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Study on the Effect of Damping Asymmetry of the Vertical Suspension on the Railway Bogie Vibrations

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Abstract: This paper has an original approach to the study of vibrations in the bogie of the railway vehicle by taking into account the asymmetry of the vertical damping of suspension. The study relies on the results derived from the numerical simulation applications, where the damping asymmetry of the bogie suspension is simulated through different degrees of reducing the damping of the suspension of one of the wheels compared to the reference value. Under such circumstances, the vibrations behaviour of the bogie, assessed in five reference chassis points—four points located against the suspension of each wheel and one point located against the bogie's center of mass—will be the result of several combined effects. A first effect is produced by the suspension asymmetry that trigger interferences between the vertical vibrations of bounce and pitch and the horizontal roll vibrations of the bogie. Another effect is introduced by the reduction of the damping in the system. To these, the geometric filtering effect due to the bogie wheelbase is added, which has a selective nature depending on velocity. The analysis of the bogie vertical vibrations aims to assess the level of vibrations and identify the dominant vibration mode in the bogie reference points, as well as to determine the critical point of the vibration mode of the bogie, in correlation with velocity and the reduction degree of the suspension damping.

Keywords: railway bogie; vibrations behaviour; damping asymmetry; dominant vibration mode; vibrations critical point

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1. Introduction

From a mechanical perspective, the railway vehicle represents a complex oscillating system, which features a vibrations behaviour with specific characteristics. The vibrations of the railway vehicle are low frequency, which develop both vertically and horizontally, as simple vibration modes—rigid modes, or complex vibration modes—flexible modes [1]. As a principle, the construction of the railway vehicle is symmetrical in terms of geometry and inertia. Also, from a theoretical point of view, the vehicle suspension is considered to comply with the conditions of elastic symmetry and damping. In these hypotheses, the vertical vibrations of the vehicle can be considered decoupled from the horizontal vibrations and therefore studied separately.

The hypothesis of the symmetrical suspension as far as the elasticity and damping is frequently accepted in the research on the vibrations of railway vehicles, approached from different perspectives, such as the reduction or the control over vibrations, the improvement the performance of ride comfort or ride quality. The adoption of this hypothesis is justified by the fact that elastic and damping elements sharing the same characteristics are indeed used in the primary suspension corresponding to each wheelset of the bogie. Similarly, the secondary suspensions of the vehicle include the same types of elastic elements and damping components.

During exploitation, however, defects can occur in the suspension components, which may lead to the change in the elastic or damping characteristics of the suspension [2]. In terms of reliability, the damper is the critical component of the railway vehicle suspension [3]. A failure in a damper will lead to the reduction in the damping of the suspension where it belongs, thus determining the asymmetry of the damping of the vehicle suspension [4]. Under these circumstances, interferences take place between the vibration modes that develop in the same plan (vertical or horizontal) or between the vertical and horizontal vibration modes, which will influence the dynamic behaviour of the vehicle, with possible consequences upon its dynamic performances—ride quality, ride comfort, safety, or upon the integrity of the rolling gear, of the load-bearing structure of the vehicle or of the track.

The issue of the damping asymmetry of the railway vehicles suspension has been mainly addressed from the perspective of detecting and identifying the suspension defects, a technique known as condition monitoring [5–10]. Many research studies in this regard is being developed using model-based methods where the asymmetry of the suspension damping is included, such as interacting multiple-model algorithm [11,12], functional model method [13], evaluating estimation residuals [14], system dynamic interactions method [15–17], cross-correlation analysis based method [18–20].

Other studies have examined the track dynamic stress [21], the behaviour of vertical vibrations of the bogie and the carbody [22,23] in case of a failure of one damper in the primary suspension of the vehicle. The failure of the damper has been represented by the asymmetry of the primary suspension. Another research in which the asymmetry of the suspension has been considered is oriented towards the identification of the possibilities to improve the ride comfort [4,24,25].

The previous research studies dedicated to the assessment of the behavior of vertical vibrations of the bogie to the failure of a damper in the primary suspension of the vehicle, developed on the basis of a simplified model with two degrees of freedom—bounce and pitch, have shown that the asymmetry of the vertical damping of suspension introduces an imbalance in the system [22,26]. This imbalance leads to dynamic interferences between the bounce and pitch vertical vibrations of the bogie, which are decoupled for the symmetrical damping of suspension. As a consequence, the level of vibrations in the bogie increases due to the damper failure, and this increase is closely linked to the failure degree and to the reduction degree of the suspension damping, respectively.

The hypothesis of decoupling the vertical vibrations from the horizontal vibrations is no longer valid for the asymmetric damping of the suspension. Consequently, the analysis of the vibrations behaviour of the bogie has therefore to take into account the fact that the roll vibrations in the horizontal plan are also excited due to the asymmetry of the suspension damping, which interfere with the bounce and pitch vertical vibrations. Such an approach of the issue of vibrations of the railway bogie, which is the subject of this paper, is not found in the literature.

The paper studies the effect of the asymmetry of the vertical suspension on the vibrations behaviour of a two-axle bogie in correlation with velocity, using for this the results derived by numerical simulations. The applications of numerical simulation are developed based on an original three-degree-of-freedom bogie model, corresponding to the rigid vibration modes in a vertical plan—bounce and pitch, and horizontal—roll, where the asymmetry of the suspension damping of the bogie is simulated through different degrees of reducing the suspension damping constant in one of the wheels, compared to a reference value.

The vibrations behaviour of the bogie is assessed via the power spectral density of acceleration and the root mean square of acceleration in five reference points of the bogie chassis—four of them are placed against the chassis support points on the suspension and one against the bogie's center of mass. During a first stage, the characteristics of the vibrations behaviour of the bogie for symmetrical damping of the suspension are examined. The results thus derived are the reference base for the second stage of the study where the effect of the asymmetry of suspension upon the vibrations behaviour of the bogie is analysed.

2. The Mechanical Model and the Equations of Motion of the Bogie

To study the vibrations of a two-axle bogie during circulation on a track with vertical irregularities, the mechanical model in Figure 1 is used. The bogie chassis is modelled via a three-degree-of-freedom rigid body, corresponding to the rigid vibration modes of the bogie in the vertical plan—bounce z_b and pitch θ_b, and in a horizontal plan—roll φ_b. The general case of the asymmetry of the suspension damping is considered, according to which the damping elements of the four Kelvin-Voigt systems have different damping constants, respectively $c_{11} \neq c_{12} \neq c_{21} \neq c_{22}$.

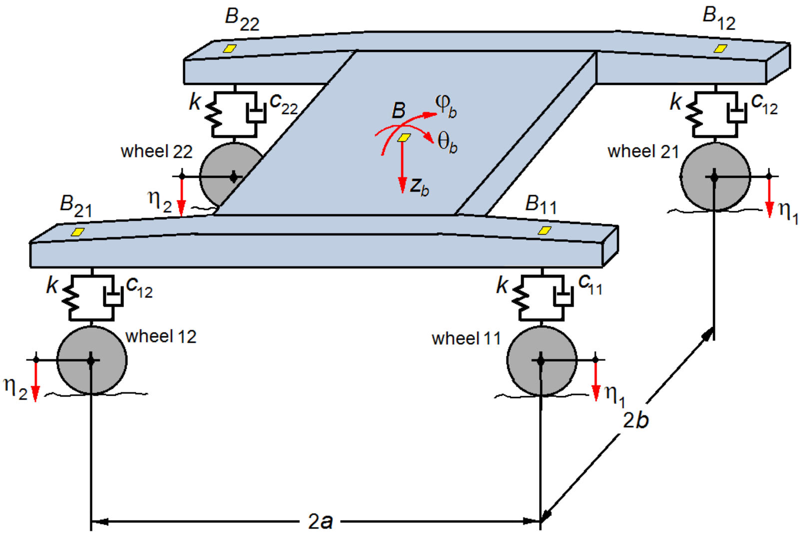

Figure 1. The mechanical model of the bogie.

As for the track model, the hypothesis of the rigid track is adopted. This simple approach is justified by the fact that the eigenfrequencies of the three rigid vibration modes of the bogie in use are much smaller than the eigenfrequencies of the wheelset-track system. For this hypothesis, wheels closely follow the vertical track irregularities, the vertical displacements of the four wheels being equal with the vertical track irregularities. While considering that the track irregularities are equal on the two threads, the vertical displacements of the wheelsets against the four wheel are $\eta_{11} = \eta_{12} = \eta_1$ and $\eta_{21} = \eta_{22} = \eta_2$.

In Figure 1, five reference points for the vibrations behaviour of the bogie are marked. Four of these points, noted with B_{11}, B_{12}, B_{21} and B_{22}, are located against the support points of the chassis on the suspension corresponding to each wheel, while the fifth point—point B, is found against the bogie center of mass.

The equations of motion describing the three vibration modes of the bogie—bounce, pitch and roll, for the general case of asymmetry of the vertical suspension damping are as follows:

$$\begin{aligned} &m_b\ddot{z}_b + (c_{11}+c_{12}+c_{21}+c_{22})\dot{z}_b + 4kz_b + a(c_{11}+c_{12}-c_{21}-c_{22})\dot{\theta}_b - \\ &-b(c_{11}-c_{12}+c_{21}-c_{22})\dot{\varphi}_b = (c_{11}+c_{12})\dot{\eta}_1 + (c_{21}+c_{22})\dot{\eta}_2 + 2k(\eta_1+\eta_2) \end{aligned} \tag{1}$$

$$\begin{aligned} &J_{b\theta}\ddot{\theta}_b + a^2(c_{11}+c_{12}+c_{21}+c_{22})\dot{\theta}_b + 4a^2k\theta_b + a(c_{11}+c_{12}-c_{21}-c_{22})\dot{z}_b - \\ &-ab(c_{11}-c_{12}-c_{21}+c_{22})\dot{\varphi}_b = a[(c_{11}+c_{12})\dot{\eta}_1 - (c_{21}+c_{22})\dot{\eta}_2] + 2ak(\eta_1-\eta_2) \end{aligned} \tag{2}$$

$$\begin{aligned} &J_{b\varphi}\ddot{\varphi}_b + b^2(c_{11}+c_{12}+c_{21}+c_{22})\dot{\varphi}_b + 4b^2k\varphi_b - b(c_{11}-c_{12}+c_{21}-c_{22})\dot{z}_b - \\ &-ab(c_{11}-c_{12}-c_{21}+c_{22})\dot{\theta}_b = -b[(c_{11}-c_{12})\dot{\eta}_1 + (c_{21}-c_{22})\dot{\eta}_2] \end{aligned} \tag{3}$$

where m_b is the bogie mass, $2a$ is the bogie wheelbase, $2b$ is the lateral base of the suspension and $J_{b\theta}$ and $J_{b\varphi}$ are the inertia moments for pitch and roll.

The equations of motion (1)–(3) show that if the damping of the vertical suspension is asymmetric, the bogie vibration modes in a vertical plan—bounce and pitch, are coupled with the roll that develops in a horizontal plan.

For the simplified case, when the damping of the suspension in the two wheelsets is symmetrical ($c_{11} = c_{12} = c_{21} = c_{22} = c$), two independent equations of motion are derived, corresponding to the vertical vibrations of bounce and pitch.

$$m_b\ddot{z}_b + 4c\dot{z}_b + 4kz_b = 2[c(\dot{\eta}_1 + \dot{\eta}_2) + k(\eta_1 + \eta_2)] \tag{4}$$

$$J_{b\theta}\ddot{\theta}_b + 4a^2c\dot{\theta}_b + 4a^2k\theta_b = 2a[c(\dot{\eta}_1 - \dot{\eta}_2) + k(\eta_1 - \eta_2)] \tag{5}$$

In this case, the roll vibrations of the bogie are not excited, as per the equation below:

$$J_{b\phi}\ddot{\phi}_b + 4b^2c\dot{\phi}_b + 4b^2k\phi_b = 0 \tag{6}$$

3. The Calculation of the Frequency Response Functions of the Bogie

To calculate the frequency response functions for the bogie corresponding to the vibrations of bounce (z_b), pitch (θ_b) and roll (φ_b), the vertical track irregularities are considered of harmonic form with the wavelength Λ and amplitude η_0. Against the wheels of the two axles, the vertical track irregularities are described by the functions

$$\eta_{1,2}(x) = \eta_0 \cos\frac{2\pi}{\Lambda}(x \pm a) \tag{7}$$

where $x = Vt$ is the coordinate of the bogie center and V is the velocity.

The functions $\eta_{1,2}$ can also be expressed as time harmonic functions, respectively

$$\eta_{1,2}(t) = \eta_0 \cos\omega\left(t \pm \frac{a}{V}\right) \tag{8}$$

where $\omega = 2\pi V/\Lambda$ stands for the pulsation induced by the vertical track irregularities, with the corresponding function $f = V/\Lambda = \omega/2\pi$.

As for the bogie response, it is assumed that it is also harmonic, with the same frequency as the frequency induced by the vertical track irregularities. The coordinates describing the bogie movement can thus be written as below

$$z_b(t) = Z_b(\cos\omega t + \alpha_z),\ \theta_b(t) = \Theta_b(\cos\omega t + \alpha_\theta),\ \phi_b(t) = \Phi_b(\cos\omega t + \alpha_\phi), \tag{9}$$

where Z_b, Θ_b, Φ_b are the amplitudes of the bounce, pitch and roll vibrations and α_z, α_θ, α_φ are the phase shifts compared to the vertical track irregularities against the bogie center.

There are introduced the complex measures, associated with the real ones, with $i^2 = -1$:

$$\overline{\eta}_{1,2}(t) = \overline{\eta}_{01,2}e^{i\omega t} \tag{10}$$

$$\overline{z}_b(t) = \overline{Z}_b e^{i\omega t},\ \overline{\theta}_b(t) = \overline{\Theta}_b e^{i\omega t},\ \overline{\phi}_b(t) = \overline{\Phi}_b e^{i\omega t}. \tag{11}$$

The complex amplitudes of the track irregularities against the wheels are as below

$$\overline{\eta}_{01} = \eta_0 e^{i\alpha_1};\ \overline{\eta}_{02} = \eta_0 e^{i\alpha_2},\ \text{for } \alpha_{1,2} = \pm(2\pi/\Lambda)a \tag{12}$$

while the complex amplitudes of the coordinates in the vehicle motions are expressed as

$$\overline{Z}_b = Z_b e^{i\alpha_z},\ \overline{\Theta}_b = \Theta_b e^{i\alpha_\theta},\ \overline{\Phi}_b = \Phi_b e^{i\alpha_\phi}. \tag{13}$$

When the relations (10) and (11) are introduced in the differential equations of motion (1)–(3), a system of three algebraic equations is obtained

$$\mathbf{A}\overline{\mathbf{X}} = \mathbf{B} \tag{14}$$

where $\overline{\mathbf{X}}$ is the vector of the frequency response functions of the displacements of the bogie corresponding to the three vibration modes—bounce, pitch and roll, $\mathbf{A}$ is the system matrix and $\mathbf{B}$ is the vector of the non-homogeneous terms, these being of the form

$$\overline{\mathbf{X}} = \begin{bmatrix} \overline{H}_z & \overline{H}_\theta & \overline{H}_\phi \end{bmatrix}^{\mathrm{T}} \tag{15}$$

where $\overline{H}_z = \overline{Z}_b/\overline{\eta}_0$, $\overline{H}_\theta = \overline{\Theta}_b/\overline{\eta}_0$, $\overline{H}_\phi = \overline{\Phi}_b/\overline{\eta}_0$ are the response functions corresponding to the vibrations of bounce (z_b), pitch (θ_b) and roll (φ_b) of the bogie;

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{16}$$

where
$a_{11} = -\omega^2 m_b + \mathrm{i}\omega(c_{11} + c_{12} + c_{21} + c_{22}) + 4k,$
$a_{12} = \mathrm{i}\omega a(c_{11} + c_{12} - c_{21} - c_{22}),$
$a_{13} = -\mathrm{i}\omega b(c_{11} - c_{12} + c_{21} - c_{22}),$
$a_{21} = \mathrm{i}\omega a(c_{11} + c_{12} - c_{21} - c_{22}),$
$a_{22} = -\omega^2 J_{b\theta} + \mathrm{i}\omega a^2(c_{11} + c_{12} + c_{21} + c_{22}) + 4a^2k$
$a_{23} = -\mathrm{i}\omega ab(c_{11} - c_{12} - c_{21} + c_{22}),$
$a_{31} = -\mathrm{i}\omega b(c_{11} - c_{12} + c_{21} - c_{22}),$
$a_{32} = -\mathrm{i}\omega ab(c_{11} - c_{12} - c_{21} + c_{22}),$
$a_{33} = -\omega^2 J_{b\phi} + \mathrm{i}\omega b^2(c_{11} + c_{12} + c_{21} + c_{22}) + 4b^2k;$

$$\mathbf{B} = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}^{\mathrm{T}} \tag{17}$$

where
$b_1 = e^{\mathrm{i}\alpha_1}[\mathrm{i}\omega(c_{11} + c_{12}) + 2k] + e^{\mathrm{i}\alpha_2}[\mathrm{i}\omega(c_{21} + c_{22}) + 2k],$
$b_2 = ae^{\mathrm{i}\alpha_1}[\mathrm{i}\omega(c_{11} + c_{12}) + 2k] - ae^{\mathrm{i}\alpha_2}[\mathrm{i}\omega(c_{21} + c_{22}) + 2k],$
$b_3 = -b[e^{\mathrm{i}\alpha_1}\mathrm{i}\omega(c_{11} - c_{12}) + e^{\mathrm{i}\alpha_2}\mathrm{i}\omega(c_{21} - c_{22})].$

Should the suspension damping of the two wheelsets is symmetrical (c_{11} = c_{12} = c_{21} = c_{22} = c), a system of two algebraic equations is obtained when replacing the relations (10) and (11) in the Equations (4) and (5)

$$\mathbf{A_s}\overline{\mathbf{Y}} = \mathbf{B_s} \tag{18}$$

where $\overline{\mathbf{Y}}$ is the vector of the frequency response functions of the displacements of the bogie corresponding to bounce and pitch, $\mathbf{A_s}$ is the system matrix and $\mathbf{B_s}$ is the vector of the non-homogeneous terms in the form of

$$\overline{\mathbf{Y}} = \begin{bmatrix} \overline{H}_z & \overline{H}_\theta \end{bmatrix}^{\mathrm{T}} \tag{19}$$

$$\mathbf{A_s} = \begin{bmatrix} -\omega^2 m_b + 4(\mathrm{i}\omega c + k) & 0 \\ 0 & -\omega^2 J_{b\theta} + 4a^2(\mathrm{i}\omega c + k) \end{bmatrix} \tag{20}$$

$$\mathbf{B_s} = \begin{bmatrix} 2(\mathrm{i}\omega c + k)(e^{\mathrm{i}\alpha_1} + e^{\mathrm{i}\alpha_2}) \\ 2a(\mathrm{i}\omega c + k)(e^{\mathrm{i}\alpha_1} - e^{\mathrm{i}\alpha_2}) \end{bmatrix} \tag{21}$$

The response functions in the reference points of the bogie can be determined when knowing the response functions corresponding to the three vibration modes of the bogie. The response functions of the bogie displacement against the suspensions of the four wheels are in the form of

$$\overline{H}_{B_{11},B_{12}}(\omega) = \overline{H}_z(\omega) + a\overline{H}_\theta(\omega) \mp b\overline{H}_\phi(\omega) \tag{22}$$

$$\overline{H}_{B_{21},B_{22}}(\omega) = \overline{H}_z(\omega) - a\overline{H}_\theta(\omega) \mp b\overline{H}_\phi(\omega) \tag{23}$$

and the response function of the displacement at the bogie center is calculated with

$$\overline{H}_B(\omega) = \overline{H}_z(\omega) \tag{24}$$

To calculate the response functions of the bogie acceleration in the five reference points, the following relations are used

$$\overline{H}_{aB_{11},aB_{12}}(\omega) = \omega^2 \overline{H}_{B_{11},B_{12}}(\omega) \tag{25}$$

$$\overline{H}_{aB_{21},aB_{22}}(\omega) = \omega^2 \overline{H}_{B_{21},B_{22}}(\omega) \tag{26}$$

$$\overline{H}_{aB}(\omega) = \omega^2 \overline{H}_B(\omega) \tag{27}$$

4. Analysis of the Vibrations Behaviour of the Bogie

As shown in the next section, the power spectral density of acceleration and the root square deviation of acceleration in five reference points of the bogie chassis will be used to study the vibrations behavior of the bogie and the effect of the suspension damping.

To calculate the power spectral density of the bogie acceleration, the vertical track irregularities are considered to be a stochastic stationary process, which can be described by means of the power spectral density defined according to the relation

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \tag{28}$$

where Ω is the wavelength number, Ω_c = 0.8246 rad/m, Ω_r = 0.0206 rad/m, and A is a coefficient depending on the track quality. For a high quality track, $A = 4.032 \times 10^{-7}$ radm, while a low quality track will have a coefficient A equal to 1.080×10^{-6} radm [27].

In dependence on the angle frequency $\omega = \pi V/\Lambda = V\Omega$, the power spectral density of the track irregularities can be expressed under the general relation

$$G(\omega) = S(\omega/V) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]} \tag{29}$$

The power spectral density of the bogie acceleration in the five reference points of the bogie chassis is calculated as a function of the power spectral density of the track irregularities and the response functions of the bogie, as in the relations below

$$G_{B_{11},B_{12}}(\omega) = G(\omega)\left|\overline{H}_{aB_{11},aB_{12}}(\omega)\right|^2 = \omega^4 G(\omega)\left|\overline{H}_z(\omega) + a\overline{H}_\theta(\omega) \mp b\overline{H}_\phi(\omega)\right|^2 \tag{30}$$

$$G_{B_{11},B_{12}}(\omega) = G(\omega)\left|\overline{H}_{aB_{11},aB_{12}}(\omega)\right|^2 = \omega^4 G(\omega)\left|\overline{H}_z(\omega) + a\overline{H}_\theta(\omega) \mp b\overline{H}_\phi(\omega)\right|^2 \tag{31}$$

$$G_B(\omega) = G(\omega)\left|\overline{H}_{aB}(\omega)\right|^2 = \omega^4 G(\omega)\left|\overline{H}_z(\omega)\right|^2 \tag{32}$$

Based on the spectral power density of the acceleration, the root mean square of the bogie acceleration at the four reference points located against the support points of the chassis on the suspension shall be calculated,

$$a_{B_{ij}} = \sqrt{\frac{1}{\pi}\int_0^\infty G_{B_{ij}}(\omega)\mathrm{d}\omega} \ \text{, for } i,\ j\ = 1,\ 2 \tag{33}$$

and the root mean deviation of the acceleration at the center of the bogie chassis,

$$a_B = \sqrt{\frac{1}{\pi}\int_0^\infty G_B(\omega)\mathrm{d}\omega} \tag{34}$$

with the remark that the superior limit of the integral corresponds to the relevant frequency range for the vibration modes of the bogie considered.

5. Results and Discussion

This section introduces the results of the numerical simulations regarding the power spectral density of acceleration and the root mean square of acceleration in the bogie reference points, based on which the characteristics of the bogie vibrations behaviour and the effect of the vertical suspension damping asymmetry are being studied.

The values of the reference parameters of the bogie model that are used in the numerical simulations are shown in Table 1. They correspond to the Minden-Deutz bogie, with a maximum speed of 160 km/h. The vertical track irregularities are described through the power spectral density (Equation (29)) for a low quality track (A = 1.080 $\times$ 10^{-6} radm).

Table 1. The reference parameters of the bogie model.

Bogie Mass	m_b = 2.700 kg
Bogie wheelset	$2a$ = 2.5 m
Lateral base of the suspension	$2b$ = 1.0 m
Pitch inertia moment	$J_{b\theta}$ = 1.536 $\times$ 10^3 kg·m^2
Roll inertia moment	$J_{b\phi}$ = 1.176 $\times$ 10^3 kg·m^2
Elastic constant of the suspension corresponding to a wheel	k = 0.616 MN/m
Reference damping constant of the suspension corresponding to a wheel (for c_{11} = c_{12} = c_{21} = c_{22} = c)	c = 9.05 kNs/m

5.1. The Study of the Characteristics of the Bogie Vibrations Behavior

To examine the basic characteristics of the bogie vibrations behaviour in correlation with velocity, the simplified case is being considered, where the suspension damping is symmetrical, namely the damping constants against the four wheels are equal (c_{11} = c_{12} = c_{21} = c_{22} = c). The vibrations behavior of the bogie is thus determined by the rigid vibration modes—bounce and pitch. While taking into account this simplification, as well as the one according to which the vertical displacements of the wheels of the same axle are equal, the dynamic response of the bogie against the two wheels of an axle emerges to be symmetrical. For this reason, only the reference points located against the first wheel of each axle will be considered to be studied, i.e., the points B_{11} and B_{21}, along with the point corresponding to the bogie center, the point B.

The diagrams in Figure 2 show the power spectral density of the acceleration at the bogie center (in point B) and the power spectral density of the bogie acceleration against the wheel 11 (in point B_{11}), and against the wheel 21 (in point B_{21}), respectively. The 3D representation in a linear scale of the power spectral density (diagrams a, b and c) points to the fact that the level of vibrations in all three reference points significantly increases along with the rise in the velocity at the resonance frequencies of the two vertical vibration modes of the bogie—bounce (at 5.10 Hz) and pitch (at 7.97 Hz).

The representation in a logarithmic scale of the power spectral density of the bogie acceleration in the three reference points—diagrams a′, b′ and c′, show that however the bogie vibrations do not amplify continuously when velocity increases, because of the geometric filtering effect given by the bogie wheelbase. The geometric filtering effect represents an important characteristic of the behaviour of vibrations of the railway vehicle, only due to the manner in which the excitations coming from the track are transmitted to the suspended masses through axles, irrespective of the characteristics of the suspension. Besides the peaks of the resonance frequencies, a series of anti-resonance frequencies corresponding to the geometric filtering frequencies depending on velocity and the bogie wheelbase develop, due to the geometric filtering effect. The geometric filtering effect has a selective nature in relation to velocity, being more visible at lower velocities [28–31]. Should the antiresonance frequencies coincide with the eigenfrequency of one of the vibration modes of the bogies, then its influence is much more diminished. This explains the change in the weight of the vibration modes of the bogie as a velocity function, an aspect pointed out at in the Figure 3 diagrams.

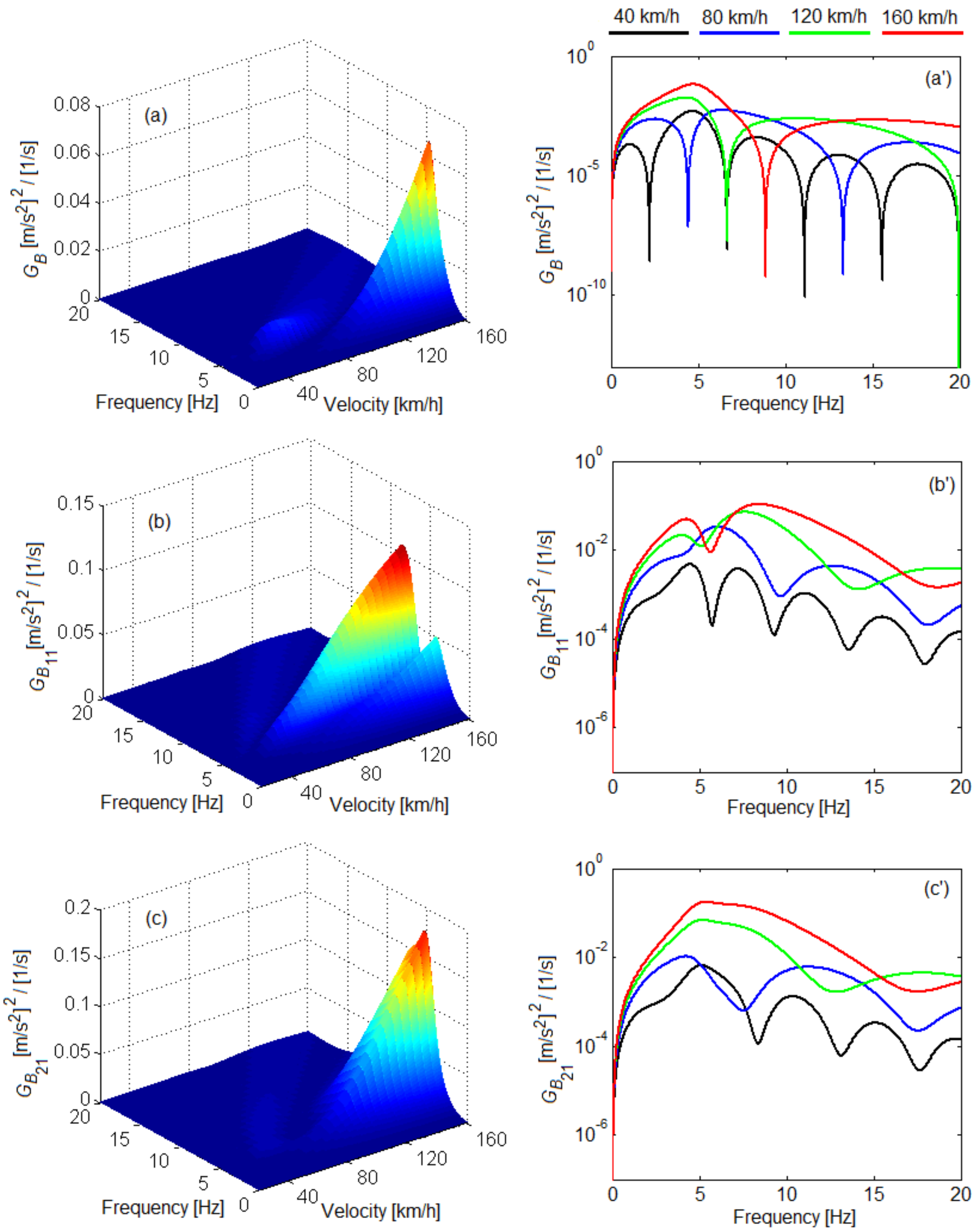

Figure 2. Power spectral density of the bogie acceleration: (**a**,**a'**) at the bogie center—in point B; (**b**,**b'**) against the wheel 11—in point B_{11}; (**c**,**c'**) against the wheel 21—in point B_{21}.

The diagrams in Figure 3 feature the power spectral density of the acceleration in the bogie reference points at the eigenfrequencies of the two vibration modes of the bogie (at 5.10 Hz—eigenfrequency of the bounce vibrations and at 7.97 Hz—eigenfrequency of the pitch vibrations). In the reference point located against the bogie center (diagram a), the vibrations behaviour is prompted by a single vibration mode—bounce. The effect of the geometric filtering effect on the bounce vibrations manifests itself up to speed of 90 km/h, being highlighted through a succession of maximum and minimum values in the curve of the power spectral density of acceleration. In the reference points of the bogie located against the wheels, the vibrations behaviour of the bogie is the result of the overlapping between the bounce and pitch vibrations. As for the dominant vibration mode, it changes depending on velocity, the geometric filtering effect respectively, which decreases either the weight of bounce or the weight of pitch (diagrams b and c). The dominant vibration mode thus alternates between bounce and pitch, as follows—in the reference point against the wheel 11, it is pitch—bounce—pitch—bounce—pitch; in the reference point against the wheel 21, the sequence is bounce—pitch—bounce—pitch—bounce.

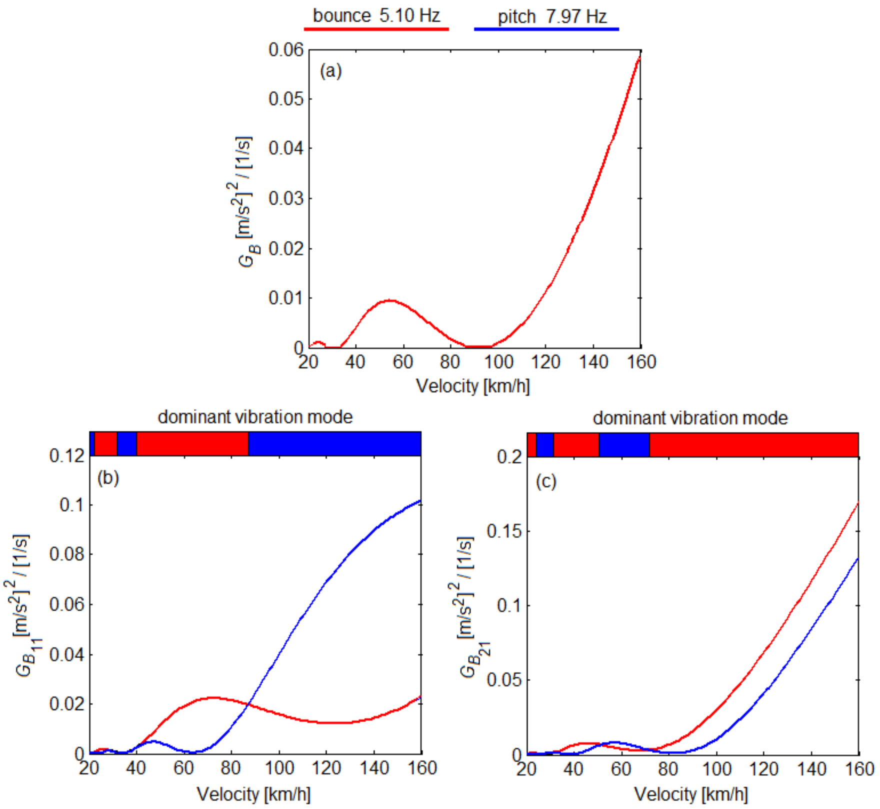

Figure 3. The dominant vibration modes of the bogie, velocity function: (**a**) at the bogie center—in point B; (**b**) against the wheel 11—in point B_{11}; (**c**) against the wheel 21—in point B_{21}.

Both diagrams b and c and diagrams b′ and c′ of the Figure 2, along with the diagrams b and c in Figure 3, show that the vibrations behaviour of the bogie above the two wheels is asymmetrical, even though the requirements of geometric, inertial, elastic and damping symmetry are met. The asymmetry of the vibrations behaviour of the bogie is an effect of the suspension damping. The diagrams in Figure 4, with the power spectral densities of the acceleration in points B_{11} and B_{21} for c = 0, show that the dynamic response of the bogie is symmetrical, in the absence of damping.

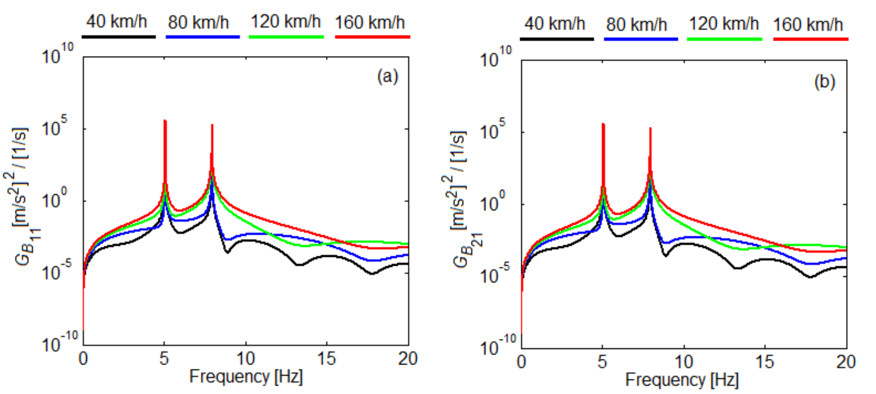

Figure 4. Power spectral density of the bogie acceleration for c = 0: (**a**) a against the wheel 11—in point B_{11}; (**b**) against the wheel 21—in point B_{21}.

Figure 5 (diagram a) presents the root mean square of the bogie acceleration in the reference point at the bogie center and in the reference points against the suspensions of wheels 11 and 21, in correlation with velocity. It should be noticed that the bogie accelerations in those three reference points do not continuously increase with velocity, due to the geometric filtering effect. Similarly, the asymmetry of the bogie dynamic response against the two wheels is again noted. For the entire velocity range, the lowest values of the root mean square of the acceleration are at the bogie center. For example, according to diagram b in the speed range 60—110 km/h, point B_{11} is the critical point. At velocities above 120 km/h, the critical point of the bogie's vibrations behaviour is point B_{21}.

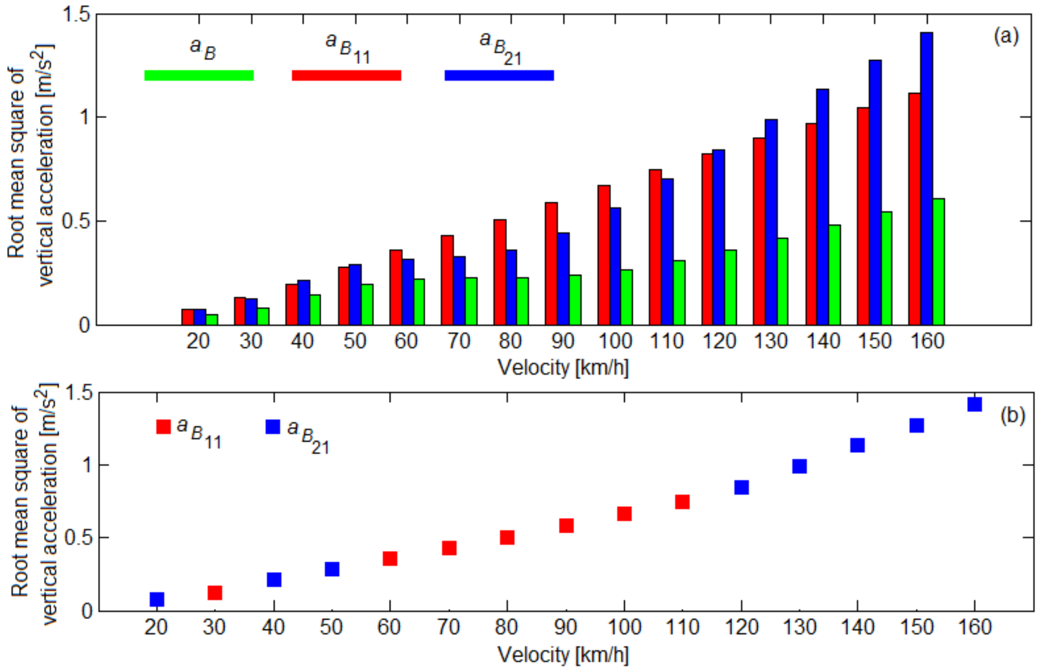

Figure 5. (**a**) Root mean square of the bogie acceleration; (**b**) The critical point of the vibrations behaviour of the bogie.

5.2. The Effect of the Damping Asymmetry of the Vertical Suspension on the Bogie Vibrations Behaviour

To study the asymmetry effect of the suspension damping of the vertical suspension on the bogie vibrations behaviour, two particular cases are being considered. In the first case, several reduction degrees of the damping constant of the suspension in wheel 11 compared to the reference value are taken into account, i.e., $c_{11} = 0.75c$, $c_{11} = 0.5c$, $c_{11} = 0.25c$, $c_{11} = 0$, whereas the reference value is maintained, $c_{12} = c_{21} = c_{22} = c$, for the damping constants of the suspensions in the other wheels. The same procedure occurs of the second case for the wheel 21: $c_{21} = 0.75c$, $c_{21} = 0.5c$, $c_{21} = 0.25c$, $c_{21} = 0$ și $c_{11} = c_{12} = c_{22} = c$.

Figure 6 shows the response functions correspondng to the three vibration modes of the bogie, at the velocity of 160 km/h for the two particular cases above. In both cases, becomes visible the amplification of the response of the bogie to the eigenfrequencies of the three vibration modes—bounce (5.1 Hz), pitch (7.97 Hz) and roll (7.28), as the increase in the reduction degree of the damping constant. The diagrams c and c' show very clearly the fact that the asymmetry of the suspension damping in one of the bogie axles triggers the excitation of the roll vibrations. The increase of the level of vibrations at the eigenfrequencies of the other two vibration modes of the bogie—bounce and pitch, is the result of a combined effect—on the one hand, the coupling between the bounce, pitch and roll vibrations and, on the other hand, the reduction in the system damping.

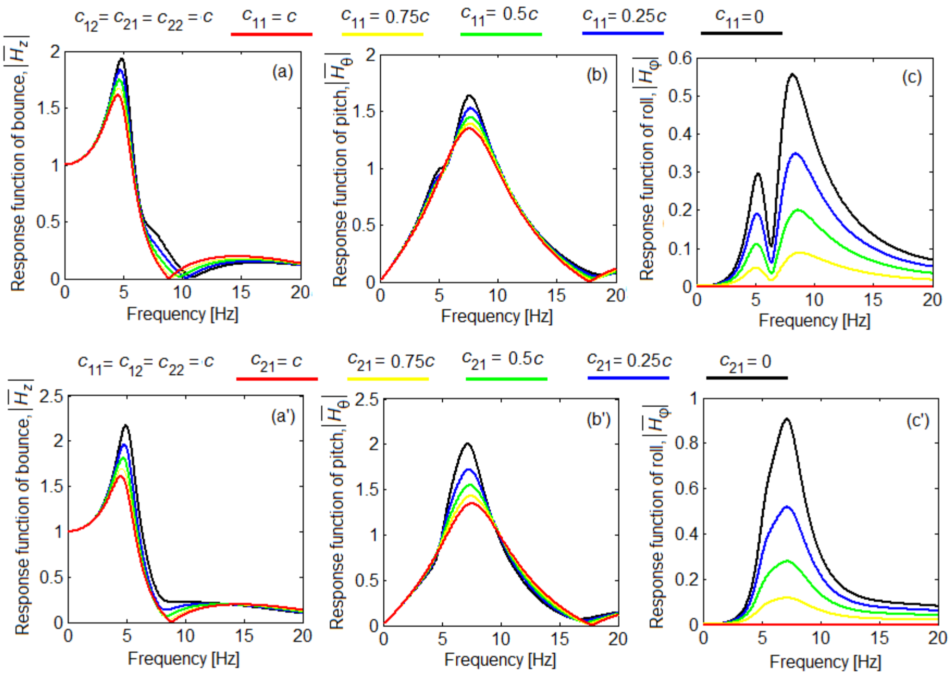

Figure 6. The response functions corresponding to the vibration modes of the bogie at velocity of 160 km/h: (**a**,**a′**)—bounce; (**b**,**b′**)—pitch; (**c**,**c′**)—roll.

Based on the diagrams in Figures 7 and 8, the effect of the asymmetry of the suspension damping on the vibrations behavior of the bogie is analyzed, depicted in the form of the power spectral density of acceleration at velocity of 160 km/h, corresponding to the two particular cases of reduction in the damping constant. In all five reference points of the bogie, the power spectral density of acceleration keeps the main characteristic of the response functions of the bogie vibration modes (see Figure 6), generated by the asymmetry of the suspension damping—the increase in the level of vibrations at the resonance frequencies.

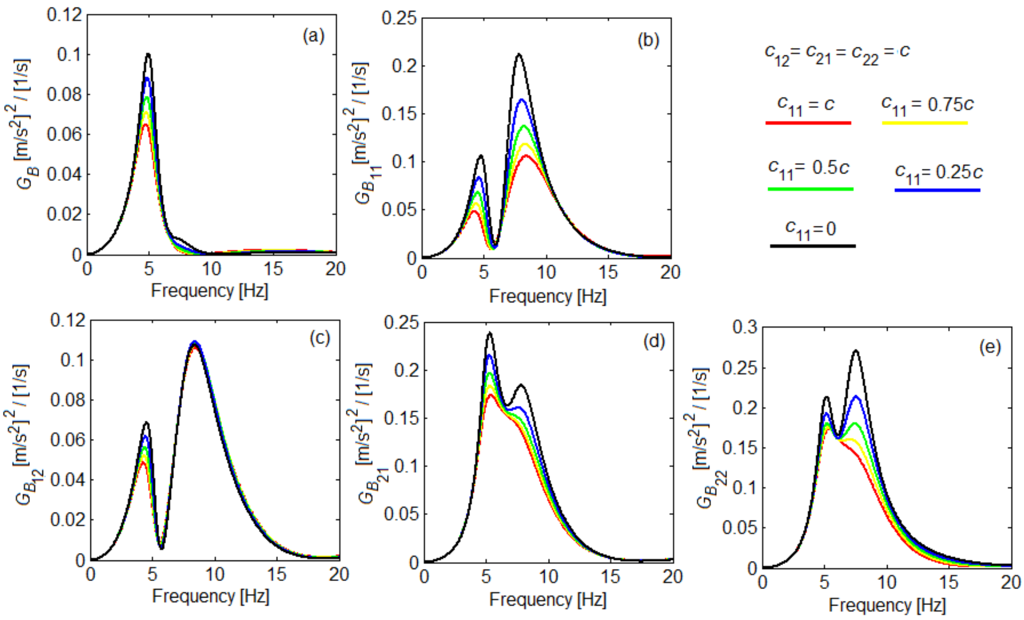

Figure 7. The effect of the suspension damping asymmetry on the power spectral density of acceleration: (**a**) at the bogie center—in point B; (**b**) against the wheel 11—in point B_{11}; (**c**) against the wheel 12—in point B_{12}; (**d**) against the wheel 21—in point B_{21}; (**e**) against the wheel 22—in point B_{22}.

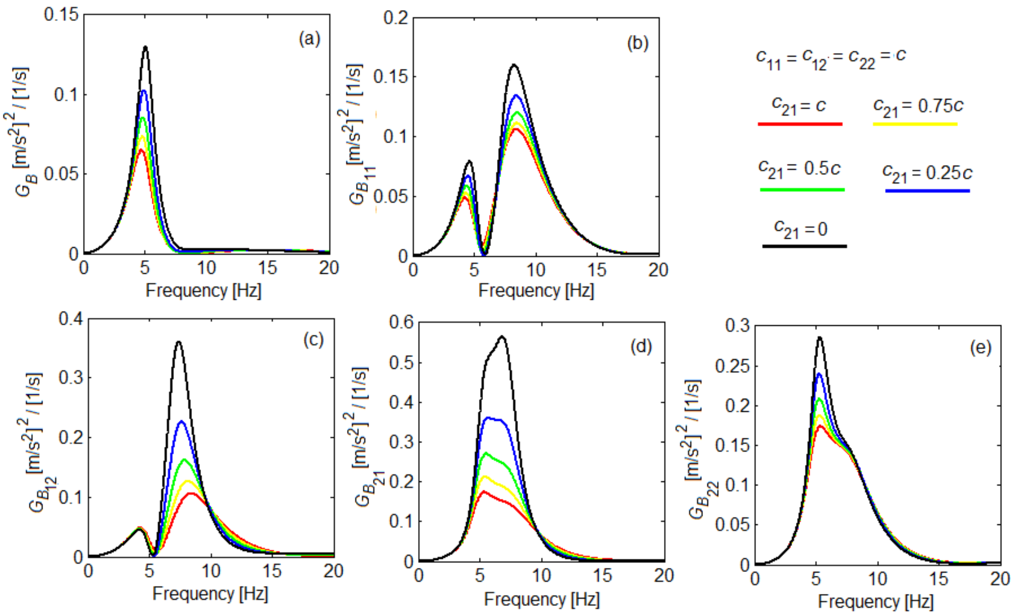

Figure 8. The effect of the suspension damping asymmetry on the power spectral density of acceleration: (**a**) at the bogie center—in point B; (**b**) against the wheel 11—in point B_{11}; (**c**) against the wheel 12—in point B_{12}; (**d**) against the wheel 21—in point B_{21}; (**e**) against the wheel 22—in point B_{22}.

As previously indicated, the vibrations behaviour at the bogie center is determined by the bounce for the symmetrical suspension damping, and vibrations behaviour in the bogie reference points located above the wheels is determined by the bounce and pitch vibrations. In case of asymmetry of the suspension damping, the vibrations behavior at the bogie center is also determined by a single vibration mode—bounce. Instead, the vibrations behavior in the reference points against the wheels is the result of the overlapping of all three vibration modes in the bogie—bounce, pitch and roll.

According to Figure 9, any of the three vibration modes can become a dominant vibration mode, depending on the reduction degree of the suspension damping constant in one of the wheels of the two axles and on the reference point of the bogie. In principle, the dominant vibration modes are the bounce and pitch. For a high degree of reduction in the damping constant (c_{12} = 0.25c or c_{21} = 0.25c) or for the null value of the damping constant (c_{12} = 0 or c_{21} = 0), the roll becomes the dominant vibration mode in some reference points of the bogie.

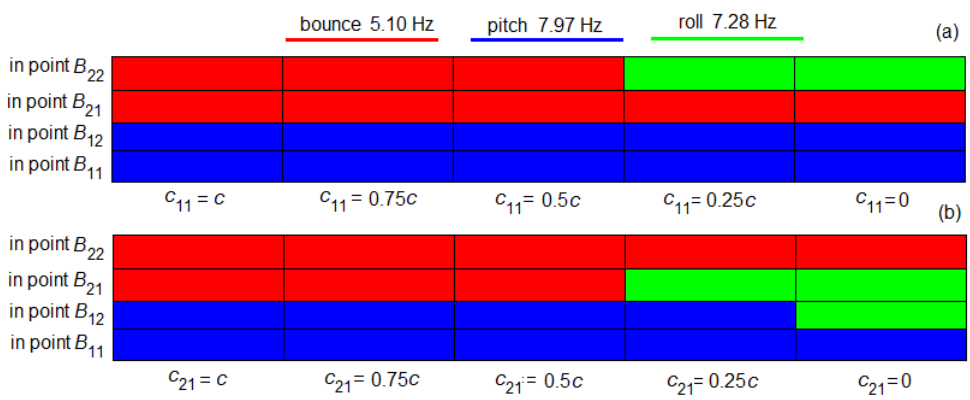

Figure 9. The effect of the suspension damping asymmetry on the dominant vibration mode at velocity of 160 km/h: (**a**) for $c_{12} = c_{21} = c_{22} = c$ and different reduction degrees of the damping constant c_{11}; (**b**) for $c_{11} = c_{12} = c_{22} = c$ and different reduction degrees of the damping constant c_{21}.

As also seen in Section 5.1, the dominant vibration mode in the reference points of the bogie depends on velocity, respectively the geometric filtering effect. Here, the velocity dependence of the dominant vibration mode is presented in correlation with different reduction degrees of the damping constant in the suspension of wheel 11 and wheel 21, respectively, in the diagrams in Figures 10 and 11. The results presented prove that it is difficult to establish an interdependence between the dominant vibration mode in the reference points of the bogie and the reduction degree in the damping constant, as velocity function. For instance, the dominant vibration mode does not change in none of the four reference points of the bogie, regardless of the reduction degree in the damping constant (at velocity of 30 km/h, as an example—see Figure 10). There are situations where the dominant vibration mode does not change in the three reference points of the bogie but it modifies in the fourth reference point (for example, at velocities of 140 km/h, 150 km/h and 160 km/h—see Figure 10) or the dominant vibration mode remains the same in two reference points of the bogie but it becomes different in the other two reference points (for example, at velocity of 130 km/h—see Figure 11).

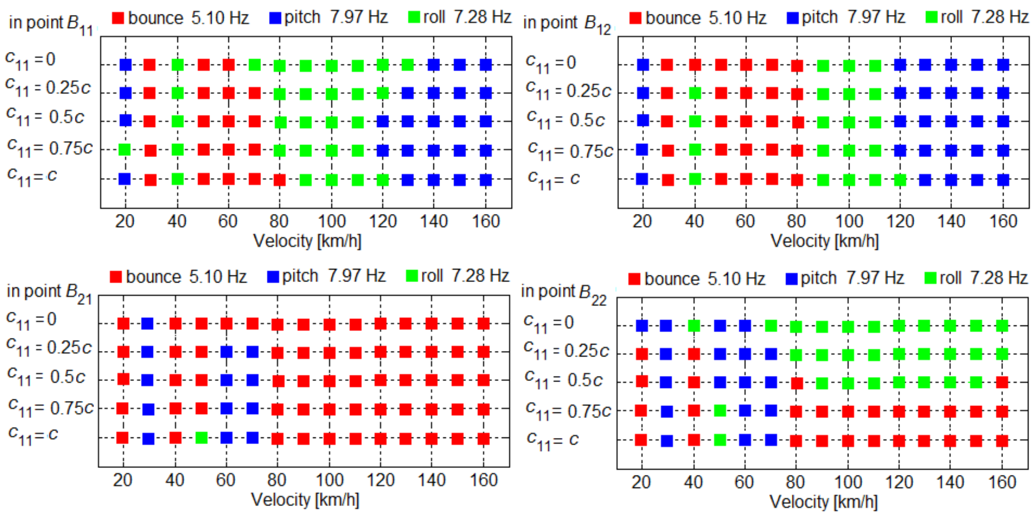

Figure 10. The effect of the suspension damping asymmetry on the dominant vibration mode for different reduction degrees in the damping constant c_{11} and $c_{12} = c_{21} = c_{22} = c$.

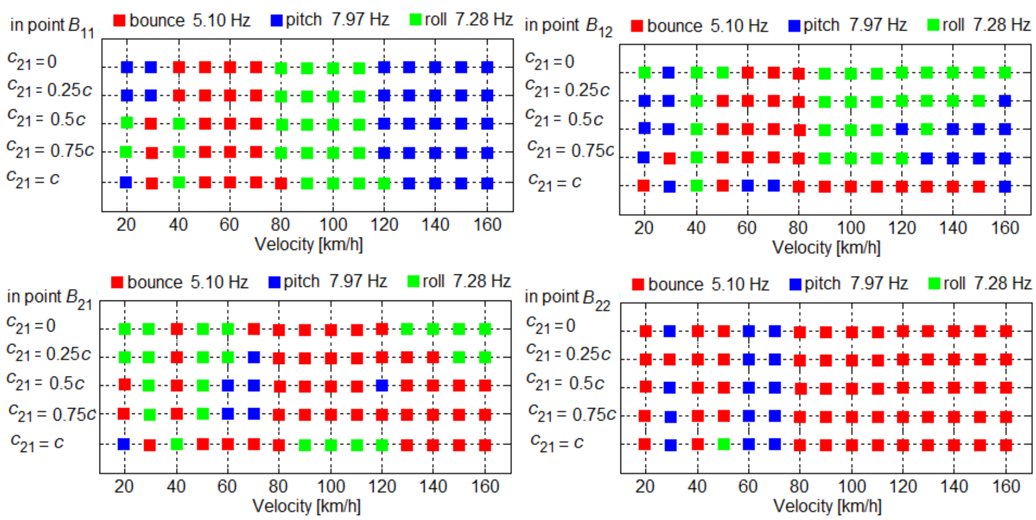

Figure 11. The effect of the suspension damping asymmetry on the dominant vibration mode for different reduction degrees in the damping constant c_{21} and $c_{11} = c_{12} = c_{22} = c$.

Figures 12 and 13 feature the values for the root mean square of acceleration in the reference points of the bogie in correlation with velocity for the two particular cases of reduction in the damping constant that determine the asymmetry of the bogie suspension. The results show that, in general, the level of vibrations increase in all the reference points of the bogie along with the rise of the reduction degree of the damping constant of the suspension in one of the two wheels. According to the Figures 14 and 15, the most significant rise in the level of vibrations is recorded when the damping constant of the suspension is reduced from the reference value to zero. This increase can reach important values, of up to 60–70% in certain reference point of the bogie. These points are located against the suspension of the wheel in which the damping constant is reduced and against the suspension of the diagonally opposite wheel (Figure 14—diagrams b and e and Figure 15—diagrams c and d).

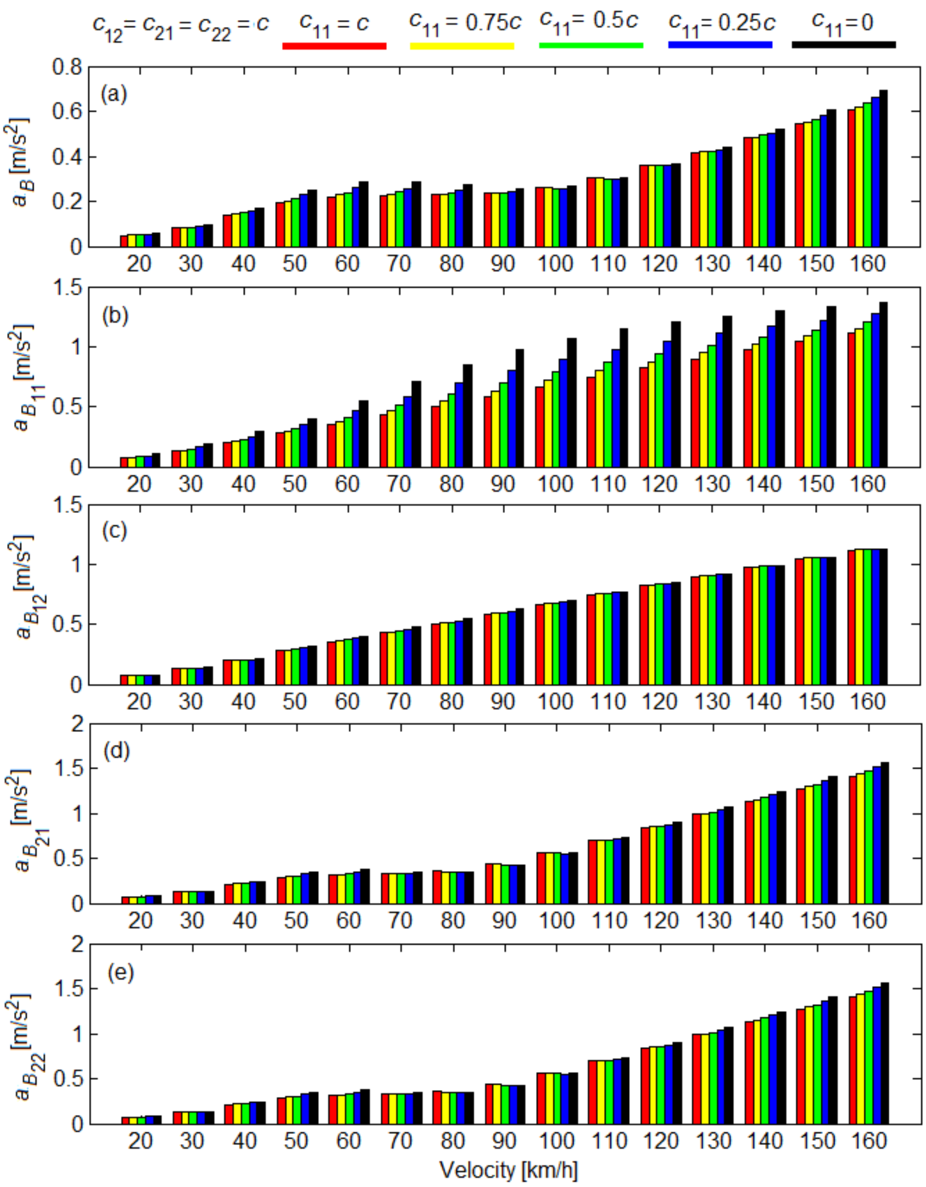

Figure 12. The root mean square of acceleration in correlation with the velocity for different reduction degrees of the damping constant c_{11}: (**a**) at the bogie center—in point B; (**b**) against the wheel 11—in point B_{11}; (**c**) against the wheel 12—in point B_{12}; (**d**) a against the wheel 21—in point B_{21}; (**e**) against the wheel 22—in point B_{22}.

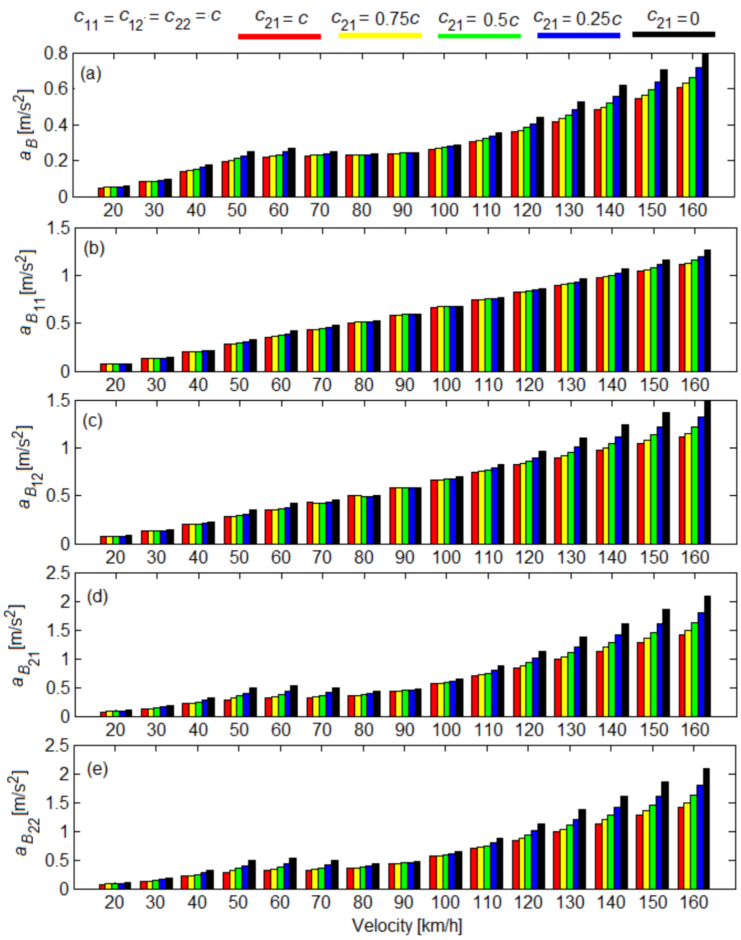

Figure 13. The root mean square of acceleration in correlation with the velocity for different reduction degrees of the damping constant c_{21}: (**a**) at the bogie center—in point B; (**b**) against the wheel 11—in point B_{11}; (**c**) against the wheel 12—in point B_{12}; (**d**) against the wheel 21—in point B_{21}; (**e**) against the wheel 22—in point B_{22}.

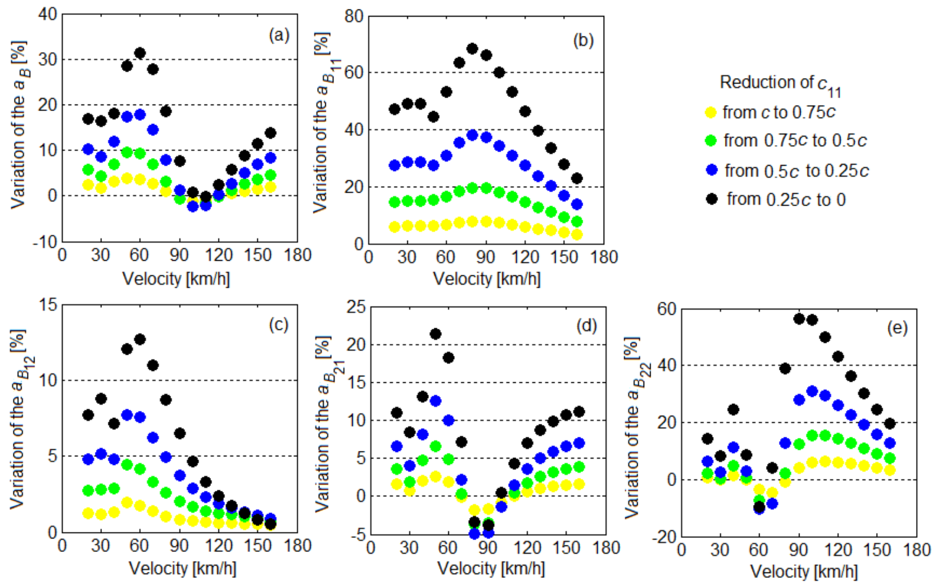

Figure 14. The variation of the root mean square of acceleration with the reduction of the damping constant c_{11} compared to the reference value: (**a**) at the bogie center; (**b**) against the wheel 11; (**c**) against the wheel 12 (**d**) against the wheel 21; (**e**) against the wheel 22.

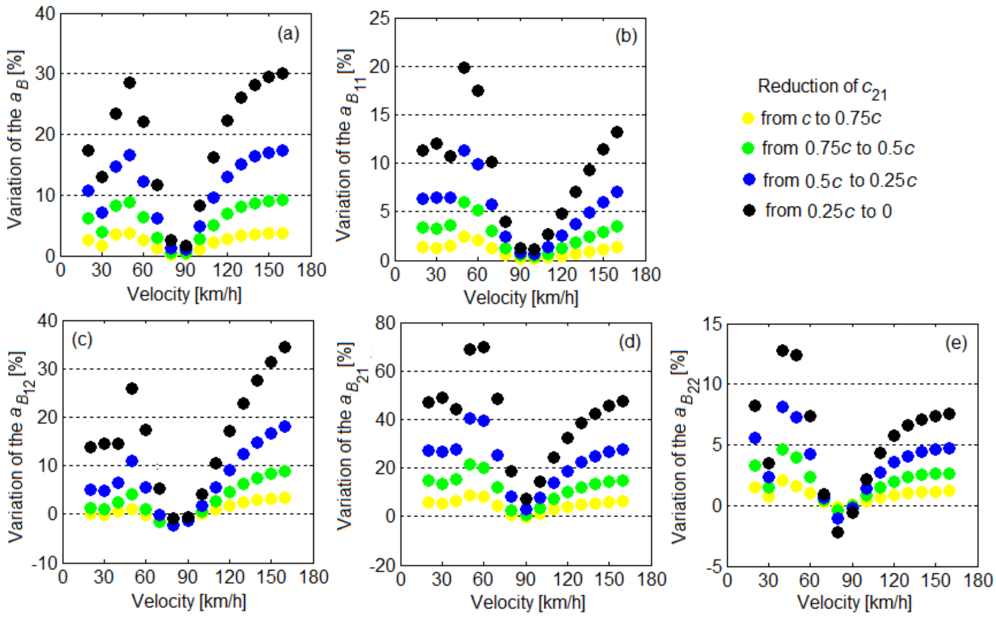

Figure 15. The variation of the root mean square of acceleration with the reduction of the damping constant c_{21} compared to the reference value: (**a**) at the bogie center; (**b**) against the wheel 11; (**c**) against the wheel 12 (**d**) against the wheel 21; (**e**) against the wheel 22.

In the previous section, the critical point of the of vibrations behaviour of the bogie has been shown to have changed depending on velocity, for the particular case of the symmetrical damping of the suspension. In case of the asymmetry of the suspension damping, the critical point of the bogie is also influenced by the reduction degree of the damping constant, as well as by the suspension of the wheel that generates the asymmetry of the bogie suspension, as shown in Figures 16 and 17.

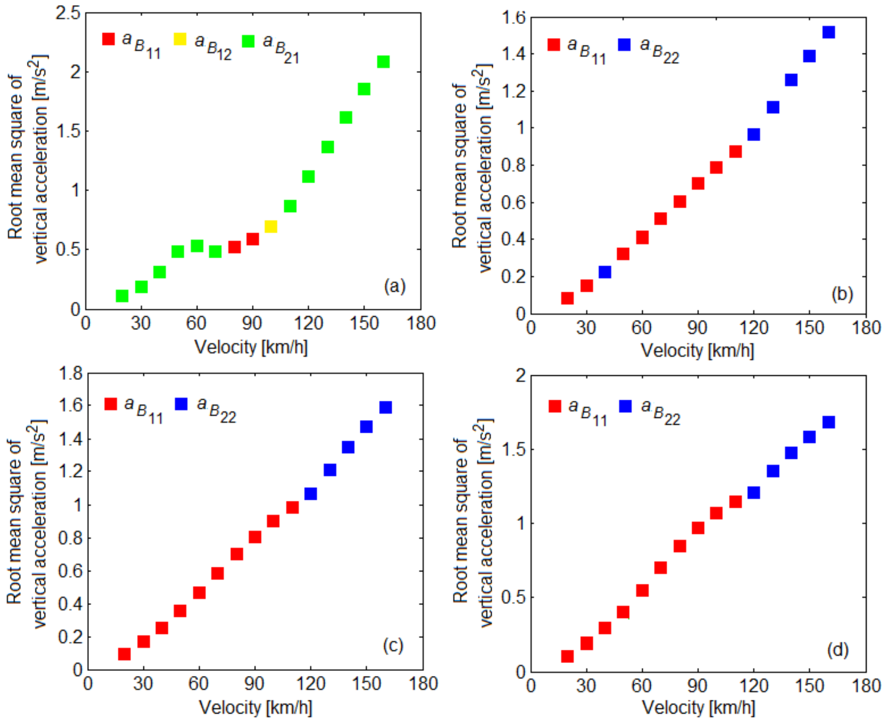

Figure 16. The effect of the suspension damping asymmetry on the critical point of the vibrations behaviour of the bogie: $c_{12} = c_{21} = c_{22} = c$ and (**a**) $c_{11} = 0.75c$; (**b**) $c_{11} = 0.5c$; (**c**) $c_{11} = 0.25c$; (**d**) $c_{11} = 0$.

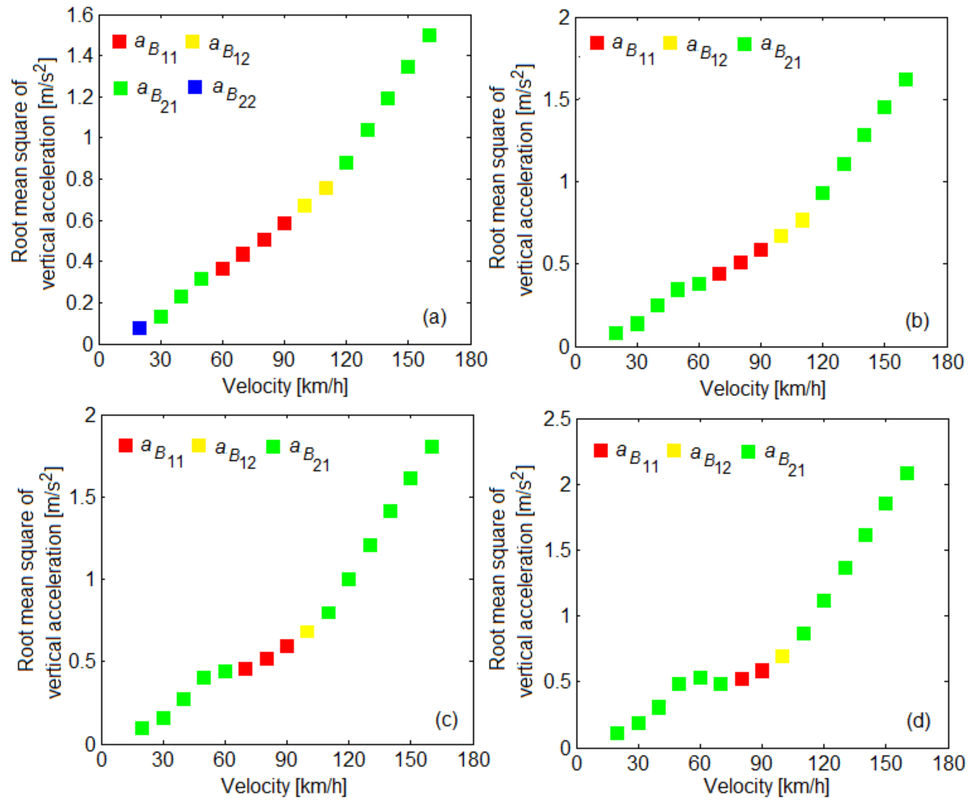

Figure 17. The effect of the suspension damping asymmetry on the critical point of the vibrations behaviour of the bogie: $c_{11} = c_{12} = c_{22} = c$ and (**a**) $c_{21} = 0.75c$; (**b**) $c_{21} = 0.5c$; (**c**) $c_{21} = 0.25c$; (**d**) $c_{21} = 0$.

For the first particular case, where the damping constant of the wheel 11 in the front wheelset is reduced, the following situations (Figure 16) are obtained: should the damping constant is reduced by 25%, the predominant critical point of the vibrations behaviour of the bogie is point B_{21} (diagram a); for higher degree of reduction in the damping constant, the critical points of the vibrations behaviour of the bogie depend on velocity—at velocities of up to 110 km/h, the critical point is B_{11}, and for higher speeds, the critical point is B_{22} (diagrams b, c and d). For the second particular case, where the damping constant of the wheel 21 in the rear wheelset is reduced, at least three reference points for each reduction degree of c_{21} are obtained (Figure 17). As an example, all four reference points of the bogie become critical points of the vibrations behaviour of the bogie (diagram a) if the damping constant is reduced by 25%, depending on velocity. For the other reduction degrees of the damping constant (50%, 75% and 100%)—the diagrams b, c and d, the maximum level of vibrations of the bogie will be recorded, velocity function, against the points B_{11}, B_{12} and B_{21}, the predominant point being point B_{21}.

6. Conclusions

The paper studies the effect of the asymmetry of the vertical suspension damping on the vibrations behaviour of a two-axle bogie, which can be triggered by the depreciation of the characteristics of the dampers during exploitation. To this purpose, the results from the numerical simulations developed on the basis of a mechanical model are used, which allows for consideration the rigid vibration modes of the bogie in a vertical plan—bounce and pitch and in the horizontal plan—roll. The vibrations behaviour of the bogie is evaluated by means of the power spectral density of acceleration and the root mean square of acceleration in five reference points of the bogie chassis. Four of the reference points are located against the support points of the chassis on the suspension corresponding to each wheel, while the fifth point is against the bogie's center of mass.

The basic characteristics of the vibrations behaviour of the bogie are analysed for the particular case of the symmetrical damping of suspension, according to which the vibrations behaviour of the bogie is only determined by the vertical vibrations of bounce and pitch. One of the important characteristics of the vibrations behaviour of the bogie has been pointed out at, namely the geometric filtering effect given by the bogie wheelbase and the selective nature of this effect as a function of velocity. Based on the geometric filtering effect, it has been explained why the level of vibrations of the bogie does not continuously amplify along with the velocity increase, as well as how the dominant vibration mode in the reference points of the bogie against the wheels change according to velocity. Characteristic of the bogie vibrations behaviour is also the fact that the level of vibrations in the reference points located against the suspension is higher than the level of vibrations in the reference point at the bogie center, irrespective of velocity. Consequently, any of the reference points located against the suspension can become critical point of the vibrations behaviour of the bogie. The position of the critical point of the vibrations behaviour of the bogie is affected, in its turn, by the geometric filtering effect. The asymmetry of the vibrations behaviour of the bogie in the reference points where all symmetry requirements are met—geometrical, inertial, elastic and of damping, represents another important characteristic, which was attributed to the suspension damping.

The effect of the damping asymmetry in the vertical suspension upon the vibrations behaviour of the bogie has been examined for four degrees of reduction in the damping constant (25%, 50%, 75% and 100%) of the suspension of one of the two wheels of the axle. On the one hand, the effect of the damping asymmetry of the suspension has was highlighted through the excitation of the roll vibrations in the bogie. On the other hand, the damping asymmetry of the suspension has been shown to have also generated the increase of the level of the bogie vertical vibrations—bounce and pitch. The general increase in the level of vibrations of the bogie is the result of a combined effect, brought about by both the coupling between the vibrations of bounce, pitch and roll coming from the asymmetry of the suspension damping, and also by the reduction of the damping in the system. The complex combined effect, to which the geometric filtering effect is added, makes it difficult to establish an interdependence between the dominant vibration mode in the reference points of the bogie, the reduction degree of the damping constant and velocity. Similarly, the critical point of the vibrations behaviour of the bogie is influenced by these effects, as well as by the position of the wheel suspension in the bogie assembly that leads to the asymmetry in the bogie suspension. Under such circumstances, the critical point can be any of the four reference points of the bogie, located against the suspension.

Preliminary results obtained by numerical simulations show that this research topic deserves further investigation. In this sense, in the next stage of the research, the results of the simulations results should be compared with the test results.

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