). \end{cases} \tag{19}$$

Then, $\tau(t)$ satisfies

$$0 < \tau_2 \le d_k \le \tau(t) < (1 + n_2)h, t \in [t_k h + d_k, t_{k+1} h + d_{k+1}),$$

where $\tau_2 = \min\{d_k\}, \tau_{M_2} = (1 + n_2)h$.

Therefore,

$$\hat{x}(t_k h) = \hat{x}(t - \tau(t)) + e_k(t), t_k h = t - \tau(t). \tag{20}$$

The event-triggered mechanism (4) is rewritten as

$$e_k^T(t)\Omega_2 e_k(t) \le \sigma_2 \hat{x}^T(t - \tau(t))\Omega_2 \hat{x}(t - \tau(t)). \tag{21}$$

Let $e(t) = x(t) - \hat{x}(t), \tau_M = \max\{\tau_{M_1}, \tau_{M_2}\}$. By (7), (15), (17) and (20), the closed-loop system can be obtained,

$$\begin{cases} \dot{x}(t) = (A + BK)x(t) - BKe(t) - BK \int_{t-\tau(t)}^t \dot{x}(s)\,ds + BK \int_{t-\tau(t)}^t \dot{e}(s)\,ds \\ + BKe_k(t) + B_\omega \omega(t), \\ \dot{e}(t) = (A - LC_2)e(t) - BK \int_{t-\tau(t)}^t \dot{x}(s)\,ds + BK \int_{t-\tau(t)}^t \dot{e}(s)\,ds \\ -(LC_2 - BK) \int_{t-\varsigma(t)}^t \dot{x}(s)\,ds + (LC_2 - BK) \int_{t-\varsigma(t)}^t \dot{e}(s)\,ds \\ + LC_2 \int_{t-\eta(t)}^t \dot{x}(s)\,ds + BKe_k(t) - Le_q(t) + B_\omega \omega(t), t \in \Phi^q. \end{cases} \tag{22}$$

To end this section, we introduce the following definition and four lemmas which are used later.

Definition 2.3. (Wang et al. [23]) The closed-loop system (22) with $\omega(t) = 0$ is asymptotically stable. If under zero initial condition, there exists an H_∞ performance index $\gamma > 0$, such that

$$\|x(t)\|_2 \le \gamma \|\omega(t)\|_2, \tag{23}$$

then, we call that (22) has H_∞ performance with γ.

Lemma 2.4. (Gu [10]) For any positive definite matrix $G \in \mathbb{R}^{n \times n}$, a scalar $d > 0$, vector function $\varpi(s) : [0, d] \to \mathbb{R}^n$, then the following inequality holds,

$$\left[\int_0^d \varpi(s) \, \mathrm{d}s \right]^T G \left[\int_0^d \varpi(s) \, \mathrm{d}s \right] \leq d \left[\int_0^d \varpi(s)^T G \varpi(s) \, \mathrm{d}s \right].$$

Lemma 2.5. (Chen and Fei [2]) For any positive definite matrix $Z \in \mathbb{R}^{n \times n}$, $Z = Z^T$, $S \in \mathbb{R}^{n \times n}$, $\begin{pmatrix} Z & S \\ * & Z \end{pmatrix} \geq 0$, a scalar function $\eta(t)$ satisfying $a \leq \eta(t) \leq b$, and a differentiable vector function $\chi(s)$, then the following inequality holds,

$$-(b-a) \int_{t-b}^{t-a} \dot{\chi}^T(s) Z \dot{\chi}(s) \, \mathrm{d}s \leq$$
$$-\left[\begin{array}{c} \chi(t-a) - \chi(t-\eta(t)) \\ \chi(t-\eta(t)) - \chi(t-b) \end{array} \right]^T \left[\begin{array}{cc} -Z & S \\ * & -Z \end{array} \right] \left[\begin{array}{c} \chi(t-a) - \chi(t-\eta(t)) \\ \chi(t-\eta(t)) - \chi(t-b) \end{array} \right].$$

Lemma 2.6. (Han [11]) For a given symmetric matrix $U \in \mathbb{R}^{n \times n}$, $U = U^T > 0$, a scalar $c > 0$, and a differentiable vector function $\vartheta(s)$, then the following inequality holds,

$$-c \int_{t-c}^{t} \dot{\vartheta}^T(s) U \dot{\vartheta}(s) \, \mathrm{d}s \leq -[\vartheta(t) - \vartheta(t-c)]^T U [\vartheta(t) - \vartheta(t-c)].$$

Lemma 2.7. (Boyd [1]) For a given symmetric matrix $W = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}$, where $W_{11} \in \mathbb{R}^{n \times n}$, then, the following three conditions are equivalent to each other,
(1)$W < 0$;
(2)$W_{11} < 0, W_{22} - W_{12}{}^T W_{11}{}^{-1} W_{12} < 0$;
(3)$W_{22} < 0, W_{11} - W_{12} W_{22}{}^{-1} W_{12}{}^T < 0$.

3. MAIN RESULTS

In this section, we construct a Lyapunov–Krasovskii functional and derive LMI-based sufficient conditions to ensure H_∞ performance of the closed-loop system (22).

Theorem 3.1. For given parameters n_1, n_2, τ_1, τ_2, σ_1, σ_2, h, the system (22) is asymptotic stable with an H_∞ performance index γ for the disturbance attenuation, if there exist symmetric matrices $P > 0$, $Q_i > 0 (i = 1, 2, 3, 4, 5)$, $R_j > 0 (j = 1, 2, 3, 4)$, $\Omega_1 > 0$, $\Omega_2 > 0$, and $S_1 > 0$, $S_2 > 0$, L and K with appropriate dimensions, such that

$$\begin{pmatrix} Q_3 & S_1 \\ * & Q_3 \end{pmatrix} \geq 0, \tag{24}$$

$$\begin{pmatrix} Q_5 & S_2 \\ * & Q_5 \end{pmatrix} \geq 0, \tag{25}$$

$$\begin{pmatrix} \Psi & \Pi_1{}^T\Theta_1 & \Pi_2{}^T\Theta_2 & \Pi_3{}^T\Theta_3 \\ * & \Sigma_1 & 0 & 0 \\ * & * & \Sigma_2 & 0 \\ * & * & * & \Sigma_3 \end{pmatrix} < 0, \tag{26}$$

where

$$\Psi = \begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ * & \Psi_4 & 0_{8\times6} \\ * & * & \Psi_5 \end{pmatrix},$$

$$\Psi_1 = \begin{pmatrix} \Psi_{11} & -PBK & 0 & 0 & 0 & 0 & Q_2 & 0 \\ * & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & Q_2 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & Q_3 - S_1{}^T & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & Q_3 - S_1{}^T \\ * & * & * & * & * & * & \Psi_{77} & 0 \\ * & * & * & * & * & * & * & \Psi_{88} \end{pmatrix},$$

$$\Psi_2 = \begin{pmatrix} Q_4 & 0 & 0 & 0 & 0 & 0 & -PBK & PBK \\ 0 & Q_4 & 0 & 0 & 0 & 0 & -PBK & PBK \\ Q_5 - S_2{}^T & 0 & 0 & 0 & Q_5 - S_2 & 0 & 0 & 0 \\ 0 & Q_5 - S_2{}^T & 0 & 0 & 0 & Q_5 - S_2 & 0 & 0 \\ 0 & 0 & Q_3 - S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_3 - S_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Psi_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & PBK & PB_\omega \\ \Psi_{2,17} & \Psi_{2,18} & PLC_2 & -PL & PBK & PB_\omega \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Psi_4 = \begin{pmatrix} \Psi_{99} & 0 & 0 & 0 & S_2 & 0 & 0 & 0 \\ * & \Psi_{10,10} & 0 & 0 & 0 & S_2 & 0 & 0 \\ * & * & \Psi_{11,11} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{12,12} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{13,13} & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{14,14} & 0 & 0 \\ * & * & * & * & * & * & -\frac{Q_1}{3\tau_M} & 0 \\ * & * & * & * & * & * & 0 & -\frac{Q_1}{2\tau_M} \end{pmatrix},$$

$\Psi_5 = diag\{-\frac{Q_1}{3\tau_M}, -\frac{Q_1}{2\tau_M}, -\frac{Q_1}{3\tau_M}, -\Omega_1, -\Omega_2, -\gamma^2 I\}$,

$\Psi_{11} = P(A + BK) + (A + BK)^T P + R_1 + R_3 - Q_2 - Q_4 + C_1^T C_1$,

$\Psi_{22} = P(A - LC_2) + (A - LC_2)^T P + R_1 + R_3 - Q_2 - Q_4$,

$\Psi_{33} = \Psi_{44} = -2Q_5 + S_2 + S_2^T, \Psi_{55} = \Psi_{66} = -2Q_3 + S_1 + S_1^T$,

$\Psi_{77} = \Psi_{88} = R_2 - R_1 - Q_2 - Q_3, \Psi_{99} = \Psi_{10,10} = R_4 - R_3 - Q_4 - Q_5$,

$\Psi_{11,11} = \Psi_{12,12} = -R_2 - Q_3, \Psi_{13,13} = \Psi_{14,14} = -R_4 - Q_5$,

$\Psi_{2,17} = -\Psi_{2,18} = -P(LC_2 - BK)$,

$\Pi_1^T = \begin{pmatrix} \Pi_{11}^T & \Pi_{12}^T \end{pmatrix}, \quad \Pi_2^T = \begin{pmatrix} \Pi_{21}^T & \Pi_{22}^T & \Pi_{23}^T & \Pi_{24}^T & \Pi_{25}^T \end{pmatrix}$,

$\Pi_3^T = \begin{pmatrix} \Pi_{31}^T & \Pi_{32}^T & \Pi_{33}^T & \Pi_{34}^T & \Pi_{35}^T \end{pmatrix}$,

$\Pi_{11} = \begin{pmatrix} C_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_2 & 0 & 0 & 0 \end{pmatrix}$,

$\Pi_{12} = \begin{pmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$,

$\Pi_{21} = \Pi_{22} = \Pi_{23} = \Pi_{24} = \Pi_{25} = \begin{pmatrix} A + BK & -BK & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -BK & BK & 0 & 0 & 0 & 0 & BK & B_\omega \end{pmatrix}$,

$\Pi_{31} = \Pi_{32} = \Pi_{33} = \Pi_{34} = \Pi_{35} = \begin{pmatrix} 0 & A - LC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -BK & BK & -(LC_2 - BK) & (LC_2 - BK) & LC_2 & -L & BK & B_\omega \end{pmatrix}$,

$\Theta_1 = \begin{pmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{pmatrix}, \Sigma_1 = \begin{pmatrix} -\frac{\Omega_1}{\sigma_1} & 0 \\ 0 & -\frac{\Omega_2}{\sigma_2} \end{pmatrix}$,

$\Sigma_2 = \Sigma_3 = diag\{-\frac{Q_1}{\tau_M}, -\frac{Q_2}{\tau_1^2}, -\frac{Q_3}{(\tau_{M_1} - \tau_1)^2}, -\frac{Q_4}{\tau_2^2}, -\frac{Q_5}{(\tau_{M_2} - \tau_2)^2}\}$,

$\Theta_2 = \Theta_3 = diag\{Q_1, Q_2, Q_3, Q_4, Q_5\}$.

P r o o f. Construct the Lyapunov–Krasovskii functional $V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$, where

$$V_1(t) = x(t)^T P x(t) + \int_{t-\tau_1}^t x(s)^T R_1 x(s)\,\mathrm{d}s + \int_{t-\tau_{M_1}}^{t-\tau_1} x(s)^T R_2 x(s)\,\mathrm{d}s$$
$$+ \int_{t-\tau_2}^t x(s)^T R_3 x(s)\,\mathrm{d}s + \int_{t-\tau_{M_2}}^{t-\tau_2} x(s)^T R_4 x(s)\,\mathrm{d}s, \tag{27}$$

$$V_2(t) = \int_{t-\tau_M}^t \int_\rho^t \dot{x}(s)^T Q_1 \dot{x}(s)\,\mathrm{d}s\mathrm{d}\rho + \tau_1 \int_{-\tau_1}^0 \int_{t+\rho}^t \dot{x}(s)^T Q_2 \dot{x}(s)\,\mathrm{d}s\mathrm{d}\rho +$$
$$(\tau_{M_1} - \tau_1) \int_{-\tau_{M_1}}^{-\tau_1} \int_\rho^t \dot{x}(s)^T Q_3 \dot{x}(s)\,\mathrm{d}s\mathrm{d}\rho + \tau_2 \int_{-\tau_2}^0 \int_\rho^t \dot{x}(s)^T Q_4 \dot{x}(s)\,\mathrm{d}s\mathrm{d}\rho \tag{28}$$
$$+ (\tau_{M_2} - \tau_2) \int_{-\tau_{M_2}}^{-\tau_2} \int_\rho^t \dot{x}(s)^T Q_5 \dot{x}(s)\,\mathrm{d}s\mathrm{d}\rho,$$

$$V_3(t) = e(t)^T P e(t) + \int_{t-\tau_1}^t e(s)^T R_1 e(s)\,\mathrm{d}s + \int_{t-\tau_{M_1}}^{t-\tau_1} e(s)^T R_2 e(s)\,\mathrm{d}s$$
$$+ \int_{t-\tau_2}^t e(s)^T R_3 e(s)\,\mathrm{d}s + \int_{t-\tau_{M_2}}^{t-\tau_2} e(s)^T R_4 e(s)\,\mathrm{d}s, \tag{29}$$

$$V_4(t) = \int_{t-\tau_M}^{t} \int_{\rho}^{t} \dot{e}(s)^T Q_1 \dot{e}(s)\, \mathrm{d}s \mathrm{d}\rho + \tau_1 \int_{-\tau_1}^{0} \int_{t+\rho}^{t} \dot{e}(s)^T Q_2 \dot{e}(s)\, \mathrm{d}s \mathrm{d}\rho +$$

$$(\tau_{M_1} - \tau_1) \int_{-\tau_{M_1}}^{-\tau_1} \int_{\rho}^{t} \dot{e}(s)^T Q_3 \dot{e}(s)\, \mathrm{d}s \mathrm{d}\rho + \tau_2 \int_{-\tau_2}^{0} \int_{\rho}^{t} \dot{e}(s)^T Q_4 \dot{e}(s)\, \mathrm{d}s \mathrm{d}\rho \qquad (30)$$

$$+ (\tau_{M_2} - \tau_2) \int_{-\tau_{M_2}}^{-\tau_2} \int_{\rho}^{t} \dot{e}(s)^T Q_5 \dot{e}(s)\, \mathrm{d}s \mathrm{d}\rho.$$

When $t \in \Phi^q$, calculating time derivatives of $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_4(t)$ along the trajectories of (22) yields

$$\dot{V}_1(t)$$
$$= 2x(t)^T P\dot{x}(t) + x(t-\tau_1)^T (R_2 - R_1)x(t-\tau_1) + x(t-\tau_2)^T (R_4 - R_3)x(t-\tau_2) \quad (31)$$
$$+ x(t)^T (R_1 + R_3)x(t) - x(t-\tau_{M_1})^T R_2 x(t-\tau_{M_1}) - x(t-\tau_{M_2})^T R_4 x(t-\tau_{M_2}),$$

$$\dot{V}_2(t) = \dot{x}^T(t)[\tau_M Q_1 + \tau_1^2 Q_2 + (\tau_{M_1} - \tau_1)^2 Q_3 + \tau_2^2 Q_4 + (\tau_{M_2} - \tau_2)^2 Q_5]\dot{x}(t)$$
$$- \int_{t-\tau_M}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\, \mathrm{d}s - \tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s)Q_4\dot{x}(s)\, \mathrm{d}s$$
$$\qquad (32)$$
$$- (\tau_{M_1} - \tau_1) \int_{t-\tau_{M_1}}^{t-\tau_1} \dot{x}^T(s)Q_3\dot{x}(s)\, \mathrm{d}s - (\tau_{M_2} - \tau_2) \int_{t-\tau_{M_2}}^{t-\tau_2} \dot{x}^T(s)Q_5\dot{x}(s)\, \mathrm{d}s,$$

$$\dot{V}_3(t)$$
$$= 2e(t)^T P\dot{e}(t) + e(t-\tau_1)^T (R_2 - R_1)e(t-\tau_1) + e(t-\tau_2)^T (R_4 - R_3)e(t-\tau_2) \quad (33)$$
$$+ e(t)^T (R_1 + R_3)e(t) - e(t-\tau_{M_1})^T R_2 e(t-\tau_{M_1}) - e(t-\tau_{M_2})^T R_4 e(t-\tau_{M_2}),$$

$$\dot{V}_4(t) = \dot{e}^T(t)[\tau_M Q_1 + \tau_1^2 Q_2 + (\tau_{M_1} - \tau_1)^2 Q_3 + \tau_2^2 Q_4 + (\tau_{M_2} - \tau_2)^2 Q_5]\dot{e}(t) -$$
$$\int_{t-\tau_M}^{t} \dot{e}^T(s)Q_1\dot{e}(s)\, \mathrm{d}s - \tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s)Q_2\dot{e}(s)\, \mathrm{d}s - \tau_2 \int_{t-\tau_2}^{t} \dot{e}^T(s)Q_4\dot{e}(s)\, \mathrm{d}s \quad (34)$$
$$- (\tau_{M_1} - \tau_1) \int_{t-\tau_{M_1}}^{t-\tau_1} \dot{e}^T(s)Q_3\dot{e}(s)\, \mathrm{d}s - (\tau_{M_2} - \tau_2) \int_{t-\tau_{M_2}}^{t-\tau_2} \dot{e}^T(s)Q_5\dot{e}(s)\, \mathrm{d}s.$$

From Lemma 2.4, we have

$$- \int_{t-\tau_M}^{t} \dot{x}^T(s)P\dot{x}(s)\, \mathrm{d}s \leq -\frac{1}{3} \int_{t-\eta(t)}^{t} \dot{x}(s)^T P\dot{x}(s)\, \mathrm{d}s - \frac{1}{3} \int_{t-\tau(t)}^{t} \dot{x}(s)^T P\dot{x}(s)\, \mathrm{d}s$$
$$- \frac{1}{3} \int_{t-\varsigma(t)}^{t} \dot{x}(s)^T P\dot{x}(s)\, \mathrm{d}s \leq -\frac{1}{3\tau_M}[\int_{t-\eta(t)}^{t} \dot{x}(s)^T \mathrm{d}sP \int_{t-\eta(t)}^{t} \dot{x}(s)\, \mathrm{d}s \qquad (35)$$
$$+ \int_{t-\tau(t)}^{t} \dot{x}(s)^T \mathrm{d}sP \int_{t-\tau(t)}^{t} \dot{x}(s)\, \mathrm{d}s + \int_{t-\varsigma(t)}^{t} \dot{x}(s)^T \mathrm{d}sP \int_{t-\varsigma(t)}^{t} \dot{x}(s)\, \mathrm{d}s],$$

and

$$-\int_{t-\tau_M}^{t} \dot{e}^T(s)P\dot{e}(s)\,\mathrm{d}s \leq -\frac{1}{2}\int_{t-\tau(t)}^{t} \dot{e}^T(s)P\dot{e}(s)\,\mathrm{d}s - \frac{1}{2}\int_{t-\varsigma(t)}^{t} \dot{e}^T(s)P\dot{e}(s)\,\mathrm{d}s$$

$$\leq -\frac{1}{2\tau_M}\int_{t-\tau(t)}^{t} \dot{e}^T(s)\,\mathrm{d}s P \int_{t-\tau(t)}^{t} \dot{e}(s)\,\mathrm{d}s - \frac{1}{2\tau_M}\int_{t-\varsigma(t)}^{t} \dot{e}^T(s)\,\mathrm{d}s P \int_{t-\varsigma(t)}^{t} \dot{e}(s)\,\mathrm{d}s. \quad (36)$$

From Lemma 2.5, we have

$$-(\tau_{M_1}-\tau_1)\int_{t-\tau_{M_1}}^{t-\tau_1} \dot{x}^T(s)Q_3\dot{x}(s)\,\mathrm{d}s \leq$$
$$- [x(t-\tau_1)-x(t-\eta(t))]^T Q_3[x(t-\tau_1)-x(t-\eta(t))]$$
$$- [x(t-\eta(t))-x(t-\tau_{M_1})]^T Q_3[x(t-\eta(t))-x(t-\tau_{M_1})] \quad (37)$$
$$- [x(t-\tau_1)-x(t-\eta(t))]^T S_1[x(t-\eta(t))-x(t-\tau_{M_1})]$$
$$- [x(t-\eta(t))-x(t-\tau_{M_1})]^T {S_1}^T[x(t-\tau_1)-x(t-\eta(t))],$$

$$-(\tau_{M_2}-\tau_2)\int_{t-\tau_{M_2}}^{t-\tau_2} \dot{x}^T(s)Q_5\dot{x}(s)\,\mathrm{d}s \leq$$
$$- [x(t-\tau_2)-x(t-\tau(t))]^T Q_5[x(t-\tau_2)-x(t-\tau(t))]$$
$$- [x(t-\tau(t))-x(t-\tau_{M_2})]^T Q_5[x(t-\tau(t))-x(t-\tau_{M_2})] \quad (38)$$
$$- [x(t-\tau_2)-x(t-\tau(t))]^T S_2[x(t-\tau(t))-x(t-\tau_{M_2})]$$
$$- [x(t-\tau(t))-x(t-\tau_{M_2})]^T {S_2}^T[x(t-\tau_2)-x(t-\tau(t))],$$

$$-(\tau_{M_1}-\tau_1)\int_{t-\tau_{M_1}}^{t-\tau_1} \dot{e}^T(s)Q_3\dot{e}(s)\,\mathrm{d}s \leq$$
$$- [e(t-\tau_1)-e(t-\eta(t))]^T Q_3[e(t-\tau_1)-e(t-\eta(t))]$$
$$- [e(t-\eta(t))-e(t-\tau_{M_1})]^T Q_3[e(t-\eta(t))-e(t-\tau_{M_1})] \quad (39)$$
$$- [e(t-\tau_1)-e(t-\eta(t))]^T S_1[e(t-\eta(t))-e(t-\tau_{M_1})]$$
$$- [e(t-\eta(t))-e(t-\tau_{M_1})]^T {S_1}^T[e(t-\tau_1)-e(t-\eta(t))],$$

$$-(\tau_{M_2}-\tau_2)\int_{t-\tau_{M_2}}^{t-\tau_2} \dot{e}^T(s)Q_5\dot{e}(s)\,\mathrm{d}s \leq$$
$$- [e(t-\tau_2)-e(t-\tau(t))]^T Q_5[e(t-\tau_2)-e(t-\tau(t))]$$
$$- [e(t-\tau(t))-e(t-\tau_{M_2})]^T Q_5[e(t-\tau(t))-e(t-\tau_{M_2})] \quad (40)$$
$$- [e(t-\tau_2)-e(t-\tau(t))]^T S_2[e(t-\tau(t))-e(t-\tau_{M_2})]$$
$$- [e(t-\tau(t))-e(t-\tau_{M_2})]^T {S_2}^T[e(t-\tau_2)-e(t-\tau(t))].$$

From Lemma 2.6, we can get

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) Q_2 \dot{x}(s)\, \mathrm{d}s \leq -[x(t) - x(t - \tau_1)]^T Q_2 [x(t) - x(t - \tau_1)], \qquad (41)$$

$$-\tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s) Q_4 \dot{x}(s) d\, \mathrm{d} \leq -[x(t) - x(t - \tau_2)]^T Q_4 [x(t) - x(t - \tau_2)], \qquad (42)$$

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s) Q_2 \dot{e}(s)\, \mathrm{d}s \leq -[e(t) - e(t - \tau_1)]^T Q_2 [e(t) - e(t - \tau_1)], \qquad (43)$$

$$-\tau_2 \int_{t-\tau_2}^{t} \dot{e}^T(s) Q_4 \dot{e}(s)\, \mathrm{d}s \leq -[e(t) - e(t - \tau_2)]^T Q_4 [e(t) - e(t - \tau_2)]. \qquad (44)$$

Substituting (35), (37), (38), (41), (42) into (32), (36), (39), (40), (43), (44) into (34), and combining (16), (21), and (31)-(34), we have

$$\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \sigma_1 [C_2 x(t) - C_2 \int_{t-\tau(t)}^{t} \dot{x}(s)\, \mathrm{d}s]^T \Omega_1 [C_2 x(t)$$

$$- C_2 \int_{t-\tau(t)}^{t} \dot{x}(s)\, \mathrm{d}s] - e_q(t)^T \Omega_1 e_q(t) + \sigma_2 [x(t) - \int_{t-\eta(t)}^{t} \dot{x}(s)\, \mathrm{d}s - e(t)$$

$$+ \int_{t-\eta(t)}^{t} \dot{e}(s) ds]^T \Omega_2 [x(t) - \int_{t-\eta(t)}^{t} \dot{x}(s)\, \mathrm{d}s - e(t) + \int_{t-\eta(t)}^{t} \dot{e}(s)\, \mathrm{d}s]$$

$$- e_k{}^T(t) \Omega_2 e_k(t) + x(t)^T C_1{}^T C_1 x(t) - z(t)^T z(t) - \gamma^2 \omega(t)^T \omega(t) + \gamma^2 \omega(t)^T \omega(t). \tag{45}$$

Define

$$\xi^T = \begin{pmatrix} \xi_1{}^T & \xi_2{}^T & \xi_3{}^T & \xi_4{}^T \end{pmatrix}, \Xi_1 = diag\{\sigma_1 \Omega_1, \sigma_2 \Omega_2\},$$

$$\xi_1{}^T = \begin{pmatrix} x(t)^T & e(t)^T & x(t-\tau(t))^T & e(t-\tau(t))^T & x(t-\eta(t))^T & e(t-\eta(t))^T \end{pmatrix},$$

$$\xi_2{}^T = \begin{pmatrix} x(t-\tau_1)^T & e(t-\tau_1)^T & x(t-\tau_2)^T & e(t-\tau_2)^T & x(t-\tau_{M_1})^T & e(t-\tau_{M_1})^T \end{pmatrix},$$

$$\xi_3{}^T = \begin{pmatrix} x(t-\tau_{M_2})^T & e(t-\tau_{M_2})^T & \int_{t-\tau(t)}^{t} \dot{x}(s)^T\, \mathrm{d}s & \int_{t-\tau(t)}^{t} \dot{e}(s)^T\, \mathrm{d}s \end{pmatrix},$$

$$\xi_4{}^T = \begin{pmatrix} \int_{t-\varsigma(t)}^{t} \dot{x}(s)^T\, \mathrm{d}s & \int_{t-\varsigma(t)}^{t} \dot{e}(s)^T\, \mathrm{d}s & \int_{t-\eta(t)}^{t} \dot{x}(s)^T\, \mathrm{d}s & e_q(t)^T & e_k(t)^T & \omega(t)^T \end{pmatrix},$$

$$\Xi_2 = \Xi_3 = diag\{\tau_M Q_1, \tau_1{}^2 Q_2, (\tau_{M_1} - \tau_1)^2 Q_3, \tau_2{}^2 Q_4, (\tau_{M_2} - \tau_2)^2 Q_5\}.$$

Thus,

$$\dot{V}(t) + z(t)^T z(t) - \gamma^2 \omega(t)^T \omega(t) \leq$$

$$\xi^T \left[\Psi + \begin{pmatrix} \Pi_1{}^T & \Pi_2{}^T & \Pi_3{}^T \end{pmatrix} \begin{pmatrix} \Xi_1 & 0 & 0 \\ 0 & \Xi_2 & 0 \\ 0 & 0 & \Xi_3 \end{pmatrix} \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{pmatrix} \right] \xi. \tag{46}$$

From Lemma 2.7, the condition (26) is equivalent to

$$\Psi + \begin{pmatrix} \Pi_1{}^T & \Pi_2{}^T & \Pi_3{}^T \end{pmatrix} \begin{pmatrix} \Xi_1 & 0 & 0 \\ 0 & \Xi_2 & 0 \\ 0 & 0 & \Xi_3 \end{pmatrix} \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{pmatrix} < 0,$$

which means that

$$\dot{V}(t) + z(t)^T z(t) - \gamma^2 \omega(t)^T \omega(t) < 0, t \in \Phi^q. \tag{47}$$

Since $\cup_{k=0}^{\infty}[t_k h + d_k, t_{k+1} h + d_{k+1}) = [t_0, +\infty)$ and $x(t)$, $e(t)$ are continuous on $[t_0, +\infty)$, $V(t)$ is continuous on $[t_0, +\infty)$. Integrating the inequality (47) from $t_k h + d_k$ to t yields

$$V(t) - V(t_k h + d_k) \leq -\int_{t_k h + d_k}^{t} z(s)^T z(s)\,\mathrm{d}s + \gamma^2 \int_{t_k h + d_k}^{t} \omega(s)^T \omega(s)\,\mathrm{d}s.$$

Then,

$$V(t) - V(0) \leq -\int_{t_0}^{t} z(s)^T z(s)\,\mathrm{d}s + \gamma^2 \int_{t_0}^{t} \omega(s)^T \omega(s)\,\mathrm{d}s.$$

Let $t \to \infty$, we can get

$$\int_{0}^{\infty} z(s)^T z(s)\,\mathrm{d}s \leq \gamma^2 \int_{0}^{\infty} \omega(s)^T \omega(s)\,\mathrm{d}s.$$

Therefore, $\|z(t)\|_2 \leq \gamma \|\omega(t)\|_2$. The proof is completed.

We can design the dynamical output feedback H_∞ controller with the two event-triggered mechanisms based on Theorem 3.1. □

Theorem 3.2. For given parameters n_1, n_2, τ_1, τ_2, σ_1, σ_2, h, there exists an H_∞ controller of (6) if there exist three symmetric matrices $P > 0$, $\Omega_1 > 0$, $\Omega_2 > 0$, $R_i > 0 (i = 1, 2, 3, 4)$, and $S_1 > 0$, $S_2 > 0$, and two matrices $\bar{L}$ and $\bar{K}$ such that

$$\begin{pmatrix} P & S_1 \\ * & P \end{pmatrix} \geq 0, \tag{48}$$

$$\begin{pmatrix} P & S_2 \\ * & P \end{pmatrix} \geq 0, \tag{49}$$

$$\begin{pmatrix} \bar{\Psi} & \bar{\Pi}_1^T & \bar{\Pi}_2^T & \bar{\Pi}_3^T \\ * & \bar{\Sigma}_1 & 0 & 0 \\ * & * & \bar{\Sigma}_2 & 0 \\ * & * & * & \bar{\Sigma}_3 \end{pmatrix} < 0. \tag{50}$$

Then, the system (22) is asymptotic stable with an H_∞ performance index γ for the disturbance attenuation under the event-triggered mechanisms (2) and (4), where

$$\bar{\Psi} = \begin{pmatrix} \bar{\Psi}_1 & \bar{\Psi}_2 & \bar{\Psi}_3 \\ * & \bar{\Psi}_4 & 0_{8 \times 6} \\ * & * & \bar{\Psi}_5 \end{pmatrix},$$

$$\bar{\Psi}_1 = \begin{pmatrix} \bar{\Psi}_{11} & -\bar{K} & 0 & 0 & 0 & 0 & P & 0 \\ * & \bar{\Psi}_{22} & 0 & 0 & 0 & 0 & 0 & P \\ * & * & \bar{\Psi}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Psi}_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Psi}_{55} & 0 & P - S_1^T & 0 \\ * & * & * & * & * & \bar{\Psi}_{66} & 0 & P - S_1^T \\ * & * & * & * & * & * & \bar{\Psi}_{77} & 0 \\ * & * & * & * & * & * & * & \bar{\Psi}_{88} \end{pmatrix},$$

$$\bar{\Psi}_2 = \begin{pmatrix} P & 0 & 0 & 0 & 0 & 0 & -\bar{K} & \bar{K} \\ 0 & P & 0 & 0 & 0 & 0 & -\bar{K} & \bar{K} \\ P - S_2{}^T & 0 & 0 & 0 & P - S_2 & 0 & 0 & 0 \\ 0 & P - S_2{}^T & 0 & 0 & 0 & P - S_2 & 0 & 0 \\ 0 & 0 & P - S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P - S_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\bar{\Psi}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & \bar{K} & PB_\omega \\ \bar{\Psi}_{2,17} & \bar{\Psi}_{2,18} & \bar{L}C_2 & -\bar{L} & \bar{K} & PB_\omega \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\bar{\Psi}_4 = \begin{pmatrix} \bar{\Psi}_{99} & 0 & 0 & 0 & S_2 & 0 & 0 & 0 \\ * & \bar{\Psi}_{10,10} & 0 & 0 & 0 & S_2 & 0 & 0 \\ * & * & \bar{\Psi}_{11,11} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Psi}_{12,12} & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Psi}_{13,13} & 0 & 0 & 0 \\ * & * & * & * & * & \bar{\Psi}_{14,14} & 0 & 0 \\ * & * & * & * & * & * & -\frac{P}{3\tau_M} & 0 \\ * & * & * & * & * & * & 0 & -\frac{P}{2\tau_M} \end{pmatrix},$$

$\bar{\Psi}_5 = diag\{-\frac{P}{3\tau_M}, -\frac{P}{2\tau_M}, -\frac{P}{3\tau_M}, -\Omega_1, -\Omega_2, -\gamma^2 I\}$,
$\bar{\Psi}_{11} = PA + A^T P + \bar{K} + \bar{K}^T + R_1 + R_3 - 2P + C_1{}^T C_1$,
$\bar{\Psi}_{22} = PA + A^T P - \bar{L}C_2 - C_2{}^T \bar{L}^T + R_1 + R_3 - 2P$,
$\bar{\Psi}_{33} = \bar{\Psi}_{44} = -2P + S_2 + S_2{}^T, \bar{\Psi}_{55} = \bar{\Psi}_{66} = -2P + S_1 + S_1{}^T$,
$\bar{\Psi}_{77} = \bar{\Psi}_{88} = R_2 - R_1 - 2P, \bar{\Psi}_{99} = \bar{\Psi}_{10,10} = R_4 - R_3 - 2P$,
$\bar{\Psi}_{11,11} = \bar{\Psi}_{12,12} = -R_2 - P, \bar{\Psi}_{13,13} = \bar{\Psi}_{14,14} = -R_4 - P$,
$\bar{\Psi}_{2,17} = -\bar{\Psi}_{2,18} = -\bar{L}C_2 + \bar{K}$.
$\bar{\Pi}_1^T = \begin{pmatrix} \bar{\Pi}_{11}^T & \bar{\Pi}_{12}^T \end{pmatrix}, \bar{\Pi}_2^T = \begin{pmatrix} \bar{\Pi}_{21}^T & \bar{\Pi}_{22}^T & \bar{\Pi}_{23}^T & \bar{\Pi}_{24}^T & \bar{\Pi}_{25}^T \end{pmatrix}$,
$\bar{\Pi}_3^T = \begin{pmatrix} \bar{\Pi}_{31}^T & \bar{\Pi}_{32}^T & \bar{\Pi}_{33}^T & \bar{\Pi}_{34}^T & \bar{\Pi}_{35}^T \end{pmatrix}, \bar{\Pi}_{11} = \Pi_{11}, \bar{\Pi}_{12} = \Pi_{12}$,
$\bar{\Pi}_{21} = \bar{\Pi}_{22} = \bar{\Pi}_{23} = \bar{\Pi}_{24} = \bar{\Pi}_{25} = \begin{pmatrix} PA + \bar{K} & -\bar{K} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -\bar{K} \quad \bar{K} \quad 0 \quad 0 \quad 0 \quad 0 \quad \bar{K} \quad PB_\omega \end{pmatrix}$,
$\bar{\Pi}_{31} = \bar{\Pi}_{32} = \bar{\Pi}_{33} = \bar{\Pi}_{34} = \bar{\Pi}_{35} = \begin{pmatrix} 0 & PA - \bar{L}C_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\quad 0 \quad 0 \quad 0 \quad -\bar{K} \quad \bar{K} \quad -\bar{L}C_2 + \bar{K} \quad \bar{L}C_2 - \bar{K} \quad \bar{L}C_2 \quad -\bar{L} \quad \bar{K} \quad PB_\omega \end{pmatrix}$,
$\bar{\Sigma}_1 = \Sigma_1, \bar{\Sigma}_2 = \bar{\Sigma}_3 = diag\{-\frac{P}{\tau_M}, \frac{P}{\tau_1{}^2}, \frac{P}{(\tau_{M_1} - \tau_1)^2}, \frac{P}{\tau_2{}^2}, \frac{P}{(\tau_{M_2} - \tau_2)^2}\}$.

Moreover, the control gain K and the observer gain L can be obtained by $K = (B^T B)^{-1} B^T P^{-1} \bar{K}$ and $L = P^{-1} \bar{L}$, respectively.

P r o o f. Let $P = Q_1 = Q_2 = Q_3 = Q_4 = Q_5$, $\bar{L} = PL$ and $\bar{K} = PBK$. Then, the condition (26) implies that the condition (50).   □

4. NUMERICAL SIMULATIONS

In order to show the effectiveness of the proposed methods, we consider a linear time-invariant system with two event-triggering conditions, which the parameters are given as follows.

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -5 & 1 \\ 0 & 0.5 & -9 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, B_\omega = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, C_1 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \omega(t) = \begin{cases} 0.5 * \sin t, t \in [0, 10], \\ 0, \text{ otherwise.} \end{cases}$$

Case 1. Let $h = 0.02$, $\sigma_1 = 0.05$, $\sigma_2 = 0.1$, $n_1 = 2$, $n_2 = 3$, $\tau_1 = 0.001$, $\tau_2 = 0.01$. The initial conditions are given as $\hat{x}(0) = (\ 0.3 \quad -0.2 \quad 0.1\ )^T$ and $x(0) = (\ 0.1 \quad -0.3 \quad 0.4\ )^T$. Applying the Matlab/LMIs toolbox, we can obtain $\gamma = 1.9095$, the controller gain $K = (\ -0.0345 \quad 0.0542 \quad 0.0506\ )$, and the observer gain
$$L = \begin{pmatrix} 0.5877 & -0.0267 & -0.0304 \\ 0.7315 & 0.4857 & 0.1825 \\ 0.8115 & 0.2810 & 0.4866 \end{pmatrix}.$$
The corresponding trigger matrices
$$\Omega_1 = \begin{pmatrix} 0.1962 & -0.0426 & -0.0184 \\ -0.0426 & 3.5459 & -4.5019 \\ -0.0184 & -4.5019 & 19.3247 \end{pmatrix} \text{ and } \Omega_2 = \begin{pmatrix} 0.3469 & 0.0331 & 0.0007 \\ 0.0331 & 3.8579 & -2.4051 \\ 0.0007 & -2.4051 & 5.2934 \end{pmatrix}.$$
The state trajectories and the release instants and release intervals are shown in Fiure 2 and Figure 3, respectively.

Case 2. Let $h = 0.02$, $\sigma_1 = 0.1$, $\sigma_2 = 0.1$, $n_1 = 2$, $n_2 = 3$, $\tau_1 = 0.001$, $\tau_2 = 0.01$. We can get $\gamma = 2.7977$, the controller gain $K = (\ -0.0318 \quad 0.0345 \quad 0.0344\ )$, and the observer gain $L = \begin{pmatrix} 0.3767 & -0.0420 & -0.0255 \\ 0.8703 & 0.4753 & 0.1446 \\ 0.9077 & 0.2358 & 0.4265 \end{pmatrix}.$
The corresponding trigger matrices
$$\Omega_1 = \begin{pmatrix} 0.1566 & -0.0086 & -0.0132 \\ -0.0086 & 1.8798 & -1.9466 \\ -0.0132 & -1.9466 & 10.3138 \end{pmatrix} \text{ and } \Omega_2 = \begin{pmatrix} 0.4182 & 0.0514 & -0.0011 \\ 0.0514 & 4.2898 & -2.0678 \\ -0.0011 & -2.0678 & 5.5935 \end{pmatrix}.$$
In Figure 4, and Figure 5, it is shown the state trajectories and the release instants and release intervals, respectively.
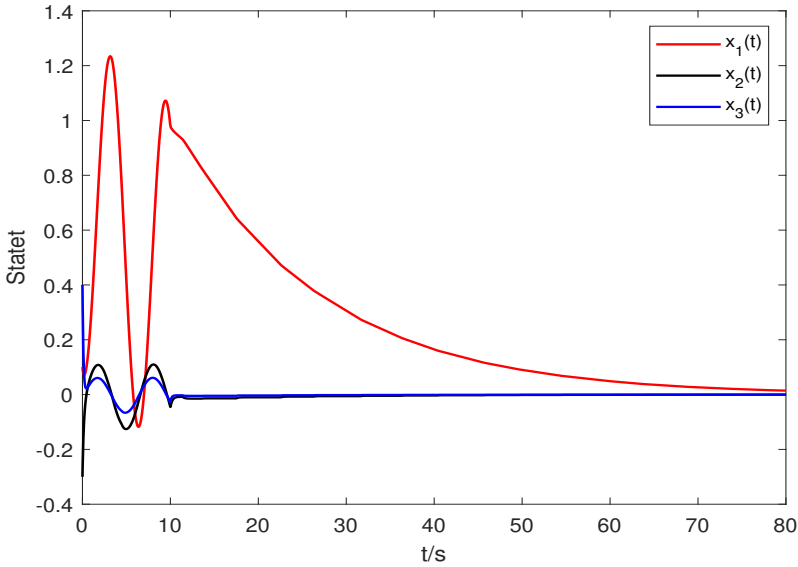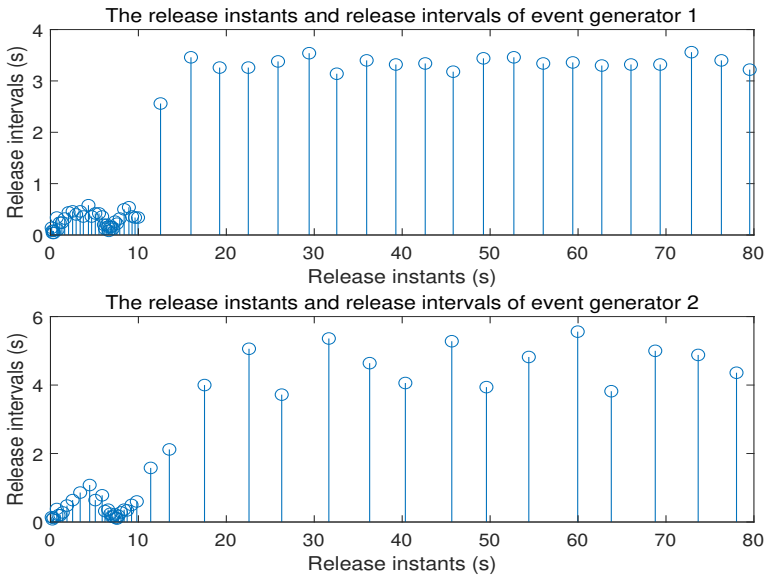
Fig. 2. The state trajectories.

Fig. 3. The release instants and release intervals of the event generator 1 and the event generator 2.
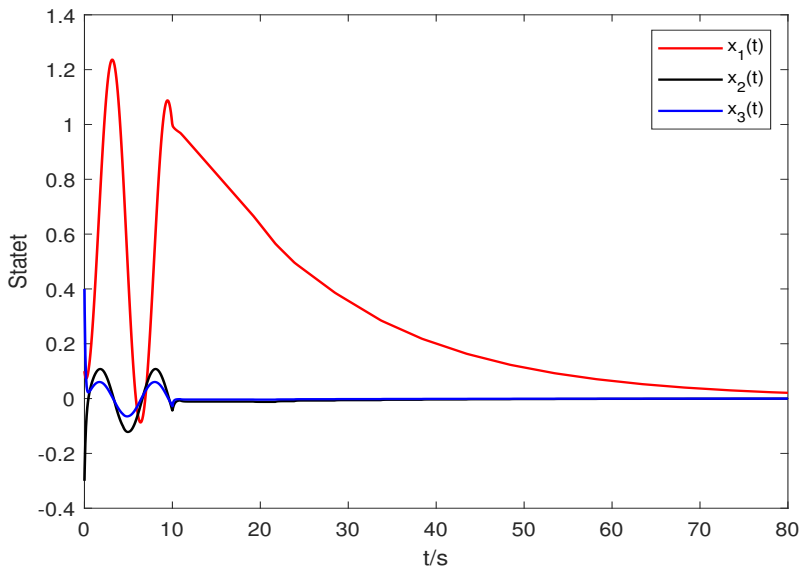
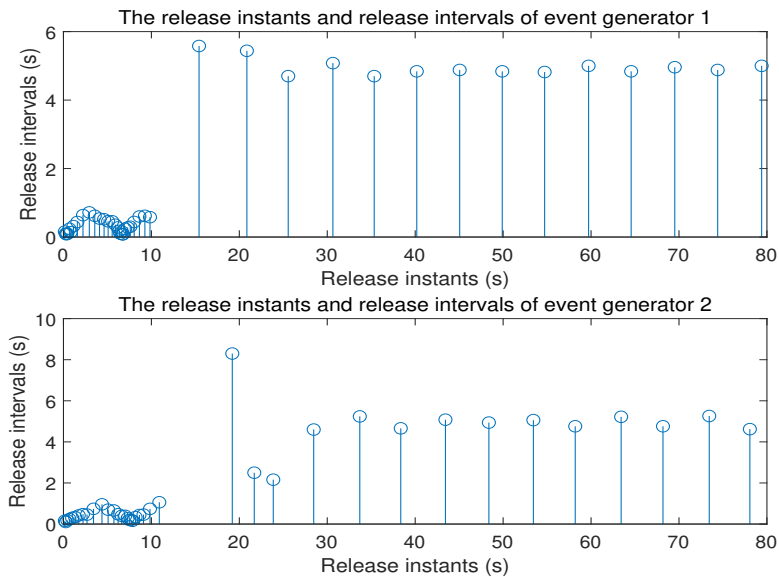**Fig. 4.** The state trajectories.

Fig. 5. The release instants and release intervals of event generator 1 and event generator 2.

In the above simulations, network delays τ_k and d_k are random in the interval $[0, 4h]$. From Figure 2 and Figure 4, we can obtain that the system under two event-triggered schemes can be stabilized while guaranteeing H_∞ performance. Moreover, in Figure 3, it is shown that only 57 sample data are transmitted from the event generator 1 to the controller, which means that 1.43% of the sample data are transmitted, and the maximum release interval is 3.56; only 1.08% sample data are transmitted from the event generator 2 to the plant, and the maximum release interval is 5.56. Similarly, Figure 5 shows that the number of release instants in the event generator 1 is 43, and the number of release instants in the event generator 2 is 39, which implies that 1.08% and 0.96% of the sample signals are transmitted in the event generator 1 and the event generator 2, respectively; the maximum release interval in the event generator 1 and the event generator 2 are 5.58 and 8.3, respectively. If only the event generator 1 is considered, the release rates of the event-triggered mechanism (2) under different value of σ_1 are shown in Table 1. As the parameter σ_1 increases, the release rate of the event-triggered mechanism (2) will decrease. If $\sigma_1 = 0$, we can get that the release rate of the event-triggered mechanism (2) is 100%. From the above analysis, the proposed two event-triggered schemes can save network bandwidth and reduce network computing resources while guaranteeing H_∞ performance.

σ_1	0	0.05	0.07	0.08	0.1	0.2
$r_t(\%)$	100	1.43	1.33	1.18	1.08	0.75

Tab. 1.   The release rate of the event-triggered mechanism (2) under different σ_1.

5. CONCLUSION

The problem of output feedback H_∞ control for NCS based on two channel event-triggered mechanisms was investigated in this paper. The considered transmission delays were random and the data buffer units were removed. The dynamical output feedback controller and the plant were updated immediately whenever the triggered data arrived. By using interval decomposition and time delay method, LMI-based sufficient conditions were obtained for H_∞ performance of the closed-loop system. Finally, numerical simulations verified the effectiveness of the proposed methods.

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