Addendum to II of "Corrections + Remarks"

We were rather careless in our treatment of the "radical anomaly" defined on p.p. 24, 25 of II. The correction was devised by Martin Zeman. We again let $\sigma : W \to M$ witness the goodness of $\langle M, W, \gamma \rangle$ and let k be as on p. 23. Let S be an iteration strategy for M and $\bar{S}$ the derived iteration strategy for $\langle M, W, \gamma \rangle$ as in p. 23. We again coiterate $\langle M, W, \gamma \rangle$ against M, using $\bar{S}, S$. Let $\mathcal{J}^W = \langle \langle W_i \rangle, \ldots, T^W \rangle$ again be the iteration of $\langle M, W, \gamma \rangle$ and $\mathcal{J}^Q = \langle \langle Q_i \rangle, \ldots, T^Q \rangle$ be the iteration on the M side. Let $\mathcal{J}^M = \sigma(\mathcal{J}^W) = \langle \langle M_i \rangle, \ldots, T^M \rangle$ again be the $\langle k, k \rangle$-copy of $\mathcal{J}^W$ with copying maps $\langle \sigma_i \rangle$. As before, we call $i = \xi + 1$ a radical anomaly if $\tau_\xi^W = \gamma$ is a superstrong index in M. (Here $\tau_\xi = \kappa_\xi^{+J^{E_\xi}_{\nu_\xi}}$, where $\kappa_\xi = \mathrm{crit}(E_{\nu_\xi})$.) Let i be such an anomaly. Then $W_i = \langle J^{E^{W_\xi}_\xi}_{\nu_\xi}, F' \rangle$, where $\pi_{-1,i} : M \| \gamma \xrightarrow[E_{\nu_\xi}]{*} W_i$.

$F' \neq \emptyset$, since $M \| \gamma = \langle J_\gamma^{E^{M_\xi}}, F \rangle$ and $F \neq \emptyset$. But $E^{Q_i}_{\nu_\xi} = \emptyset$, since this is a comparison iteration. Hence $\nu_i = \nu_\xi$ where $\xi < i$. Thus the iteration $\mathcal{I}^W$ is not normal and our iteration strategy S is not directly applicable to $\mathcal{I}^M$. We get around this by defining a modified iteration

$$\mathcal{I}^{M'} = \langle \langle M_i' \rangle, \langle \nu_i' \mid i \in D' \rangle, \langle \eta_i' \rangle, \langle \pi_{ij}' \rangle, T' \rangle$$

with the properties:

(i) $\mathcal{I}^{M'}$ is normal.

(ii) $M_i = M_i'$ unless M_i is a radical anomaly.

(iii) Let $M_i = M_i'$, $M_j = M_j'$. Then $i T_j^M \leftrightarrow i T' j$ and $\pi_{ij}^M \simeq \pi_{ij}'$.

(iv) If i is a radical anomaly, then $T'(i) = -1$.

Now let b' be a branch of limit length μ through $\mathcal{I}^{M'}$. There is at most one radical anomaly on b' by (iv). Hence $b = \{h \mid \bigvee_j (M_j' = M_j \wedge j \in b' \wedge h T_j)\}$ is a branch through $\mathcal{I}^M$ with $M_b = M_b'$. Hence at limit points λ we apply S to $\mathcal{I}^{M'} | \lambda$ to get $M_\lambda' = M_\lambda$. This gives us a strategy for $\mathcal{I}^M$ and the derived strategy for $\mathcal{J}^W$. Our first observation is that if $i = \xi + 1$ is a radical anomaly and $F = E^M_\gamma$, $F' = E^{W_i}_{\nu_\xi}$ are as above. Then $i \neq T(j+1)$ for all $j \geq i$, since $\lambda_i = \lambda_\xi$ and $T(j+1)$ is the least t s.t. $\kappa_j < \lambda_t$. Thus $i \not<_T j$ for $i < j$. We have:

$\mathrm{crit}(E_{\nu_i}) = \mathrm{crit}(F) < \lambda$, where λ = the largest cardinal $< \gamma$ in M. But $\mathrm{crit}(E_{\nu_\xi}) \geq \lambda$. Hence ξ cannot be an anomaly. Now let $\mathcal{J}^{M'}|i$ be defined. Define $\mathcal{J}^{M'}|i+1$ by setting $M'_i = M_{i+1}$ (i.e. $\pi^{M'}_{-1,i} : M \longrightarrow_{F^*} M'_i$, where

$$F^* = E^{M_i}_{\nu_i^M} = \pi^M_{0,i}(F). \text{ Thus}$$

$$F^* = E^{M_\xi}_{\nu_i^M}, \text{ since } \nu_i^{M_i} = \pi_{0\xi}(\gamma) < \nu_\xi^M =$$

$$= \pi^M_{0\xi}(\lambda^{+M}), \text{ where } \gamma < \lambda^{+M}.$$

We then set: $\nu'_\xi = \nu_i^M = \pi_{0,\xi}(\gamma)$. This defines $\mathcal{J}^{M'}|i+1$, which is easily seen to be normal. We define $y^{M'}|i+2$ by setting: $i \notin D'$ (thus $M'_i = M'_{i+1}$, $\pi^{M'}_{i,i+1} = id$). Finally, if j is neither an anomaly or the successor of an anomaly, then we set: $M'_j = M_j$. At $j = h+1$, we set: $\nu'_h = \nu_h$. This defines $y^{M'}|j+1$. The inductive verification of the properties (i) – (iv) is straightforward. (For $j = h+1$ we observe that $\xi = T(j)$ is not an anomaly. We also have $\lambda'_\xi = \lambda_\xi$, even if $\xi+1$ is an anomaly. Hence $T'(j) = \xi$ and $\pi'_{\xi j} = \pi^M_{\xi j}$.

A second problem remains. It can be shown as before that the coiteration $\langle yW, yQ \rangle$ terminates (Lemma The proof of Lemma 2 (p. 28) must be amended, however: A mistake occurs in the proof of Case 2.1.5, The proofs of (1), (2) go through as before as do the proofs of $i \neq j$ and $i \not< j$. A new argument is needed, however, to show $j \not< i$. (This, too, is due to Martin Zeman.) Suppose not. Then $E^{W_i}_{\nu_i}$ fails to satisfy the initial segment condition, since otherwise we could repeat the argument for $i \not< j$. This can only happen if some $i' \leq_T w\ i$ is a radical anomaly. But then $i = i' = \xi + 1$ and $E^{W_i}_{\nu_i}$ is the top extender. This means, in particular, that $\pi_{-1,i} : M\|\gamma \xrightarrow[E^{W_\xi}_{\nu_\xi}]{} M_i$ and hence $E^M_\gamma \neq \emptyset$. But γ is a cardinal in W; hence $E^W_\gamma = \emptyset$. Hence $\nu_0 = \gamma$, $E^M_{\nu_0} \neq \emptyset$. But we cannot have $\nu_j > \nu_0$, since then $E^{Q_j}_{\nu_j}$ would fail to satisfy the initial segment condition (since then $\langle J^{E^{Q_j}}_\gamma, E^{E^{Q_j}}_{\nu_j}, a_\gamma^{E^{Q}} \rangle$ $M\|\gamma$ is a premouse and ν_0 is a cardinal in W). Hence $\nu_j = \nu_0$

and $j = 0$, since $0, i \in D^Q$ and y^Q is a normal iteration. Thus $\theta \geq_{T^Q} 1$. Let $\xi \leq_{T^Q} \theta$ be s.t. $1 = T^Q(\xi + 1)$. Let $X \in \mathcal{P}(\lambda) \cap M \| \delta$. Then $X = \pi^Q_{-1,1}(f)(\alpha)$, where $\alpha < \lambda$, $f : \kappa \to \mathcal{P}(\kappa)$, $f \in M$, and $\kappa = \operatorname{crit}(E^M_\gamma)$. But $\pi^Q_{-1,\theta}(f) =$

$= \pi^W_{-1,\theta}(f)$. Hence $\pi^Q_{1,\theta}(X) =$

$= \pi^W_{-1,\theta}(f)(\alpha) = \pi^W_{i+1,\theta}(\pi^W_{-1,i+1}(f)(\alpha))$,

where $\pi^W_{-1,i+1}(f) = \pi^W_{-1,i}\,\pi^Q_{-1,1}(f)$.

Hence $\pi^W_{-1,i+1}(f)(\alpha) = \pi^W_{-1,i}(\pi^Q_{-1,1}(f)(\alpha)) =$

$= \pi^W_{-1,i}(X)$. Hence for $\beta < \lambda_\zeta$:

$$\beta \in E^W_{\nu_\zeta}(X) \longleftrightarrow \beta \in \pi^W_{-1,i}(X) \longleftrightarrow \beta \in \pi^W_{-1,\theta}(f)(\alpha)$$

and for $\beta < \lambda_\xi$:

$$\beta \in E^Q_{\nu_\xi}(X) \longleftrightarrow \beta \in \pi^Q_{1,\xi+1}(X) \longleftrightarrow \beta \in \pi^Q_{-1,\theta}(f)(\alpha)$$

Since $E^W_{\nu_\zeta}$, $E^Q_{\nu_\xi}$ both satisfy the initial segment condition, it follows from the previous argument that each of the cases $\zeta = \xi$, $\zeta < \xi$, $\xi < \zeta$ yields a contradiction.

QED