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Data, Information, and Knowledge

Mark Burgin
Department of Mathematics, *University of California, Los Angeles*, Los Angeles, CA 90095, *USA*
mburgin@math.ucla.edu

Abstract

In the paper, a mathematical theory of knowledge is developed, explicating a hierarchy of data types that exists between raw data and knowledge. The aim is to construct such a theory that expresses basic invariant properties of knowledge, which are independent of knowledge representation. In addition, this theory allows us to describe relations between knowledge, data, and information, demonstrating that the conventional approach lacks many important aspects, is too inexact, and thus, misleading in some cases. This research is oriented on utilization of the obtained results for developing huge distributed knowledge bases and efficient computer knowledge processing. Mathematical apparatus for the theory of knowledge is taken primarily from the theory of named sets and theory of abstract properties.

Keywords : knowledge, information, data, knowledge representation, interpreted data, attributed data, information processing

1 Introduction

The principal problem for computer science as well as for computer technology is to process not only data but also knowledge. Knowledge processing and management make problem solving much more efficient (Ueno, 1987; Osuga, 1989). To achieve this goal, it is necessary to distinct knowledge and knowledge representation, to know regularities of knowledge structure, functioning and representation, and to develop software (and in some cases, hardware) that is based on theses regularities.

Knowledge has been always important in society. However, now importance of knowledge grows very fast as society becomes more advanced. There is a great deal of different books and papers treating various problems and studying different issues of knowledge (cf., for example, Pollock, 1986). A lot of ideas, models and several theories have been suggested in this area. All research of knowledge can be separated into the following directions:

- *Structural analysis* of knowledge that strives to understand how knowledge is built;
- *Axiological analysis* of knowledge that aims at explanation of those features that are primary for knowledge;
- *Functional analysis* of knowledge that tries to find how knowledge functions, is produced, and acquired.

Structural analysis is the main tool for the system theory of knowledge, knowledge bases, and artificial intelligence.

Axiological analysis is the core instrument for philosophy of knowledge, psychology, and social sciences.

Functional analysis is the key device for epistemology, knowledge engineering, and cognitology.

Theoretical research in this direction is done in this paper where a mathematical theory of knowledge, which is introduced in (Burgin, 2002), is developed further with the emphasis on structural and axiological analysis.

In addition, it is necessary to understand relations between data, information, and knowledge. Computers are information processing systems. However, as it is written in the introduction of one authoritative book on information policy, "*Our main problem is that we do not really know what information is*." in spite of a multitude of papers and books concerning information and a lot of studies in this area, many important properties of information are unknown. As writes Tom Wilson (1993), " '*Information*' *is such a widely used word such a commonsensical word, that it may seem surprising that it has given 'information scientists' so much trouble over the years*."

There have been a lot of discussions and different approaches have been suggested trying to answer the question what information is. According to (Flückiger, 1995), in modern information theory a distinction is made between structural-attributive and functional-cybernetic types of theories. While representatives of the former approach conceive information as structure, like knowledge or data, variety, order, and so on; members of the latter understand information as functionality, functional meaning or as a property of organized systems. However, the advancement of science is very fast and a new theory appeared recently. It is called the general theory of information. It comprises all other known theories of information and contains much more.

The principal achievement of the general theory of information is that it explains and determines what information is. The new approach changes drastically our understanding of information, this one of the most important phenomena of our world. It displays that what people call information is, as a rule, only a container of information but not information itself. This theory reveals fascinating relations between matter, knowledge, energy, and information.

2 General Preliminaries

So, the first question is what knowledge is. Logical analysis shows that knowledge is difficult to define. There is no consensus on what knowledge is. Over the millennia, the dominant philosophies of each age have added their own definition of knowledge to the list. Science has added to this list as well. Consequently, there are many definitions of knowledge. For example, we can find in a dictionary:

Knowledge, *n*.
1. The act or state of knowing; clear perception of fact, truth, or duty; certain apprehension; familiar cognizance; cognition. "**Knowledge**, which is the highest degree of the speculative faculties, consists in the perception of the truth of affirmative or negative propositions." *Locke.*
2. That which is or may be known; the object of an act of knowing; a cognition; -- chiefly used in the plural. "There is a great difference in the delivery of the mathematics, which are the most abstracted of **knowledges**." *Bacon.* "**Knowledges** is a term in frequent use by Bacon, and, though now obsolete, should be revived, as without it we are compelled to borrow "cognitions" to express its import." *Sir W. Hamilton.* "To use a word of Bacon's, now unfortunately obsolete, we must determine the relative value of **knowledges**." *H. Spencer.*
3. That which is gained and preserved by knowing; instruction; acquaintance; enlightenment; learning; scholarship; erudition. "**Knowledge** puffeth up, but charity edifieth." *1 Cor. viii. 1.* "Ignorance is the curse of God; - **Knowledge**, the wing wherewith we fly to heaven." *Shak.*
4. That familiarity which is gained by actual experience; practical skill; as, a *knowledge* of life. "Shipmen that had **knowledge** of the sea." *1 Kings ix. 27.*
5. Scope of information; cognizance; notice; as, it has not come to my *knowledge*. "Why have I found grace in thine eyes, that thou shouldst take **knowledge** of me?" *Ruth ii. 10.*

In monographs on knowledge engineering (Osuga, S. *et al*., 1990), we find the following definitions. 1. Knowledge is a result of cognition. 2. Knowledge is a formalized information, to which references are made or which is utilized in logical inference.

This gives some general ideas about knowledge but is not constructive enough even to distinguish knowledge from knowledge representation and from information. The following example demonstrates differences between knowledge and knowledge representation. Some event may be described in several articles written in different languages, for example, in English, Spanish, and Chinese, but by the same author. These articles convey the same semantic information and contain the same knowledge about the event. However, representation of this knowledge is different.

Distinctions between knowledge and information are explained in detail in Section 6 on the base of the general theory of information.

If we take formal definitions of knowledge, we see that they determine only some specific knowledge representation. For example, in logic knowledge is represented by logical propositions and predicates. On one hand, informal definitions of knowledge provide little opportunities for computer processing of knowledge because computers can process only formalized information. On the other hand, there are a great variety of formalized knowledge representation schemes and techniques: semantic and functional networks, frames, productions, formal scenarios, relational and logical structures. However, without explicit knowledge about knowledge structures *per se*, these means of representation are used inefficiently.

Knowledge, as whole, constitutes a huge system, which is organized hierarchically and has many levels. It is possible to separate three main levels: microlevel, macrolevel, and megalevel (Burgin, 1997). On the megalevel, we consider the whole system of knowledge and its commensurable subsystems such as mathematical or physical knowledge. On the macrolevel, we have such systems of knowledge as formal theories and abstract models. Scientific and mathematical theories form a transition from the macrolevel to the megalevel. Small theoriesin the initial stage, such as non-Diophantine arithmetics now or non-Euclidean geometries in the middle of the 19^{th} century, are on the macrolevel, while mature theories, such as geometry, algebra or quantum physics, are on the megalevel. The microlevel contains such “bricks” and “blocks” of knowledge out of which other knowledge systems are constructed. For example, such knowledge macrosystems as formal theories in logic are constructed out of knowledge microsystems or elements: propositions and predicates. Their “bricks” or elementary logical units are atomic formulas, i.e., as simple logical functions and simple propositions, such as “*Knowledge is power*,” are, while composite propositions, logical functions and predicates are “blocks” or compound logical units.

Here we consider the microlevel of knowledge, aiming at construction of a mathematical model of knowledge units, disclosure of elementary knowledge units, and study of their composition into knowledge systems. This study is oriented to provide means for separation of data and knowledge as well as for improving efficiency of information processing by computers.

3 Mathematical Preliminaries

We base our study on the theory of named sets (Burgin, 1990) and the theory of abstract properties, which is developed in (Burgin, and Gorsky, 1991; Burgin and Kuznetsov, 1993).

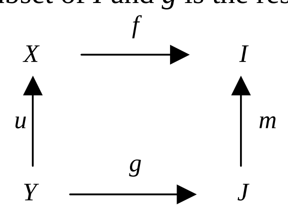
Definition 1. A *named set* or a *fundamental triad* **X** has the form (*X*, *f*, *I*) where *X* and *I* are some essences (entities, may be sets or classes) and *f* is a correspondence (connection) between *X* and *I* . The named set **X** has the following structure:

$$\textbf{Set 1} \xrightarrow{\text{correspondence}} \textbf{Set 2} \qquad (1)$$

The set *X* is called the support, *I* is called the set of names and *f* is called the naming relation of the fundamental triad **X**.

Many mathematical structures are particular cases of named sets. The most important of such structures are fuzzy sets (Zadeh, 1969) and multisets (Knuth, 1981). Any ordinary set is, as a matter of fact, some named set, and namely, a singlenamed set.

Definition 7. A named set **Y** = (*Y*, *g*, *J*) is called a named subset of a named set **X =** (*X*, *f*, *I*) if *Y* is a subset of *X*, *J* is a subset of *I* and *g* is the restriction of *f* onto *Y* and *J*.



(2)

Let U be some class (universe) of objects, and **M -** be an abstract class of partially ordered sets, i.e., such a class that with any partially ordered set contains all partially ordered sets isomorphic to it.

Definition 3. A property P of objects from some universe U is a named set P = (U, *p*, *L*). In it, *p* is a partial function (*L*-predicate) called the evaluation function Ev(P) of P. The partial function *p* has U **=** Un(P) as its domain and $L \in$ **M** as its codomain. That is, *p* is defined in **U** and takes values in *L*. *L* is called the scale Sc(P) of the property P.

For example, when you want to know something about a computer, you look into a list of its specifications, which contains many properties of a computer. Here is one of such specifications:

System memory installed: 128MB
Hard disk capacity: 30GB
Monitor diagonal size: 18 in.
Graphics memory amount: 64MB
Graphics chipset: nVidia GeForce II GTS
Supported operating systems: Windows 98 SE
Graphics card: Guillemot 3D Prophet II GTS
Monitor type: CRT
Processor model: Athlon
Processor clock speed: 1000 MHz,

Some properties of computers are now incorporated into their names. For example, when you see the name " *Dell Dimension L866r Pentium III 866MHz*," you know that this computer has the processor clock frequency 866MHz. In its turn, the processor clock frequency determines how many instructions it can execute per second.

The most popular property in logic is '*truth*' defined for logical expressions. In classical logics only one such property as *'truth'* with the scale *L* = {T, F} is considered. In multivalued logics, initial intervals of the naturals with the natural ordering on them are used as the scales for this properly. For modal logics, which have only one truth property that is determined for logical expressions, modalities are expressed by means of modal operators. Another possibility to express modality is to determine different modal truth properties: *'the truth'*, *'the necessary truth'*, and *'the possible truth'*. For modal tense logics the quantity of such truth properties is much bigger because an individual truth property is corresponded to any operator.

The mathematical theory of properties includes logic as its particular case (Burgin, 1989).

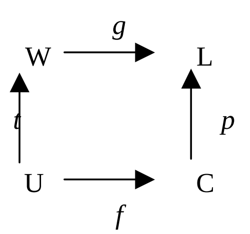
4 Knowledge and Knowledge Units

To consider knowledge, we have to begin with an important observation that there is no knowledge *per se* but we always have knowledge about something. In other words, knowledge always involves some object. Plato was may be the first to formulate this explicitly in his dialogue Republic. However, to distinct an object, we have to name it. Here it is necessary to remark that a name may be a label, number, idea, text, and even another object of a relevant nature. For example, a name may be a state of a cell in computer memory.

Besides, the simplest knowledge about an object is some property of this object. The simplest property is existence of the object in question. However, speaking about properties, we have to distinct intrinsic properties of objects and ascribed properties. Ascribed properties are obtained by measurement, calculation or inference. In addition, we consider a property for a set U of some objects, even if this set consists of single object. We call this set U the knowledge domain.

Thus, we assume that knowledge about some object F involves: 1) some class U containing the object F; 2) an intrinsic property that is represented by an abstract property T = (U, t, W) with the scale W, which is defined for objects from U; 3) some class C, which includes a name «F» of the object F; and 4) an ascribed property that is represented by an abstract property P = (C, p, L) with the scale W, which is defined for names from C. This property P is ascribed to objects from U, although not directly, but through their names. Thus, we come to the following definition.

Definition 4. *An elementary unit* K of *general knowledge* about an object F is defined by the diagram (3):



(3)

In it: the correspondence *f* relates each object H from U to its name (system of names or more generally, conceptual image, (Burgin and Gorsky, 1991)) «H» from C and the correspondence *g* relates values of the property T to values of the property P. In other words, *g* relates values of the intrinsic property to values of the ascribed property. For example, when we consider such property of material things as weight (the intrinsic property), in weighting any thing, we can get only an approximate value of the real weight, or weight with some precision (the ascribed property).

Relation *f* has the form of some algorithms/procedures of object recognition, construction or acquisition. Relation *g* has the form of some algorithms/procedures of measurement, evaluation or prediction.

Remark 1. It is possible that objects from U are characterized by a system of properties. However, this does not demand to change our representation of an elementary unit of property due to the construction of composition of properties and the result of Theorem 1.

Let $P_1, \ldots, P_n$, P be some properties on a class **U** with scales $L_1, \ldots, L_n$, and L, correspondingly. All these scales belong to M , and a mapping $\omega: L_1 \times \ldots \times L_n \rightarrow L$ is defined. Here X×Y denotes the direct product of sets X and Y.

Definition 5. A property P with a scale L is called an n-ary ω-composition of the properties $P_1, \ldots, P_n$, if for any $a \in$**U,** we have $P(a) = \omega(P_1(a), \ldots, P_n(a))$, when $P_i(a)$ are defined for a and for all i = 1, ..., n; and $P(a) = *$ in other cases. Here $P(a) = *$ means that $P(a)$ is undefined.

The ω-composition of the properties is denoted by $P = P_1, \ldots, P_n\omega$.

The main logical operations: conjunction &, disjunction ∨, and implication ⇒, are examples of ω-compositions of those properties that are determined by propositions. Thus, we have the &-composition, ∨-composition, and ⇒-composition of logical properties. For example, if P and Q are logical predicates, then P&Q is the &-composition and P∨Q is the ∨-composition of P and Q.

Another class of examples is provided try physical laws, which in a lot of cases describe some equalities for properties of studied systems and phenomena, while these properties are compositions of other properties. For instance, the law of Newtonian dynamics $F = ma$ shows that the property of material bodies called "*force*" is a composition of the properties "*mass*" and "*acceleration*".

Definition 6. A property P = (**U**, p, L) is equivalent to a system **Z** of properties $\{P_i = (\mathbf{U}, p_i, L_i);\ i \in I\}$ if the validity of the inequality $P_i(a) \neq P_i(b)$ for some $i \in I$ implies $P(a) \neq P(b)$ for any $a, b \in \mathbf{U}$, and vice versa.

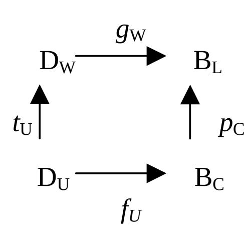
Composition of properties makes possible to prove the following result (cf. Burgin, 1997).

Theorem 1. *For any system* **Z** $= \{P_i = (\mathbf{U}, p_i, L_i);\ i \in I\}$ *of properties, there is a property* $P = (\mathbf{U}, p, L)$ *equivalent to* **Z**.

Some suggest that knowledge does not exist outside some knowledge system. Elementary units of knowledge form such minimal knowledge systems.

Knowledge may range from general to specific (Grant, 1996). General knowledge is broad, often publicly available, and independent of particular events. Specific knowledge, in contrast, is context-specific. General knowledge, its context commonly shared, can be more easily and meaningfully codified and exchanged, especially among different knowledge or practice communities. Codifying specific knowledge so as to be meaningful across an organization requires its context to be described along with the focal knowledge. This, in turn, requires explicitly defining contextual categories and relationships that are meaningful across knowledge communities. Thus, we have to define specific knowledge.

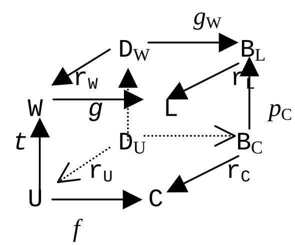
Definition 7. *An elementary unit* K of *specific knowledge* about an object F is represented by the diagram (4):



(4)

In it: D_U is a subset of U that contains F; D_W is the set of values of the property T on objects from D_U, i.e., $D_W = \{ t(u) ;\ u \in D_W\}$; B_C is a subset of C that consists of the

names of objects from D_U, i.e., $B_C = \{f(u) \; ; \; u \in D_W\}$; B_L is the set of values of the property P on the names of objects from D_U ; f_U , t_U , p_C , and g_W are corresponding restrictions of relations f_U , t_U , p_C , and g_W .
As a result, we obtain a commutative cube (5), in which all mappings r_W , r_L , r_U , r_C are inclusions.



(5)

Any system of knowledge is built from such elementary units by means of relations, which glue these knowledge bricks together. However, it is possible that some larger blocks are constructed from elementary units and then systems of knowledge are built from such blocks.

Elementary unit of knowledge (1) has specific components. The first one is the *attributive* or *estimate component* of knowledge. It reflects relation of the intrinsic property T to the ascribed property P and is represented by the diagram (6).

$$W \xrightarrow{\;g\;} L \qquad (6)$$

The second is the *naming component of knowledge*. It reflects the process of naming of an object when this object is separated, discovered or constructed. It is represented by the diagram (7).

$$U \xrightarrow[\;f\;]{} C \qquad (7)$$

The third is the *object component of knowledge*. It is the intrinsic property of objects from the knowledge domain U. It is reflected by the diagram (8).

$$U \xrightarrow[\;t\;]{} W \qquad (8)$$

The object component may be material, for example, U consists of all elementary particles or of computers, or may be ideal, for example, U consists of some texts or of real numbers.

The fourth is the *information* or *representation component of knowledge*. It is the ascribed property of objects from the knowledge domain U. The information component of knowledge renders information about the knowledge domain U. It is reflected by the diagram (9).

$$C \xrightarrow{\;p\;} L \qquad (9)$$

Any knowledge system has two parts: cognitive and substantial. The elementary unit of knowledge the object component (U, *t*, W) as its substantial part and all other elements, that is, the information component (C, *p*, L) and two relations *f* and *g*, as its cognitive component. Cognitive parts of knowledge form knowledge systems *per se*, while adding to substantial parts make extended knowledge systems.

5 Types of Knowledge and Data

Giving an exact definition of knowledge by building its mathematical model allows us to consider data also in a more exact form than before, discern data from knowledge, and specify several types of data.

The first type is *raw* or *uninterpreted data*. They are represented by the diagram (10).

$$B \xrightarrow{\;q\;} M \qquad (10)$$

In this diagram, B consists of names of some objects, M consists of some semiotic objects such as numbers, vectors or other texts, and *q* is a relation between B and M.

Raw data resemble the information component of knowledge. The difference is that raw data are not related to any definite property.

Having raw data, a person or a computer system can transform them into knowledge by means of other knowledge that this person or computer system has.

Remark 2. All other types of data may be also treated as incomplete knowledge.

The second type is *formally interpreted data*. They are related to the abstract property P, forming a named subset of the information component of a knowledge unit K, and are represented by the diagram (11).

$$B_C \longrightarrow B_L \qquad (11)$$

The third type is *attributed data*. They are related not to the abstract property P but to values of an intrinsic property T. Attributed data are represented by the diagram (12).

$$D_W \xrightarrow{\;q\;} B_L \qquad (12)$$

The fourth type is *object interpreted data*. They are represented by the diagram (13).

$$\begin{array}{ccc} & & B_L \\ & & \uparrow p \\ D_U & \xrightarrow{\;q\;} & B_C \end{array} \qquad (13)$$

The fifth type is *object attributed data*. They are represented by the diagram (14).

$$\begin{array}{ccc} D_W & \xrightarrow{\;q\;} & B_L \\ & & \uparrow p \\ & & B_C \end{array} \qquad (14)$$

It is necessary to emphasize that here we consider only the simplest units of knowledge. More extended systems are formed by means of specific relations and estimates that glue these small “bricks” and larger “blocks” of knowledge together.

6 Evaluation of knowledge

We have demonstrated that knowledge on the microlevel has the structure presented in the diagrams (3) and (4). However, this does not mean that other essences cannot have this structure. As a matter of fact, cognitive essences related to knowledge have on the microlevel the same structure. Namely, knowledge is closely related to beliefs and fantasy. To distinct knowledge, beliefs and fantasy, we need some criteria. There are two types of such criteria: *object-dependent* and *attitude-dependent*. According to the first approach, we have the following definitions.

Definition 8. *An elementary unit* K of *specific knowledge* (of *general knowledge*) about an object F is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is highly validated.

There are different systems of validation: science, for which the main validation technique is experiment; mathematics, which is based on logic with its deduction and induction; religion with its postulates and creeds; history, which is based on historical documents and archeological discoveries.

Definition 9. *An elementary unit* K of *specific belief* (of *general belief*) about an object F is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is insufficiently validated.

As writes Bem (1970), "beliefs and attitudes play an important role in human affairs. And when public policy is being formulated, beliefs *about* beliefs and attitudes play an even more crucial role." As a result, beliefs are thoroughly studied in psychology and logic. Belief systems are formalized by logical structures that introduces structures in belief spaces and calculi, as well as by belief measures that evaluate attitudes to cognitive structures and are built in the context of fuzzy set theory. There are developed methods of logics of beliefs (cf., for example, Munindar and Nicholas, 1993; or Baldoni, Giordano, and Martelli, 1998) and belief functions (Shafer, 1976). Logical methods, theory of possibility, fuzzy set theory, and probabilistic technique form a good base for building cognitive infological systems in computers.

Definition 10. *An elementary unit* K of *specific fantasy* (of *general fantasy*) about an object F is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is not validated.

According to the attitude-dependent approach, we have the following definitions.

Definition 11. *An elementary unit* K of *specific knowledge* (of *general knowledge*) about an object F for a system *R* is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is estimated (believed) by the system *R* to represent with high extent of confidence true relations.

Definition 12. *An elementary unit* K of *specific belief* (of *general belief*) about an object F for a system *R* is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is estimated (believed) by the system *R* to represent with moderate extent of confidence true relations.

Definition 13. *An elementary unit* K of *specific fantasy* (of *general fantasy*) about an object F for a system *R* is the entity that has the structure represented by the diagram (4) (by the diagram (3)) that is estimated (believed) by the system *R* to represent with low extent of confidence true relations.

If confidence depends on some validation system, then there is a correlation between the first and the second stratification of cognitive structures: knowledge, beliefs, and fantasy.

Separation of knowledge from beliefs and fantasies leads to the important question whether knowledge gives only true/correct representation of object properties. Many think that knowledge has to be always true. What is not true is called misconception. However, history of science and mathematics shows that what is considered knowledge at one time may be a misconception at another time. For example, the Euclidean geometry was believed for 2200 years to be unique (both as an absolute truth and a necessary mode of human perception). People were not even able to imagine something different. The famous German philosopher Emmanuil Kant claimed that (Euclidean) geometry is given to people *a priory*, i.e., without special learning. In spite of this, almost unexpectedly some people began to understand that geometry is not unique. Trying to improve the axiomatic system suggested for geometry by Euclid, three great mathematicians of the 19^{th} century (C.F.Gauss, N.I. Lobachewsky, and Ja. Bolyai) discovered a lot of other geometries. At first, even the best mathematicians opposed this discovery and severely attacked Lobachewsky and Bolyai who published their results. Forecasting such antagonistic attitude, the first mathematician of his times Gauss was afraid to publish this. Nevertheless, progress of mathematics brought understanding and then recognition. This discovery is now considered as one of the highest achievements of the human genius. It changed to a great extent understanding of mathematics and improved comprehension of the whole world.

In the 20^{th} century, a similar situation existed in arithmetic. For thousands of years, much longer than that for the Euclidean geometry, only one arithmetic existed. Mathematical establishment treated arithmetic as primordial entity. For example, such prominent mathematician as Kronecker (1825-1891) wrote: "*God made the integers, all the rest is the work of man*". By H.J.S. Smith, arithmetics (the Diophantine one) is one of the oldest branches, perhaps the very oldest branch, of human knowledge. His older contemporary C.O. Jacobi (1805-1851) said: " *God ever arithmetizes*".

But in spite of such a high estimation of the Diophantine arithmetic, its uniqueness and indisputable authority has been recently challenged. A family of non-Diophantine arithmetics was discovered (Burgin, 1997). Like geometries of Lobatchewsky, these arithmetics depend on a special parameter, although this parameter is not a numerical but a functional one.

These examples, show that it is necessary to consider true knowledge, approximate knowledge, and false knowledge or misconception. To separate them, we consider the object and representation components of knowledge (cf. diagrams 8 and 9).

Definition 11. An elementary unit K of knowledge (3) or (4) is:

a) *exact* if the properties of and relations between intrinsic properties are the same, or more exactly, isomorphic to the properties of and relations between ascribed properties;

b) *approximate* if properties of and relations between intrinsic properties are similar, i.e., are not essentially different from the properties of and relations between ascribed properties

c) *false* if the properties of and relations between intrinsic properties are essentially different from the properties of and relations between ascribed properties;

To correspond an exact meaning to the expressions *similar* and *essentially different* in this definition, it is possible to use the concept of tolerance (Zeeman and Buneman, 1968) and different metrics in systems of cognitive structures.

Definition 12. A binary relation between X and Y is called tolerance if it is reflexive and symmetric.

Let a tolerance T is defined for elements from X and Y.

Definition 13. Elements x from X and y from Y are similar if xTy is true and are essentially different if xTy is not true.

An important case of tolerance is defined through the distance between structures: some threshold value k is introduced and two structures belong to the corresponding tolerance relation if the distance between them is less than *k*. In general, we can introduce a measure of relevance to estimate similarity.

Thus, we have three issue of estimate for cognitive structures: groundedness, confidence, and relevance. For each of them, a corresponding measure exists that allows one to find numerical values representing differences between separate cognitive structures and their classes.

7 Cognitive information

Now let us look where information fits in this picture of knowledge and data.

The most popular in computer science approach to information is expressed by Rochester (1996). According to him, *information is an organized collection of facts and data*. Rochester develops this definition through building a hierarchy in which data are transformed into information into knowledge into wisdom. Thus, information appears as an intermediate level of similar phenomena leading from data to knowledge.

Ignoring that an "*organized collection*" is not a sufficiently exact concept, it is possible to come to a conclusion that we have an appropriate definition of information. This definition and similar ones are used in a lot of monographs and textbooks on computer science. Ignoring slight differences, we may assume that this is the most popular definition of information. This gives an impression that we actually have a working concept.

To explain why this definition is actually incoherent, let us consider some examples where information is involved.

The first example is dealing with a text that contains a lot of highly organized data. However, this text is written in Chinese. An individual, who does not know Chinese, cannot understand this text. Consequently, it contains no information for this person because such a person cannot distinct this text from a senseless collection of hieroglyphs. However, we have one and the same collection of organized data, while it contains information only for those who know Chinese. Thus, we come to a conclusion that information is something different from this collection of organized data.

It is possible to speculate that this collection of data is really information but it is accessible only by those who can understand the text. In our case, they are those who know Chinese.

Nevertheless, this is not the case. To explain this, we consider the second example. We have another text, which is a review paper in mathematics. Three people, a high level mathematician **A**, a mathematics major **B**, and a layman **C**, encounter this paper, which is in the field of expertise of **A**. After all three of them read or tried to read the paper, they come to the following conclusion. The paper contains very little information for **A** because he already knows what is written in it. The paper contains no information for **C** because he does not understand it. The paper contains a lot of information for **B** because he can understand it and knows very little about the material that is presented in it.

So, the paper contains different information for each of them. At the same time, data in the paper are not changing as well as their organization.

This vividly shows that data, even with a high organization, and information have an extremely distinct nature. Structuring and restructuring cannot eliminate these distinctions.

These inconsistencies that appear when we try to apply the given definition explicitly demonstrate that this definition is not adequate. It does not reflect real situations, and we need an essentially dissimilar definition. In the general theory of information this definition is achieved through the system of principles (Bur4gin, 2001).

Ontological Principle O1. *It is necessary to separate information in general from information (or a portion of information) for a system* ***R***. In other words, *empirically, it is possible to speak only about information (or a portion of information) for a system.*

Definition 14. The system ***R*** is called the *receiver* of the information ***I***.

The first principle explicates an important property if information, but says nothing what information is. This is done by the second principle that exists in two forms.

Ontological Principle O2. *In a broad sense, information* ***I*** *for a system* ***R*** *is any essence causing changes in the system* ***R*** .

This principle explains why information influences society and individuals. Namely, reception of information implies transformation. In this sense, information is similar to energy. Moreover, according to Principle O2, energy is a kind of information in a broad sense. This exactly corresponds to the Carl Friedrich von Weizsäcker's idea (cf., for example, Flükiger, 1995) that *energy might in the end turn out to be information*. In its turn, the von Weizsäcker's conjecture explains the exact correspondence between such characteristic of energy as the thermodynamic entropy, which is given by the Boltzmann-Planck formula $S = k \cdot \ln P$, and such characteristic of information as the quantity of information, which is given by a similar Hartley-Shannon formula $\mathrm{I} = K \cdot \ln N$.

In addition, Principle O2 makes it possible to separate different kinds of information. For example, information that is considered in theory and practice is only cognitive information. At the same time, there are two other types: emotional and effective information (Burgin, 2001). For example emotional information is very important for intelligence. As said Minsky (1998), “Emotion is only a different way to think. It may use some of the body functions, such as when we prepare to fight (the heart beats faster, etc.). Emotions have a survival value, so that we are able to behave efficiently in some situations.

Therefore, truly intelligent computers will need to have emotions. This is not impossible or even difficult to achieve. Once we understand the relationship between thinking, emotion and memory, it will be easy to implement these functions into the software.”

This distinction is based on the concept of an infological subsystem of a system. Infological system plays the role of a free parameter in the general theory of information, providing for representation in this theory different kinds and types of information.

Definition 15. A subsystem IF(***R***) of the system ***R*** is called an infological system of ***R*** if IF(***R***) contains infological elements.

Infological elements are different kinds of structures (Burgin, 1991; 1997). Let us take as a standard example of infological elements knowledge, data, images, ideas, fantasies, abstractions, beliefs, etc. The infological system that consists of these elements is called cognitive. This system is very important, especially, for intelligent systems. The majority of researchers believe that information is intrinsically connected to knowledge (cf. Flückiger, 1995). Consequently, we take the system of knowledge of ***R*** as a model example of an infological system IF(***R***) of an intelligent system ***R***. It is called in cybernetics the thesaurus Th(***R***) of the system ***R***. A thesaurus is a part of a cognitive infological system. Another example of an infological system is the memory of a computer. Such a memory is a place in which data and programs are stored.

Ontological Principle O2a. *Information in the strict sense or, simply, information for a system **R**, is everything that changes the infological system* IF(***R***) *of the system **R**.*

This principle implies that for a complex system there are different kinds of information. Each kind of infological systems determines a specific kind of information. For example, information that causes changes in the cognitive infological system is called cognitive information. This kind of information is crucial for information technology, although recently computer scientists have begun experiments in developing such computers that process emotional information.

Ontological Principle O2a implies that information is not of the same kind as knowledge and data, which are structures (Burgin, 1997). Actually, if we take that *matter* is the name for all substances as opposed to *energy* and the *vacuum*, we have the relation that is represented by the following diagram.

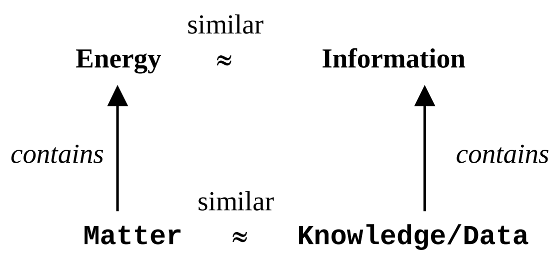


Fig. 1. Information/Energy Diagram

In other words, ***information is related to knowledge and data as energy is related to matter***.

Distinction between knowledge and cognitive information implies that transaction of information (for example, in a teaching process) does not give knowledge itself. It only causes such changes that may result in the growth of knowledge. In other words, it is possible to transmit from one system to another only information that allows a corresponding infological system to transform data into knowledge. In microphysics, the main objects are subatomic particles and quantum fields of interaction. In this context, knowledge and data play role of particles, while information realizes interaction.

However, the general theory of information differs from this conception of information because it demonstrates that information transaction may result also in the decrease of knowledge (Burgin, 1994).

Let ***I*** be some portion of information for a system ***R*** .

Ontological Principle O3. *There is always some carrier **C** of the information **I**.*

Really, people get information from books, magazines, TV and radio sets, computers, and from other people. To store information people use their brains, paper, tapes, and computer disks.

Carriers of information belong to three classes: material, mental, and structural. For example, let us consider a book. It is a physical carrier of information. However, it contains information only because some meaningful text is printed in it. Without this text it would not be a book. The text is the structural carrier of information in the book. Besides, the text is understood if it represents some knowledge. This knowledge is the mental carrier of information in the book.

In a mathematical model of information, a cognitive infological system is modeled by a mathematical structure, for example, a space of knowledge, beliefs, and fantasies. Information or more exactly, a unit of information is an operator acting on this structure. A global unit of information is also an operator. Its place is in a higher level of hierarchy as it acts on the space of all (or some) cognitive infological systems.

8 Conclusion

It is necessary to remark that the approach to knowledge evaluation considered in this paper is relevant to all types of knowledge. According to systemic typology developed in the structure-nominative model of developed knowledge systems (Burgin, and Kuznetsov, 1993), there are four main types of knowledge: *logic-linguistic knowledge* that reflects relations and connections existing in the object domain; *model knowledge* is knowledge in a form of a model (for instance, knowledge of a place or a person); *operational knowledge* describes, or prescribes, how to do something; and *problem knowledge* exposes absence of knowledge about something. This typology stays true for microknowledge considered in this paper as properties may be composite and have an extensive structure.

The suggested approach allows us to get traditional directions in the theory of knowledge as special cases. For example, when the validation system in Definition 8 is object oriented, we obtain the externalist approach to prepositional knowledge (cf., Chisholm, 1989). When the confidence system in Definition 11 is based on personal attitudes and estimates, we obtain the internalist approach to prepositional knowledge (cf., Steup, 2001).

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