Iteration Theorems for Subcomplete and Related Forcings

§0 Introduction

This paper is intended not to present new results, but rather to give a clearer exposition of some results we had announced previously. The centerpiece of the paper is §1, which gives a new proof of the main iteration theorem for subcomplete forcing. It differs from the proof in [SPSC] §2 (Thm 5) and is, we hope, simpler and more transparent. It is based on the proof of the rather special iteration theorem in [EN] §1. (However, the version of [EN] §1 which we originally placed on our website was marred by some very confusing slips of the pen. We have since replaced it by a corrected version.) In §2 we then tinker with the proof to get some variations which we alluded to in earlier work. (Among the variations is a generalization of the theorem in [EN] §1.) In §3 we then modify the proof to get the main iteration theorem for

subproper forcing. The variations in §3 can be similarly modified to become theorems about subproper forcing. In §4 we further modify the argument to get the iteration theorem for semi-subproper forcing. The earlier notes which we have made available did not contain a proof of this. To our embarassment we discovered that, in order to prove it, we had to slightly change the definition of "semi-subproper" (replacing "full" by "almost full"). (Thus the version stated in [SPSC] §2 (Lemma 7) is, in its literal formulation, still unproven and quite possibly unprovable.) The variations in §2 can again be suitably modified

Bibliography

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[SPSC] " Subproper and Subcomplete Forcing (handwritten notes)

[EN] Jensen, R. The Extended Namba Problem (handwritten notes)

[F] Fuchs, Gunter A Characterization of Generalized Prikry Sequences Archive for Math. Logic 44 (8) 935-971 (2005)