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Analyzing the Impact of Information Asymmetry on Strategy Adaptation in Swarm Robotics: A Game-Theoretic Approach

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Abstract: In dynamic environments characterized by information asymmetry, swarm robots encounter significant challenges in efficiently collaborating to complete tasks. This study investigates the effects of factors such as resource information, shared costs, transmission efficiency, and strategy-switching probabilities arising from uneven information sharing among robots from the perspective of information disparity. A payoff matrix is developed to model the selection between search and exploration strategies under conditions of information asymmetry. Utilizing evolutionary game theory and replicator dynamics, the study analyzes how robots adapt their strategies in response to variations in resource information and shared costs. The findings reveal that the system ultimately evolves toward one of two dominant strategies: search or exploration. Numerical simulations demonstrate that information disparity, shared costs, transmission efficiency, and strategy-switching probabilities collectively drive the transition of robots from a search strategy to an exploration strategy, enabling them to acquire unknown environmental information more effectively and expedite task completion. The results suggest that in environments with balanced information, the system predominantly favors the search strategy to optimize resource utilization. Conversely, in environments with pronounced information asymmetry, the system is more inclined to adopt the exploration strategy, enhancing adaptability to environmental changes and accelerating task completion.

Keywords: information asymmetry; swarm robotics; evolutionary game theory; strategy selection; interaction

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1. Introduction

With the rapid development of artificial intelligence and multi-agent systems (MASs), Swarm Robotics Systems (SRSs) have gradually become effective tools for solving complex problems [1,2]. The research inspiration for swarm robotics systems comes from collective behaviors in nature, such as the collective decision-making mechanisms of insects like ants and bees [3]. In these systems, multiple robots collaborate to perform tasks, leveraging distributed computing and local decision-making to achieve global objectives [4]. The advantages of swarm robotics systems lie in their high flexibility, robustness, and adaptability, enabling them to efficiently complete tasks such as search [5], exploration [6], and monitoring [7] in dynamic and uncertain environments. Research over the past few decades has shown that collective intelligence and distributed control are among the core strengths of swarm robotics systems [8]. MASs, as a core component of swarm robotics systems, provide the theoretical foundation and technical support for cooperation and decision-making among multiple agents [9]. In these systems, multiple agents must collaborate, exchange information, and make distributed decisions to achieve collective goals. In the presence of information asymmetry and communication delays, research on MASs has provided effective solutions to help robots make efficient decisions in complex environments [10].

However, in practical applications, the issue of information asymmetry among robots has become increasingly prominent. The evolutionary game dynamics driven by information disparity represent a core challenge in swarm robotics systems. Information asymmetry [11] refers to the fact that each robot in the group does not have perfectly symmetrical information when performing tasks due to perception range, communication delays, and sensor accuracy. This disparity may lead robots to make decisions based on local information, affecting the coordination of the group and the efficiency of task execution.

In swarm robotics systems, the evolutionary dynamics driven by information asymmetry involve interactions and cooperation between robots. In these interactions, each robot makes decisions based on its local information and needs to exchange information and collaborate with other robots to achieve a common goal [12]. This information-driven evolutionary game is not merely a simple cooperation-versus-defection issue but a dynamic and adaptive strategy optimization process. Game theory [13,14], particularly evolutionary game theory (EGT) [15–17], provides a powerful analytical framework for this evolution, as it can describe the strategy adjustment process of swarm robots in multi-round interactions and analyze how individuals evolve optimal cooperative strategies based on historical actions and local feedback in information asymmetry environments. In swarm robotics systems, information asymmetry means that each robot can only rely on limited local information when making decisions. Therefore, robots must continuously interact with others and adjust their behavior strategies. The key to this evolutionary process lies in how to optimize the current strategy based on past experiences and real-time feedback, thereby achieving the optimal collaboration of the entire group [18]. Strategy evolution in evolutionary game theory not only depends on the choices of individual robots but is also influenced by the strategies of other robots in the group, thus forming a "collective game". In this process, interactions between robots and the flow of information play a crucial role.

Information asymmetry is particularly prominent when swarm robots perform search and exploration tasks. Each robot can only perceive a portion of the surrounding environment, and how to share information with other robots to complement local perceptions and form global knowledge becomes a key aspect of group collaboration. In this process, the evolutionary dynamics driven by information asymmetry exhibit complex and dynamic characteristics. When making task-related decisions, each robot must consider not only the local task progress but also engage in information exchange or collaboration with other robots to allocate global tasks and schedule resources [19]. However, due to information asymmetry, robots may face uncertainty in collaboration and even encounter strategic conflicts. For instance, a robot might choose to "defect" instead of cooperating, leading to resource waste or task duplication, which negatively affects the overall collaboration of the group [20]. Designing effective game mechanisms to coordinate cooperation and competition between robots and optimize the strategy evolution process is critical to improving the performance of swarm robotics systems.

Although existing studies have explored individual strategy choices, they have yet to comprehensively consider factors such as information asymmetry, communication delays, strategy selection, and the interaction relationships between robots. This gap has created challenges in understanding the micro-level mechanisms influencing task completion efficiency in dynamic environments. Based on this, the main contributions of this paper are as follows:

1. This paper investigates the strategy selection problem in swarm robotics under conditions of information asymmetry. Focusing on the role of information sharing, it

comprehensively examines multiple influencing factors, including shared resource information, shared costs, communication delays, and strategy switching among robots. To model interactions between a robot and its neighbors, a payoff matrix is developed to evaluate the selection of search and exploration strategies.

2. Using evolutionary game theory and replicator dynamics, this paper examines the stable strategy combinations and evolutionary trajectories emerging from interactions among robots. The analysis reveals that when the payoff from switching strategies exceeds the difference between resource information and shared costs, the robots stabilize in either the (search, search) or (exploration, exploration) state. Conversely, when the payoff from switching strategies is lower than the difference in resource information and shared costs, the robots stabilize in the (search, search) state.
3. Numerical simulations are used in this study to examine how variations in different parameters affect the probabilities of robots selecting search or exploration strategies. The findings reveal that factors such as the proportion of shared resource information, the shared cost ratio, communication delays, and the probability of strategy switching play a crucial role in shaping the evolution of robot strategies. Specifically, adjusting these factors can effectively drive robots to transition from a search strategy to an exploration strategy, enabling them to acquire more environmental information and complete tasks more efficiently. This approach not only enhances task completion efficiency but also improves the system's ability to respond to diverse emergencies.

The structure of this paper is as follows: Section 2 introduces the decision-making choices of swarm robots and related work in game theory research. Section 3 elaborates on the swarm robotics system model established in this study, covering the construction of both the general model and the evolutionary game model. Section 4 presents an analysis of evolutionary stability, discussing strategy choices and their stability under different conditions. Section 5 verifies and analyzes the proposed model through simulation results. Finally, Section 6 summarizes the research findings and discusses future work.

2. Related Work

This section presents related research on strategy selection, exploring the impact of information asymmetry on different strategy choices and the application of evolutionary game theory in swarm robotics. It provides insights into the evolution of information-sharing strategies among swarm robots in dynamic scenarios, which has inspired the development of an evolutionary game model for information-sharing in such systems.

2.1. Research Status on Strategy Selection Based on Information Asymmetry

In recent years, the issue of strategy selection based on information asymmetry has received widespread attention across multiple disciplines, particularly in economics [21,22], management [23], decision science [24], and medicine [25]. In decision-making processes, differences in the amount and quality of information possessed by different decision-makers or systems often lead to biases in the decision outcomes. In many practical scenarios, information asymmetry directly impacts decision-making efficiency and the optimization of resource allocation [26,27]. For example, in market transactions, differences in the perceptions of product quality, price, and risk between buyers and sellers often lead to divergent choices. This information asymmetry can result in market failure or suboptimal resource allocation [28]. With the development of information technology, research on information asymmetry has expanded beyond economic transactions to include areas such as data privacy and multi-source information fusion.

Many studies have attempted to design effective decision-making strategies in the context of information asymmetry, particularly in intelligent decision systems where the impact of information disparity on strategy selection is more complex. Fu et al. [29] proposed a framework based on agent methods and evolutionary algorithms, aiming to optimize the decision-making process in project-driven supply chains by integrating information asymmetry. Lin et al. [30] investigated the information design problem in reinforcement learning agents, introducing the concepts of Markov signaling games and signal gradients to address the impact of information provision on agent behavior and its non-stationarity. Liu et al. [31] introduced a new multi-agent system (iAgents) that resolves information asymmetry through an information reasoning mechanism (InfoNav), allowing agents to actively exchange human information in collaboration. Additionally, H. Tavafoghi et al. [32] studied dynamic multi-agent decision problems with asymmetric information and non-strategic agents, proposing the concept of "full information" to effectively compress agent information. Through sequential decomposition and backward induction methods, they addressed the interdependence between agent strategies and beliefs and developed a globally optimal dynamic plan.

These findings provide profound insights into strategy selection and optimization in swarm robotics systems. However, current research primarily focuses on bridging information gaps, with less attention given to how to leverage information asymmetry to optimize strategy selection. For swarm robotics systems, effective strategy selection in the context of information asymmetry is crucial. Future research should focus on exploring how to utilize information asymmetry to optimize collaborative decision-making in robot swarms, particularly in multi-robot collaboration, by enhancing information sharing and optimizing local decision-making to improve group decision efficiency and overall performance.

2.2. Research Status of Evolutionary Game Theory in Swarm Robotics

Evolutionary game theory plays a crucial role in complex social environments, particularly in the strategy selection of swarm robotics. This theoretical framework allows us to deeply analyze the behavioral strategies of swarm robots using mathematical methods and to examine the mechanisms behind their emergence and evolution. Especially in dynamic environments, where individual decisions are influenced by factors such as resources, competition, and cooperation, evolutionary game theory helps to understand these social interactions [33].

Game theory has been increasingly applied inMASs, particularly in areas such as robot coordination and emergency rescue, providing effective decision support. Du et al. [34] used evolutionary game theory to achieve coordinated control inMASs by modeling the local interactions of agents as a game process, enabling task division and the achievement of overall goals through evolutionary dynamics. The Strategy Dynamics Particle Swarm Optimization (SDPSO) method incorporates evolutionary game theory to control population states, thereby improving optimization performance [35]. In the absence of communication, game theory models allow multi-robot systems to navigate toward target locations in a decentralized manner. Robots estimate the behaviors of local teammates to avoid obstacles, prevent collisions, and maintain team cohesion [36]. Researchers have also applied evolutionary game models to analyze strategies for loot box sales in games, revealing that interactions between companies and players can lead to win–win or lose–lose situations and that enhanced regulation contributes to the healthy development of the industry [37].

In emergency rescue, evolutionary game models have been used to analyze the cooperation between government rescue teams, social emergency organizations, and government support agencies. The results show that shared rescue goals and effective resource allocation are key to improving cooperation efficiency [38]. N. Fontbonne [39] proposed the

Horizontal Information Transfer (HIT) and Centralized Cooperative Co-evolutionary Algorithm (CCEA) to optimize robot coordination and collective behavior and demonstrated its effectiveness in multi-robot tasks. KAR Youssefi et al. [40] introduced a decentralized asynchronous swarm robot search algorithm that combines game theory to optimize robot decentralized strategies, effectively preventing early convergence and improving search efficiency. Xiong et al. [41] proposed a target search model combining bio-inspired techniques and evolutionary game theory, analyzing the task allocation problem in multi-robot systems within unknown areas. The model demonstrated that under weak selection conditions, it can promote the coexistence of two strategies, providing theoretical support for the design of self-organizing collective dynamics inMASs. These research findings indicate that the application of game theory can help robot teams collaborate efficiently, enhancing overall work efficiency and quality, which is of significant importance in emergency rescue operations.

The above research provides valuable insights from different perspectives into the motivations behind strategy selection in swarm robotics systems based on game theory. In swarm robot tasks, the choice between search and exploration strategies is influenced by both environmental conditions and the behaviors of other robots, necessitating consideration of the strategy evolution and cooperative adaptation process. Using evolutionary game theory for analysis can reveal the stability, adaptability, and efficiency of swarm robot systems under different strategy combinations, helping to optimize the overall effectiveness of task completion.

3. System Model

To further explore the strategy selection problem in swarm robotics systems and optimize the efficiency and performance of task completion, this section constructs both a general model and an evolutionary game model. The general model provides an understanding of the overall framework and basic rules of the swarm robotics system, enabling better analysis and description of the interactions between robots. The evolutionary game model, on the other hand, reveals the evolutionary patterns of strategy selection and the formation path of optimal strategies by considering the dynamic evolution of individual robot strategies. To facilitate a better understanding of the system, Figure 1 illustrates the key models and their interrelationships.

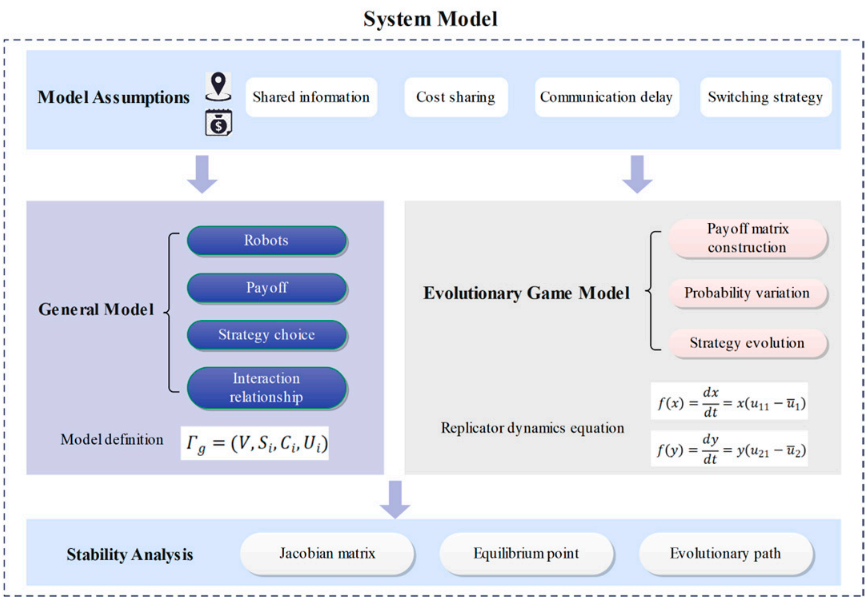

Figure 1. The system model framework diagram.

3.1. General Model

Consider a swarm consisting of N robots, where each robot is denoted by i $(i \in N)$. The swarm $G = (V,\ \varepsilon)$ can be defined by a set of nodes $V = \{\nu_1,\ \nu_2,\ \cdots,\ \nu_n\}$ and a set of edges $\varepsilon \subseteq V \times V$. The neighbor set of node N_i is denoted as $N_i = \{j \in V(i,\ j) \in \varepsilon\}$, assuming i is not its own neighbor.

The network game model can be defined as the following four-tuple:

$$\Gamma_g = (V,\ S_i,\ C_i,\ U_i), \tag{1}$$

- A set of nodes $V = \{\nu_1,\ \nu_2,\ \cdots,\ \nu_n\}$, where each node represents a participant, with a total of N robots, who can randomly choose their own strategies; for convenience, they are divided into two populations: n and m. The set of search robots n is represented as $S = (1,\ 2,\ \ldots,\ S)$, and the set of exploration robots m is represented as $E = (1,\ 2,\ \ldots,\ E)$.
- S_i represents the strategy set $S_i \in \{S,\ E\}$, where in this scenario, the strategies include search and exploration.
- C_i denotes the interaction relationships between robots and their neighbors, including the transmission, sharing, and collaboration of information. The information connections between robots are represented by defining a neighbor set.
- U_i represents the payoff for a robot under a chosen strategy. By assessing the benefits of different strategies, the robot adjusts its strategy to achieve higher payoffs. The payoffs here are defined and evaluated based on the amount of resource information obtained, which includes the cost of acquiring resources and the sum of resource information obtained after interacting with neighbors.

3.2. Model Assumptions and Payoff Matrix Construction

In emergency scenarios, swarm robots need to rationally select between exploration and search strategies to achieve higher payoffs for rescue operations. The search strategy focuses on locating trapped individuals and beneficial resource information within known areas, incurring negligible costs. In contrast, the exploration strategy may involve finding new rescue routes or resources, thus requiring higher costs to obtain new resource information, leading to asymmetric information in strategy selection.

Assume the participants in this evolutionary game are robot i and its neighbor j. Suppose robot i chooses the exploration strategy with a probability of x and the search strategy with a probability of $1-x$; neighbor j chooses the exploration strategy with a probability of y and the search strategy with a probability of $1-y$. When robot i chooses the search strategy, the amount of information it gains is E_{is}. If robot i chooses the exploration strategy, it will incur a cost C_{ie} (the cost of obtaining information, including resource consumption, time consumption, energy consumption, etc.), and it will receive a fixed amount of information I_{ie}. Similarly, when neighbor robot j chooses the search strategy, the amount of information it gains is E_{js}. If neighbor j chooses the exploration strategy, it will incur a cost C_{je}, and it will receive a fixed amount of information I_{je}.

3.2.1. Information Interaction

In order to obtain higher resource information, robots typically need to engage in information interaction, with robot i being able to interact only with its neighbors to learn about relevant local information. Based on this, the robot may change its strategy or maintain its current strategy. If the two parties choose different strategies at a certain moment, robot i will share the resource information it has obtained with its neighbor at a certain proportion $\left(I_{je} = \alpha \cdot I_{ie}\right)$, and the neighbor robot will also bear a certain proportion of the consumption cost $\left(C_{je} = \beta \cdot C_{ie}\right)$. When both parties choose the exploration strategy, although the exploring robot incurs a cost, it cannot be shared with other robots; when both parties choose the search strategy since they are both searching for resources in a known environment, the benefits obtained are relatively low.

Therefore, they will incur additional costs to support other strategy choices during this period, and the probability of them choosing to switch strategies is μ, $\{\mu \in (0, 1)\}$. All parameters are greater than 0, and $C_{ij} < C_{ie}$ or C_{je}.

3.2.2. Environmental Information Parameters

When robots interact with each other, the surrounding environment imposes a communication delay d on them. Here, we define e^{-d} as the attenuation factor due to the communication delay d. Thus, $\left(D = 1 - e^{-d}\right)$ represents the proportion of communication information that can be transmitted normally, which is the successful transmission efficiency. Additionally, the urgency of the event, denoted as γ, $\{\gamma \in (0, 1)\}$, affects the equilibrium of strategy selection. The higher the urgency, the higher the payoff of information obtained when robots choose the exploration strategy.

Based on this, the payoff situation for robot i and its neighbor robot j under the choice of two strategies can be represented by the following 2×2 payoff matrix, as shown in Table 1.

Table 1. Evolutionary game payoff matrix.

		Neighbor Robot *j*	
	Strategy Selection Probabilities	Explore (y)	Search $(1-y)$
Robot *i*	Explore (x)	$E_{is} + I_{ie} - C_{ie} + D \cdot I_{je}$, $E_{js} + I_{je} - C_{je} + D \cdot I_{ie}$	$E_{is} + I_{ie} - C_{ie} + D \cdot E_{js}$, $E_{js} + D \cdot \alpha \cdot I_{ie} - \beta \cdot C_{ie}$
	Search $(1-x)$	$E_{is} + D \cdot \alpha \cdot I_{je} - \beta \cdot C_{je}$, $E_{js} + I_{je} - C_{je} + D \cdot E_{is}$	$E_{is} + D \cdot E_{js} - \mu \cdot C_{ij}$, $E_{js} + D \cdot E_{is} - \mu \cdot C_{ij}$

3.3. *Evolutionary Game Model*

In the evolutionary game model, the dynamic evolution of individual strategy selection is described by the replicator dynamic equation. Specifically, the probability of an individual selecting a particular strategy changes over time, depending on the difference between its expected payoff and the average expected payoff. When robot i and its neighbor j choose between exploration or search strategies, they continuously adjust the probability of selecting each strategy based on their respective payoff situations. Figure 2 systematically illustrates this dynamic process and its interrelationships, providing a framework for the subsequent detailed calculations.

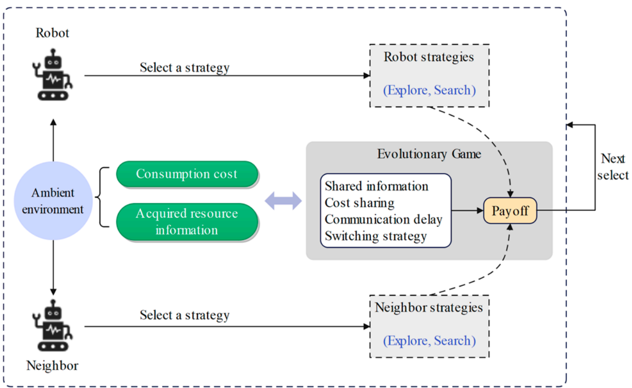

Figure 2. Schematic diagram of the dynamic evolution process in the evolutionary game model.

According to evolutionary game theory and the replicator dynamics equation, the rate of change in the probability x that robot i chooses the exploration strategy should be proportional to the difference between the expected payoff of choosing the exploration strategy and the average expected payoff. At time t, when robot i chooses the exploration strategy, its payoff is u_{11}:

$$u_{11} = y\left(E_{is} + I_{ie} - C_{ie} + D\cdot I_{je}\right) + (1-y)\left(E_{is} + I_{ie} - C_{ie} + D\cdot E_{js}\right), \tag{2}$$

When robot i chooses the search strategy, its payoff is u_{12}:

$$u_{12} = y\left(E_{is} + D\cdot\alpha\cdot I_{je} - \beta\cdot C_{je}\right) + (1-y)\left(E_{is} + D\cdot E_{js} - \mu\cdot C_{ij}\right), \tag{3}$$

The expected payoff for robot i is $\overline{u}_1$:

$$\overline{u}_1 = xu_{11} + (1-x)u_{12}, \tag{4}$$

The replication dynamics equation for robot i can be further expressed as

$$f(x) = \frac{dx}{dt} = x(u_{11} - \overline{u}_1) = x(1-x)\left(y\cdot D\cdot I_{je} + I_{ie} - C_{ie} - y\cdot D\cdot\alpha\cdot I_{je} + y\cdot\beta\cdot C_{je} + \mu\cdot C_{ij} - y\cdot\mu\cdot C_{ij}\right), \tag{5}$$

Similarly, if neighbor j chooses the exploration strategy, the payoff is u_{21}:

$$u_{21} = x\left(E_{js} + I_{je} - C_{je} + D\cdot I_{ie}\right) + (1-x)\left(E_{js} + I_{je} - C_{je} + D\cdot E_{is}\right), \tag{6}$$

When neighbor robot j chooses the search strategy, its payoff is u_{22}:

$$u_{22} = x\left(E_{js} + D\cdot\alpha\cdot I_{ie} - \beta\cdot C_{ie}\right) + (1-x)\left(E_{js} + D\cdot E_{is} - \mu\cdot C_{ij}\right), \tag{7}$$

The expected payoff for neighbor j is $\overline{u}_2$:

$$\overline{u}_1 = yu_{21} + (1-y)u_{22}, \tag{8}$$

By the same reasoning, the replication dynamics equation for neighbor j can be obtained as

$$f(y) = \frac{dy}{dt} = y(u_{21} - \overline{u}_2) = y(1-y)\left(x\cdot D\cdot I_{ie} + I_{je} - C_{je} - x\cdot D\cdot\alpha\cdot I_{ie} + x\cdot\beta\cdot C_{ie} + \mu\cdot C_{ij} - x\cdot\mu\cdot C_{ij}\right), \tag{9}$$

By solving the system of two-dimensional dynamic Equations (5) and (9), we can obtain the following five equilibrium points: $(0,\ 0)$, $(0,1)$, $(1,0)$, $(1,1)$, and $\left(\frac{C_{je} - I_{je} - \mu\cdot C_{ij}}{D\cdot I_{ie}\cdot(1-\alpha) + \beta\cdot C_{ie} - \mu\cdot C_{ij}}, \frac{C_{ie} - I_{ie} - \mu\cdot C_{ij}}{D\cdot I_{je}(1-\alpha) + \beta\cdot C_{je} - \mu\cdot C_{ij}}\right)$. Notably, the fifth equilibrium point is abbreviated as $(x^*,\ y^*)$ here.

Equations (5) and (9) describe the dynamic process of strategy selection probabilities. Furthermore, to analyze the stability of these dynamic equations at the equilibrium points, the Jacobian matrix is constructed, with its elements being the partial derivatives with respect to the strategy probabilities, reflecting the interrelationships between strategies. The Jacobian matrix is shown in Equation (10):

$$J = \begin{bmatrix} \frac{\partial f(x)}{\partial x} & \frac{\partial f(x)}{\partial y} \\ \frac{\partial f(y)}{\partial x} & \frac{\partial f(y)}{\partial y} \end{bmatrix}, \tag{10}$$

The elements of the Jacobian matrix are given in Equation (11).

$$\begin{cases} \frac{\partial f(x)}{\partial x} = (1-2x)\left[y\cdot\left(D\cdot I_{je}(1-\alpha)+\beta\cdot C_{je}-\mu\cdot C_{ij}\right)+I_{ie}-C_{ie}+\mu\cdot C_{ij}\right] \\ \frac{\partial f(x)}{\partial y} = x(1-x)\left[D\cdot I_{je}(1-\alpha)+\beta\cdot C_{je}-\mu\cdot C_{ij}\right] \\ \frac{\partial f(y)}{\partial x} = y(1-y)\left[D\cdot I_{ie}(1-\alpha)+\beta\cdot C_{ie}-\mu\cdot C_{ij}\right] \\ \frac{\partial f(y)}{\partial y} = (1-2y)\left[x\cdot\left(D\cdot I_{ie}(1-\alpha)+\beta\cdot C_{ie}-\mu\cdot C_{ij}\right)+I_{je}-C_{je}+\mu\cdot C_{ij}\right] \end{cases}, \quad (11)$$

According to the stability theorem of differential equations, the stability of an equilibrium point as an Evolutionarily Stable Strategy (ESS) can be determined by calculating the eigenvalues of the Jacobian matrix. Specifically, when the determinant (det) is greater than zero and the trace (tr) is less than zero, the equilibrium point is an ESS, which applies to two-player games. The det is the product of the eigenvalues, and the tr is the sum of the eigenvalues. If the trace is zero, the corresponding equilibrium point is a saddle point. The calculation results of the determinant and trace of the Jacobian matrix at different equilibrium points are shown in Table 2.

Table 2. Determinant and trace values for each equilibrium point.

Equilibrium Point	Determinant	Trace
$(0,0)$	$\left(\mathrm{I_{ie}}-C_{ie}+\mu\cdot C_{ij}\right)\left(\mathrm{I_{je}}-C_{je}+\mu\cdot C_{ij}\right)$	$\left(\mathrm{I_{ie}}-C_{ie}+\mu\cdot C_{ij}\right)+\left(\mathrm{I_{je}}-C_{je}+\mu\cdot C_{ij}\right)$
$(0,1)$	$\left(C_{je}-\mathrm{I_{je}}-\mu\cdot C_{ij}\right)$ $\left(\mathrm{I_{ie}}-C_{ie}+\beta\cdot C_{je}+D\cdot\mathrm{I_{je}}-D\cdot\alpha\cdot\mathrm{I_{je}}\right)$	$\left(C_{je}-\mathrm{I_{je}}-\mu\cdot C_{ij}\right)+$ $\left(\mathrm{I_{ie}}-C_{ie}+\beta\cdot C_{je}+D\cdot\mathrm{I_{je}}-D\cdot\alpha\cdot\mathrm{I_{je}}\right)$
$(1,0)$	$\left(C_{ie}-\mathrm{I_{ie}}-\mu\cdot C_{ij}\right)$ $\left(\mathrm{I_{je}}-C_{je}+\beta\cdot C_{ie}+D\cdot\mathrm{I_{ie}}-D\cdot\alpha\cdot\mathrm{I_{ie}}\right)$	$\left(C_{ie}-\mathrm{I_{ie}}-\mu\cdot C_{ij}\right)+$ $\left(\mathrm{I_{je}}-C_{je}+\beta\cdot C_{ie}+D\cdot\mathrm{I_{ie}}-D\cdot\alpha\cdot\mathrm{I_{ie}}\right)$
$(1,1)$	$\left(\mathrm{I_{je}}-C_{je}+\beta\cdot C_{ie}+D\cdot\mathrm{I_{ie}}-D\cdot\alpha\cdot\mathrm{I_{ie}}\right)$ $\left(\mathrm{I_{ie}}-C_{ie}+\beta\cdot C_{je}+D\cdot\mathrm{I_{je}}-D\cdot\alpha\cdot\mathrm{I_{je}}\right)$	$\left(\mathrm{I_{je}}-C_{je}+\beta\cdot C_{ie}+D\cdot\mathrm{I_{ie}}-D\cdot\alpha\cdot\mathrm{I_{ie}}\right)+$ $\left(\mathrm{I_{ie}}-C_{ie}+\beta\cdot C_{je}+D\cdot\mathrm{I_{je}}-D\cdot\alpha\cdot\mathrm{I_{je}}\right)$
$(x^*,\ y^*)$	$-x^*\cdot y^*\left(D\cdot\mathrm{I_{ie}}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+\mathrm{I_{je}}\right)$ $\left(D\cdot\mathrm{I_{je}}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+\mathrm{I_{ie}}\right)$	0

4. Evolutionary Stability Analysis

By analyzing the signs of the determinant and trace of the Jacobian matrix, the Evolutionarily Stable Strategy (ESS) can be effectively determined. Based on the ESS determination criteria, we can solve for the stable equilibrium points under different evolutionary paths. That is, $(0,0)$ and$(1,1)$ are the stable strategies of the evolutionary game, while $(0,1)$ and $(1,0)$ are unstable points, and $(x^*,\ y^*)$ is a saddle point of the evolutionary game. Next, a detailed analysis of these conditions will be provided.

4.1. Pur Strategy Stability Analysis

In the context of pure strategies, the strategy selection and evolutionary paths of robots under specific payoff conditions exhibit differences in stability. By constructing the Jacobian matrix and analyzing the stability of the equilibrium points, we discuss two scenarios separately.

4.1.1. Switching Payoff Greater than Resource Difference

Theorem 1. *When the payoff obtained by both robots from switching strategies is greater than the difference between the resource information and shared cost in the interaction process, both the robot and its neighbor will choose to follow the strategy. Specifically, if one robot chooses the search strategy, the other will also choose the search strategy; if one robot chooses the exploration strategy, the other will also choose the exploration strategy.*

Proof. When $I_{ie} + \mu \cdot C_{ij} < C_{ie}$, $I_{je} + \mu \cdot C_{ij} < C_{je}$, $I_{je} + \beta \cdot C_{ie} + D \cdot I_{ie} < D \cdot \alpha \cdot I_{ie} + C_{je}$, $I_{ie} + \beta \cdot C_{je} + D \cdot I_{je} < C_{ie} + D \cdot \alpha \cdot I_{je}$, that is $I_{ie} - C_{ie} + \mu \cdot C_{ij} > D \cdot \alpha \cdot I_{ie} - \beta \cdot C_{ie} - D \cdot I_{ie}$ and $I_{je} - C_{je} + \mu \cdot C_{ij} > D \cdot \alpha \cdot I_{je} - \beta \cdot C_{je} - D \cdot I_{je}$. The stability of each equilibrium point is shown in Table 3. Under this condition, the points $(0,0)$ and $(1,1)$ are stable strategies. The point $(0,0)$ indicates that both robot i and its neighbor j have a probability of 0 for choosing the exploration strategy, meaning they will both choose the search strategy. The robots aim to maximize the amount of resource information. Although under this condition, the robots can always find better strategies to replace their current choices, these strategies are unstable due to evolutionary pressures or competition that arise during interactions with neighbors. This leads to adjustments in strategies, and ultimately, the system stabilizes at the point $(0,0)$, where the main payoff from the search strategy is better than the payoffs from other strategies. Thus, the robots will not change their strategies for a certain period of time. Similarly, the point $(1,1)$ is a stable strategy, indicating that both robot i and its neighbor j choose the exploration strategy with a probability of 1, meaning they will both choose the exploration strategy. In the current environment, the payoff from choosing exploration is higher than the payoff from other strategies, so the robots will not change their strategies. □

Table 3. Stability of the Jacobian matrix at equilibrium points under the conditions of Theorem 1.

Equilibrium Point	Determinant Symbol	Trace Symbol	Stability
$(0,0)$	+	−	ESS
$(0,1)$	−	+	Unstable
$(1,0)$	−	+	Unstable
$(1,1)$	+	−	ESS
(x^*, y^*)	−	0	Saddle point

4.1.2. Switching Payoff Less than Resource Difference

Theorem 2. *When the payoff obtained by both robots from switching strategies is less than the difference between the resource information and shared cost in the interaction process, the robots will still choose the same strategy, but the result is more likely to stabilize in the (search, search) state.*

Proof. When $I_{ie} + \mu \cdot C_{ij} < C_{ie}$, $I_{je} + \mu \cdot C_{ij} < C_{je}$, $I_{je} + \beta \cdot C_{ie} + D \cdot I_{ie} > D \cdot \alpha \cdot I_{ie} + C_{je}$, $I_{ie} + \beta \cdot C_{je} + D \cdot I_{je} > C_{ie} + D \cdot \alpha \cdot I_{je}$, that is $I_{ie} - C_{ie} + \mu \cdot C_{ij} < D \cdot \alpha \cdot I_{ie} - \beta \cdot C_{ie} - D \cdot I_{ie}$ and $I_{je} - C_{je} + \mu \cdot C_{ij} < D \cdot \alpha \cdot I_{je} - \beta \cdot C_{je} - D \cdot I_{je}$. The stability of each equilibrium point is shown in Table 4. Under this condition, the point $(0,0)$ is the only stable strategy, meaning that both the robot and its neighbor will choose the search strategy because, under this condition, the cost of choosing exploration exceeds the resource information gained. Although there is an amount of exploration information greater than the resource information exchanged during the evolutionary process, these strategies are unstable due to evolutionary pressures or competition. This leads to adjustments in strategies, and ultimately, the system will stabilize at the point $(0,0)$, where the main payoff from the search strategy is greater than the payoff from other strategies. Therefore, for a certain period of time, the robot will not change its strategy. □

Table 4. Stability of the Jacobian matrix at equilibrium points under the conditions of Theorem 2.

Equilibrium Point	Determinant Symbol	Trace Symbol	Stability
$(0,0)$	+	−	ESS
$(0,1)$	+	+	Unstable
$(1,0)$	+	+	Unstable
$(1,1)$	+	+	Unstable

4.1.3. Comprehensive Comparison

Both Theorem 1 and Theorem 2 exhibit stability for the same strategy, but the mechanisms influencing them differ significantly. In Theorem 1, the system stabilizes with both search and exploration strategies. This is because the benefits derived from switching strategies are sufficient to offset the differences between resource information and shared costs, allowing the robots to flexibly switch between these two strategies when interacting with their neighbors. This flexibility enables the robots to fully utilize environmental information and resources in various scenarios, thereby optimizing task execution efficiency in dynamic and uncertain environments. For example, when there is a large amount of unknown information in the environment, the exploration strategy helps the robots discover new resources and information, while in relatively resource-rich situations, the search strategy efficiently utilizes the available resources.

In Theorem 2, since the benefits of switching strategies are insufficient to offset the differences between resource information and shared costs, the robots tend to adopt the more conservative search strategy. In this case, the robots choose the search strategy to maximize efficiency under known resources and information, avoiding unnecessary costs and risks. The conservative strategy choice helps reduce resource waste and unnecessary expenses in uncertain environments while ensuring efficient operation under known conditions. The search strategy effectively locates the target in a known environment, reducing the uncertainty and potential risks associated with exploration.

It can be observed that the benefit of switching strategies plays a crucial role in the strategy selection process, particularly in relation to the difference between resource information and shared costs. When the benefits are higher, the robot can maintain flexibility between the search and exploration strategies, balancing the advantages and disadvantages of both to adapt to different environmental changes and task requirements. Conversely, when the benefits are lower, the robot tends to adopt the more conservative search strategy to minimize uncertainty and risk. This strategy selection mechanism reflects the robot's adaptability and decision optimization capability when facing dynamic environments, helping to enhance the overall efficiency and performance of the system in complex and ever-changing environments.

4.2. Mixed Strategy Stability Analysis

In the analysis of mixed strategy stability, we determine the stability by studying the local dynamics of the equilibrium points. According to the determinant and trace results in Table 2, the Jacobian matrix at the saddle point $(x^*,\ y^*)$ is used to describe the behavior of the system near this point. The diagonal elements of the matrix are zero, meaning that at this equilibrium point, the probability values of variables x and y do not directly affect their own changes but rather are influenced by the interaction with the other player's strategy. Therefore, the stability of the system is determined by the cross terms (i.e., the interaction effects between strategies). The specific form of the Jacobian matrix at this point is given by Equation (12).

$$J = \begin{bmatrix} 0 & \frac{\partial f(x)}{\partial y} \\ \frac{\partial f(y)}{\partial x} & 0 \end{bmatrix}, \tag{12}$$

The values of the elements in the matrix are given in Equation (13).

$$\begin{cases} \frac{\partial f(x)}{\partial y} = \frac{x^*\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+I_{je}\right)\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-\mu\cdot C_{ij}\right)}{D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-\mu\cdot C_{ij}} \\ \frac{\partial f(y)}{\partial x} = \frac{y^*\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+I_{ie}\right)\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-\mu\cdot C_{ij}\right)}{D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-\mu\cdot C_{ij}} \end{cases}, \tag{13}$$

It can be further concluded that the determinant and trace at this point are given by Equations (14) and (15), respectively.

$$detJ = -x^*\cdot y^*\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+I_{je}\right)\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+I_{ie}\right), \tag{14}$$

$$trJ = 0, \tag{15}$$

At this point, the characteristic equation of the determinant is: the characteristic equation is:

$$\lambda^2 - trJ\cdot\lambda + detJ = 0, \tag{16}$$

Here, λ represents the eigenvalue of the characteristic equation, which is the eigenvalue of the Jacobian matrix at the saddle point P. It is used to analyze the local stability of the model at the saddle point. When $trJ^2 - 4detJ > 0$, the eigenvalues at the saddle point P are real numbers, meaning the solutions to the characteristic equation of the Jacobian matrix are real. In this case, according to the Jury condition [42], the sufficient and necessary condition for the stability of the saddle point P is given by Equation (17).

$$\begin{cases} 1 + trJ + detJ > 0 \\ 1 - trJ + detJ > 0 \\ |detJ| < 1 \end{cases}, \tag{17}$$

Based on the above conditions, Equation (18) can be derived as follows:

$$\begin{cases} 1 + \left(-x^*\cdot y^*\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+I_{je}\right)\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+I_{ie}\right)\right) > 0 \\ \left|-x^*\cdot y^*\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+I_{je}\right)\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+I_{ie}\right)\right| < 1 \end{cases}, \tag{18}$$

Then, the stability range is obtained:

$$-1 < x^*\cdot y^*\left(D\cdot I_{ie}\cdot(1-\alpha)+\beta\cdot C_{ie}-C_{je}+I_{je}\right)\left(D\cdot I_{je}\cdot(1-\alpha)+\beta\cdot C_{je}-C_{ie}+I_{ie}\right) < 1. \tag{19}$$

That is, the saddle point $P\ (x^*, y^*)$ is a stable equilibrium point within the range defined by the parameters; however, if the parameters exceed this range, the point becomes unstable. When the point is unstable, the robot will choose search and exploration strategies with a certain probability distribution. The phase diagram of the system evolution is shown in Figure 3.

From the diagram, it can be understood that when the equilibrium point $P\ (x^*,\ y^*)$ moves within the region *SNOE*, different initial states of $P\ (x^*,\ y^*)$ will result in different ultimate evolutionary outcomes of the strategy.

If $x^* < \frac{1}{2}$, $y^* < \frac{1}{2}$, at this point, the area of the quadrilateral *SPEN* will be greater than the area of the quadrilateral *SOEP*, and the system's final strategy will evolve stably to point $N\ (1,\ 1)$, meaning that the robot and its neighbor's final strategy choice will be (explore, explore). If $x^* = \frac{1}{2}$, $y^* = \frac{1}{2}$, the area of the quadrilateral *SPEN* will be equal to the area of the quadrilateral *SOEP*, and the system's final strategy may evolve stably to point $N\ (1,\ 1)$, meaning that the robot and its neighbor's final strategy choice will be (explore, explore), or it may evolve stably to point $O\ (0,\ 0)$, meaning that the robot and its neighbor's final strategy choice will be (search, search). Therefore, under this condition, the strategies of the robot and its neighbor can be either exploration or search. If $x^* > \frac{1}{2}$, $y^* > \frac{1}{2}$, the area of the quadrilateral *SPEN* will be less than the area of the quadrilateral *SOEP*, and the system's final strategy will evolve stably to point $O\ (0, 0)$, meaning that the robot and its neighbor's final strategy choice will be (search, search).

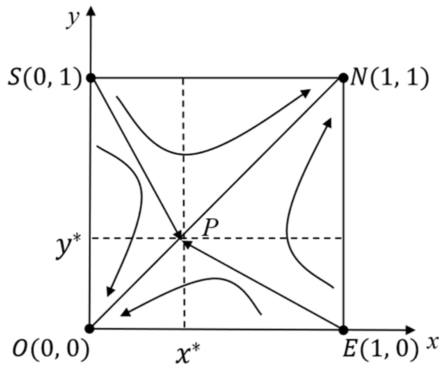

Figure 3. System evolution diagram at the Nash equilibrium point.

5. Numerical Simulation Analysis

In this section, we will validate the feasibility of our model using numerical simulations and explore the dynamic behavior of strategy selection and the evolutionary process in swarm robotics systems. We used MATLAB R2016a for the simulations and set the initial parameters. First, we will simulate the evolutionary path, demonstrating the evolutionary process of individual robots under different strategy choices, thus revealing how the system tends toward equilibrium. Next, we will set different parameters to analyze the impact of various factors on the system's evolutionary path, including communication delay, strategy selection probability, and information-sharing efficiency. Through the simulation results with parameter variations, we will further investigate the impact of these factors on collaboration in swarm robotics.

5.1. Parameter Settings and Selection

In order to validate the effectiveness and feasibility of the proposed evolutionary game model, numerical simulations were conducted. Table 5 summarizes the parameters used in this study. The simulation settings and selections of these parameters are based on empirical observations, logical assumptions, and the characteristics of the modeled scenario while considering the differences in strategy selection, resource consumption, and information sharing between the robots and their neighbors. The meaning of each parameter and the rationale for its selection are briefly explained in the table. The initial ratio between the robot and its neighbor is set as $x_0 = y_0 = 0.5$.

Table 5. Parameters used in numerical simulations and their setting rationale.

Parameter Symbol	Parameter Description	Basis or Source of Parameter Selection
C_{ie}	Cost of the exploration strategy for the robot	Empirically set based on the resource consumption of the robot's tasks
C_{je}	Cost of the exploration strategy for the neighbor	Assumes the neighbor has slightly higher resource costs

Table 5. *Cont.*

Parameter Symbol	Parameter Description	Basis or Source of Parameter Selection
I_{ie}	Resource information obtained by the robot	Effective resource information acquired in the simulation scenario
I_{je}	Resource information obtained by the neighbor	Assumes the neighbor has a higher resource acquisition capability
x_0, y_0	Initial probability of strategy selection	Assumes equal initial probabilities for both strategies
D	Proportion of resource information successfully exchanged under communication delay	Reflects transmission efficiency under communication delay
C_{ij}	Cost of switching from one strategy to another	Set based on the resource cost of strategy switching
μ	Probability of switching strategies	Assumes a relatively low probability of switching strategies
α	Proportion of shared resources when different strategies are chosen	Assumes a high level of resource-sharing
β	Proportion of costs borne when both choose the search strategy	Reflects the imbalance in resource cost-sharing

5.2. System Evolution Path Simulation

In this section, the data selected are based on different scenarios to simulate the evolution paths of the robot and its neighbor. First, based on Theorem 1, we choose the initial ratio of both to be less than $\frac{1}{2}$. For example, the initial parameters chosen are as follows: $D = 0.9$, $I_{ie} = 20$, $I_{je} = 25$, $C_{ie} = 25$, $C_{je} = 30$, $C_{ij} = 10$, $\mu = 0.4$, $\alpha = 0.8$, $\beta = 0.3$, and the replication dynamic equations for this scenario are $\frac{dx}{dt} = x(1-x)\,(9.5y-1)$ and $\frac{dy}{dt} = y(1-y)\,(8.6x-1)$, where $x^* = 0.1163$, $y^* = 0.1053$. The simulation results are shown in Figure 4.

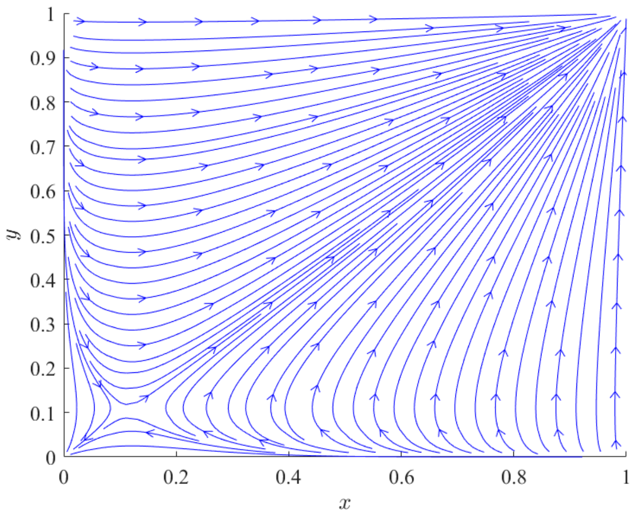

Figure 4. The system evolution diagram for Theorem 1.

As shown in Figure 4, in Theorem 1, there is one saddle point, two stable points, and two unstable points. The evolution path of the system will change depending on the initial x* and y*. In the case of (search, search), both robots choose the conservative search strategy to maximize the resource utilization efficiency of the known environment, whereas in the case of (explore, explore), both robots use the exploration strategy to more effectively discover new information and adapt to environmental changes. This is similar to the Hawk–Dove game: if the value of the resource is higher than the cost of competing for it, the robots are more likely to adopt a consistent strategy to optimize their gains. Whether stabilizing at (search, search) or (explore, explore), this indicates that the system demonstrates higher consistency and stability in this scenario.

Furthermore, in Theorem 2, we choose the initial ratio of robots and neighbors to be greater than $\frac{1}{2}$. For example, the chosen initial parameters are as follows: $D = 0.9$, $I_{ie} = 20$, $I_{je} = 25$, $C_{ie} = 25$, $C_{je} = 30$, $C_{ij} = 10$, $\mu = 0.4$, $\alpha = 0.8$, $\beta = 0.2$, and the replication dynamic equations for this scenario are $\frac{dx}{dt} = x(1-x)\ (1.96y - 2.2)$, $\frac{dy}{dt} = y(1-y)\ (1.44x - 2.2)$, where $x^* = 1.5278$, $y^* = 1.1224$. The simulation results are shown in Figure 5.

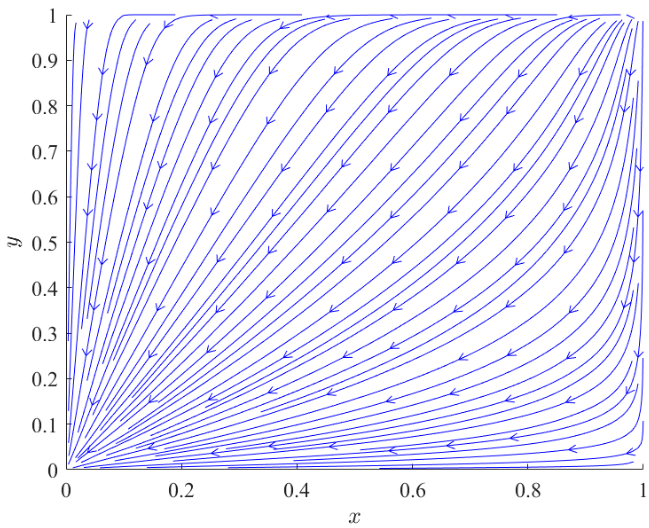

Figure 5. The system evolution diagram for Theorem 2.

As shown in Figure 5, there is one stable point and three unstable points. The system's evolution path will move from the unstable points toward the stable point. In other words, the robots may initially attempt the exploration strategy, but as the interactions evolve and the payoffs are calculated, they will eventually converge toward the stable state of (search, search). This conservative choice effectively avoids uncertainty and high-cost risks, representing a rational strategy under conditions of limited resources.

5.3. Simulation Analysis of Factor Variations

To thoroughly analyze the impact of various factors on the evolutionary game model, this section will conduct a simulation analysis to observe how changes in key parameters affect the system's behavior. By altering critical factors, the goal is to examine how these factors influence the evolutionary path of the group robots, the final decision strategy, and the number of steps required to reach the final decision. Further, it aims to reveal the influence of different factors on the system's evolutionary path and verify the model's reliability and applicability. The initial parameter values in the model are set as follows: the cost of exploration for both the robot and its neighbor is set as $C_{ie} = 25$ and $C_{je} = 30$, the size of the resource information obtained is set as $I_{ie} = 20$ and $I_{je} = 25$, the initial probability of choosing each strategy is $x_0 = y_0 = 0.5$, assuming the proportion of information resources that can be normally interacted with under communication delay conditions is $D = 0.9$, the cost of switching from search to exploration strategy when both parties choose search is $C_{ij} = 10$, and the probability of switching strategies is $\mu = 0.4$, the proportion of shared resource information when choosing different strategies is $\alpha = 0.8$, and the cost proportion borne by the party choosing search strategy when both choose search is $\beta = 0.3$. Based on the above numerical values, this paper explores the impact of various parameters on the strategy choices of both the robot and its neighbor.

5.3.1. Analysis of the Variation in Shared Information Ratio

When the shared resource information ratios for robot i and its neighbor j are set to 0.8, 0.84, 0.88, 0.92, and 0.96, the results showing the impact of the shared information resource ratio on the system's evolutionary path are illustrated in Figure 6.

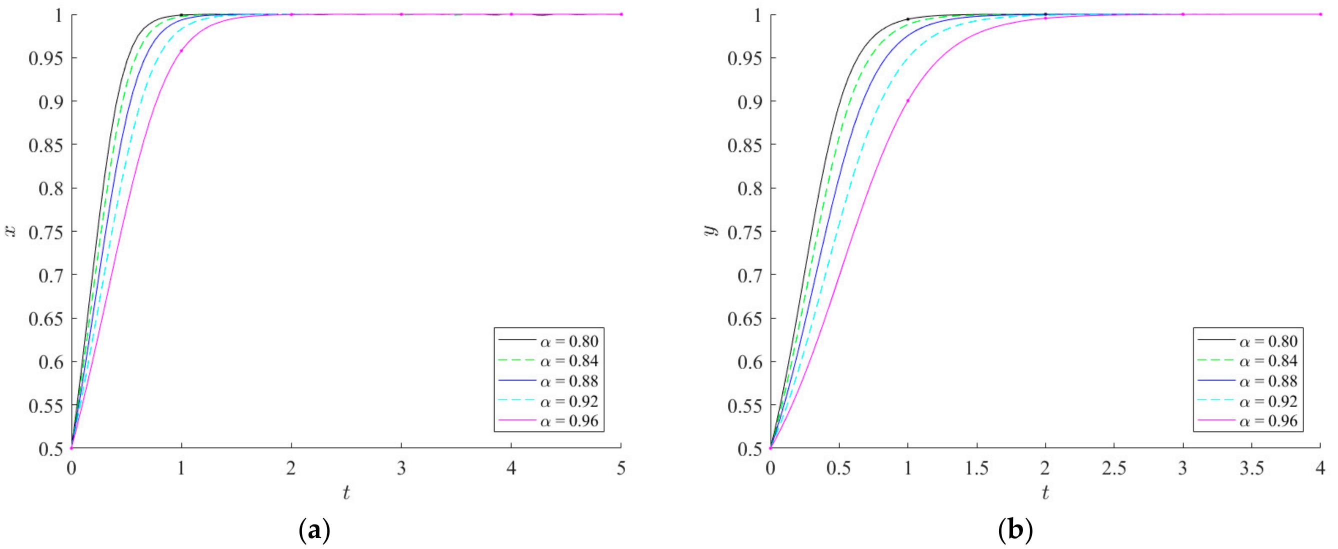

Figure 6. The impact of the proportion of shared information on evolutionary pathways. (**a**) The impact of shared information ratio on the path evolution of robot i; (**b**) the impact of shared information ratio on the path evolution of neighbor j.

From Figure 6a, it can be observed that as the system's shared information ratio decreases from 0.96 to 0.80, the rate at which robot i transitions to the exploration state gradually increases. From Figure 6b, it is evident that as the system's shared information ratio decreases from 0.96 to 0.80, the rate at which neighbor j transitions to the exploration strategy also gradually increases. This indicates that during the dynamic strategy evolution process, as the amount of shared information among robots decreases, they are more inclined to choose the exploration strategy as their ultimate strategy. When both the robot and its neighbor choose the exploration strategy, the system's transition step size to the exploration strategy shortens from 3 to 2.

5.3.2. Analysis of the Variation in the Cost-Sharing Ratio

When the cost-sharing ratios for robot i and neighbor j are set to 0.10, 0.12, 0.14, 0.16, and 0.18, the impact of the cost-sharing ratio on the system's evolutionary path is shown in Figure 7.

From Figure 7a, it can be observed that as the system's cost-sharing ratio increases from 0.10 to 0.18, the probability of robot i's strategy shifting toward the exploration strategy gradually increases. Similarly, from Figure 7b, it is evident that as the system's cost-sharing ratio increases from 0.10 to 0.18, the probability of neighbor j's strategy shifting toward the exploration strategy also gradually increases. This implies that during the dynamic strategy evolution process, as the shared cost among robots increases, they are more inclined to choose the exploration strategy as their ultimate strategy. When both the robot and its neighbor choose the exploration strategy, the system's transition step size to the exploration strategy shortens from 8 to 4.

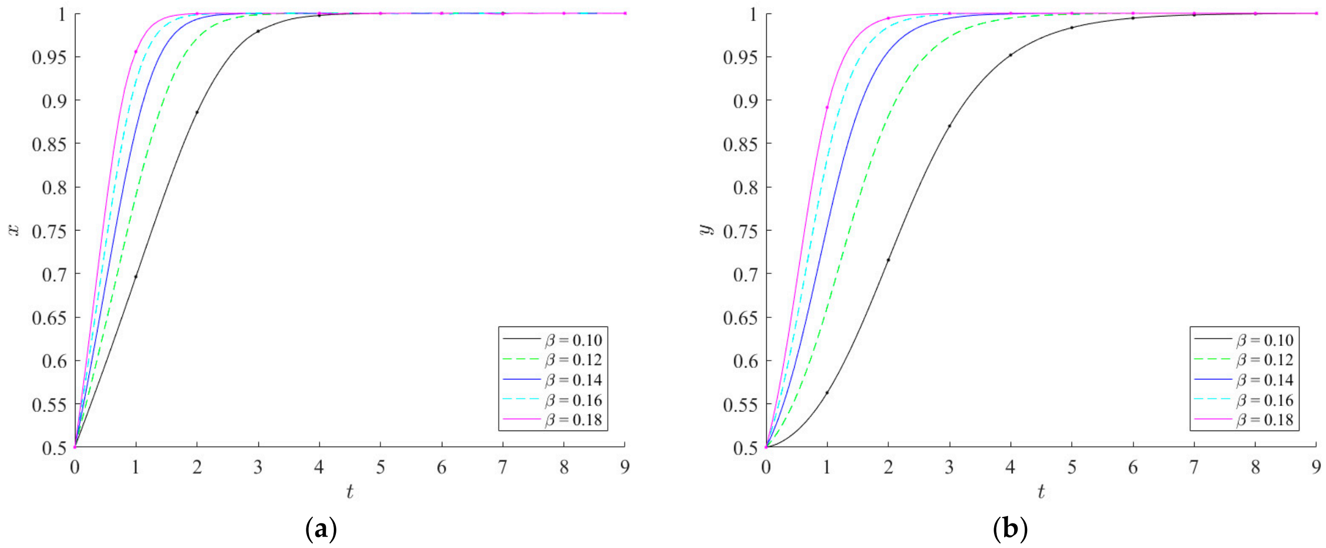

Figure 7. The impact of the proportion of shared costs on evolutionary pathways. (**a**) The impact of cost-sharing ratio on the evolutionary path of robot *i*; (**b**) the impact of the cost-sharing ratio on the evolutionary path of neighbor *j*.

5.3.3. Analysis of Communication Delay Variation

When the successful transmission efficiency between robot *i* and neighbor *j* is set to 0.5, 0.6, 0.7, 0.8, and 0.9, the impact of the successful transmission efficiency on the system's evolutionary path is shown in Figure 8.

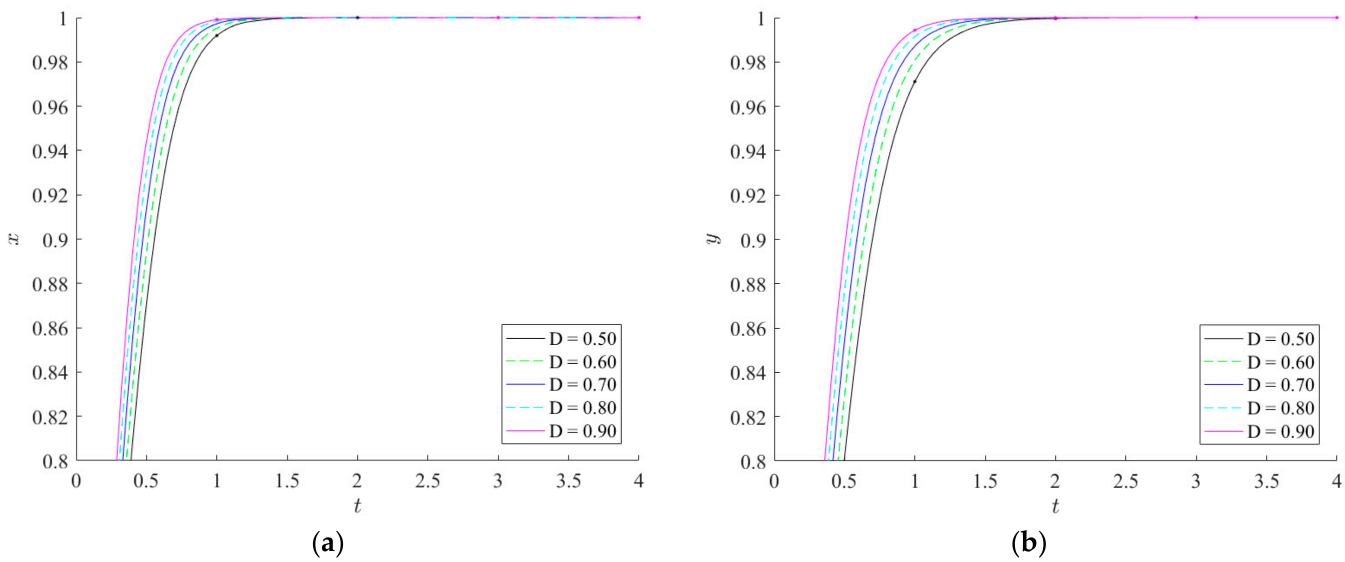

Figure 8. The impact of successful transmission efficiency on evolutionary pathways. (**a**) The impact of successful transmission efficiency on the evolutionary path of robot *i*; (**b**) the impact of successful transmission efficiency on the evolutionary path of neighbor *j*.

From Figure 8, it can be observed that as the system's successful transmission efficiency increases from 0.5 to 0.9, the probability of both robot *i* and neighbor *j* transitioning to the exploration strategy gradually increases. This means that during the dynamic strategy evolution process, as the communication delay in the surrounding rescue environment gradually decreases, i.e., as the transmission efficiency in the environment increases, both robots are more likely to choose the exploration strategy as the final strategy. The system's transition step length to the exploration steady state is 2.

5.3.4. Analysis of Switching Strategy Probability Variation

When the switching strategy probability for robot i and neighbor j is set to 0.1, 0.3, 0.5, 0.7, and 0.9, the impact of the switching strategy probability on the system's evolutionary path is shown in Figure 9.

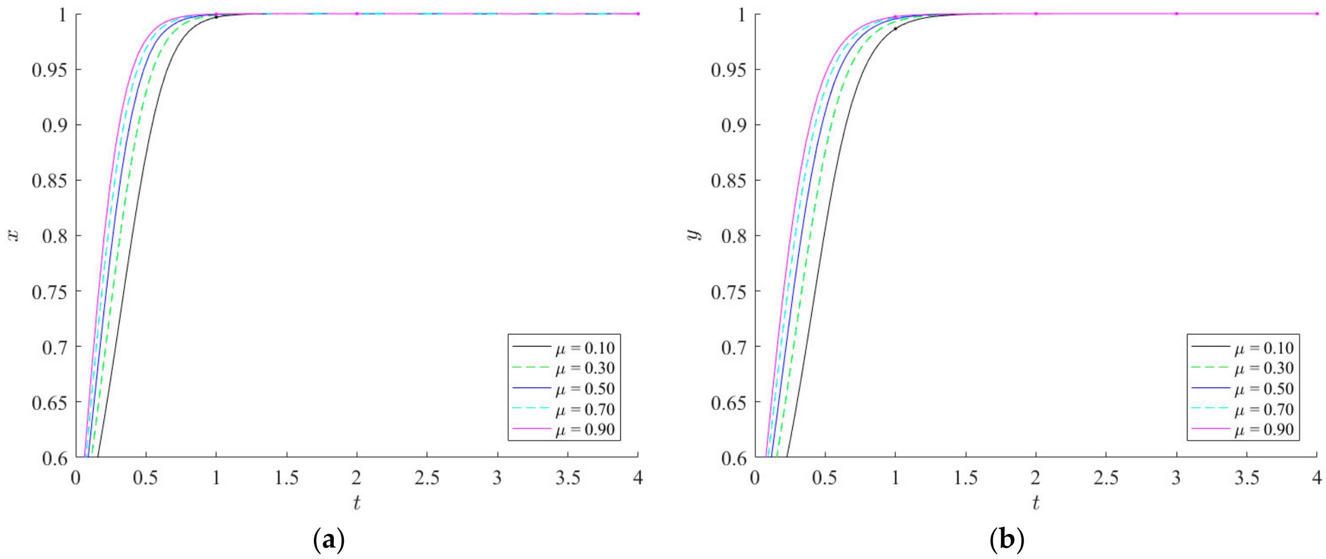

Figure 9. The impact of switch strategy probability on evolutionary pathways. (**a**) The impact of switching strategy probability on the evolutionary path of robot i; (**b**) the impact of switching strategy probability on the evolutionary path of neighbor j.

Based on the observations in the figures, it can be noted that as the switching strategy probability between robot i and neighbor j increases from 0.1 to 0.9, the rate at which the system transitions to the exploration state gradually increases. This indicates that during the dynamic strategy evolution process, as the probability of switching strategies gradually increases, the adaptability of the robots and their neighbors in dynamic decision-making is enhanced. As a result, the system can transition to the exploration state more quickly, thereby improving task execution efficiency. The system's transition step length to the exploration steady state is 1.5.

5.4. Discussion

Based on the simulation analysis results in Section 5.2, under conditions of information disparity, a decrease in the ratio of shared resource information and an increase in shared costs lead to improved transmission efficiency and a higher probability of strategy switching. Consequently, the robot system transitions gradually from a search strategy to an exploration strategy. This shift enhances the system's ability to acquire unknown environmental information more effectively, thereby accelerating task completion.

During strategic interactions among swarm robots, a decrease in the proportion of shared information leads the system's evolution to favor the exploration strategy. This shift is driven by the potential benefits arising from information asymmetry. Limited information exchange reduces the robots' ability to perceive their environment, making the search strategy insufficient for acquiring the necessary environmental data to address dynamic changes. In contrast, the exploration strategy enhances the system's overall effectiveness and task completion efficiency by actively uncovering new information or resources and adapting existing strategies. Consequently, under conditions of information asymmetry, the exploration strategy offers greater long-term benefits.

Further analysis reveals that as the proportion of shared resource information decreases and the shared cost ratio among neighbors increases, the relative advantage of the exploration strategy becomes more pronounced, encouraging robots to adopt this approach. Notably, when neighbors bear higher costs, robots can effectively reduce their own burden and enhance collective coordination by selecting the exploration strategy. Additionally, improvements in successful transmission efficiency further reinforce the system's preference for the exploration strategy. Efficient communication mechanisms enable robots to quickly and accurately acquire environmental feedback, facilitating rapid adaptation to dynamic conditions and accelerating task execution. By enhancing adaptability to unknown environments and optimizing decision-making with newly acquired environmental data, the exploration strategy significantly boosts overall task completion efficiency.

Additionally, an increase in the probability of strategy switching steers the system's evolutionary path toward the exploration strategy. This trend is largely influenced by the incentive mechanism. As the incentives for switching strategies increase, robots and their neighbors are more inclined to adopt the exploration strategy, particularly in environments characterized by significant information disparities. This is because strategy switching provides higher potential rewards, encouraging robots to actively explore unknown areas, gather additional information and resources, and ultimately improve the system's overall performance.

6. Conclusions

This paper investigates the strategy selection of group robots in dynamic environments characterized by information differences, with a particular focus on the evolutionary process of search and exploration strategies. By constructing an evolutionary game model and employing numerical simulation analysis, the study explores the impact of factors such as the proportion of shared resource information, shared costs, transmission efficiency, and the probability of strategy switching on robot strategy selection. The findings indicate that as the proportion of shared information decreases and as shared costs, successful transmission efficiency, and strategy-switching probability increase, the robot system gradually shifts from a search strategy to an exploration strategy. In environments with balanced information, the system tends to optimize resource utilization through the search strategy. However, under conditions of information asymmetry, the exploration strategy emerges as the dominant choice due to its potential for higher rewards. The exploration strategy enhances the robots' adaptability and task completion efficiency by actively acquiring new information and adjusting to environmental changes. This research highlights the critical role of information disparities in the strategy selection of group robots, offering theoretical support for decision-making in dynamic, information-asymmetric environments. Future research will explore more complex strategy selection models, integrating optimization algorithms such as reinforcement learning to enhance the efficiency and adaptability of strategy selection. Additionally, the applicability of the model in multi-robot collaborative tasks will be further validated to optimize the application of group robotic systems. From a strategy validation perspective, evaluating resource allocation efficiency through backward deduction of Nash equilibrium points will provide strong support for performance evaluation in multi-robot systems.

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