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Estimation of Reservoir Storage Capacity Using the Gould-Dincer Formula with the Aid of Possibility Theory

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Abstract: This paper presents a method for estimating reservoir storage capacity using the Gould–Dincer normal formula (G-DN), enhanced by the possibility theory. The G-DN equation is valuable for regional studies of reservoir reliability, particularly under climate change scenarios, using regional statistics. However, because the G-DN formula deals with measured data, it introduces a degree of uncertainty and fuzziness that traditional probability theory struggles to address. Possibility theory, an extension of fuzzy set theory, offers a suitable framework for managing this uncertainty and fuzziness. In this study, the G-DN formula is adapted to incorporate fuzzy logic, and the possibilistic nature of reservoir capacity is translated into a probabilistic framework using α-cuts from the possibility theory. These α-cuts approximate probability confidence intervals with high confidence. Applying the proposed methodology, in the present crisp case with the storage capacity D = 0.75, the value of the capacity C was found to be 1271×10^6 m^3, and that for D = 0.5 was 634.5×10^6 m^3. On the other hand, in the fuzzy case using the possibility theory, the value of the capacity for D = 0.75 is the internal $[315, 5679] \times 10^6$ m^3 and for D = 0.5 the value is interval $[158, 2839] \times 10^6$ m^3, with a probability of $\geq$95% and a risk level of α = 5% for both cases. The proposed approach could be used as a robust tool in the toolkit of engineers working on irrigation, drainage, and water resource projects, supporting informed and effective engineering decisions.

Keywords: reservoir capacity; probability; fuzzy logic; confidence intervals; fuzzy estimators; possibility theory

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1. Introduction

Determining the required reservoir storage capacity to meet demand with an acceptable level of reliability has been a longstanding challenge. Traditionally, hydraulic engineers have used Rippl's mass curve method [1] or the sequent peak algorithm [2] for designing water storage systems. Today, there are numerous techniques available for this purpose [3–8], with the Gould–Dincer method being one of the simplest.

The Gould–Dincer approach is a method used for analyzing reservoir capacity, yield, and reliability. Originally developed by Löf and Hardison in 1966 [9], it was later refined by Professor T. Dincer from the Middle East Technical University in Turkey. This modification, which assumed that reservoir inflows were normally distributed and serially uncorrelated, became known as the Dincer method [2]. In 1964, Gould (1964) [10] independently derived a similar reservoir storage–yield relationship but incorporated inflows that were Gamma distributed. This method has become known as the Gould Gamma method [11]. To adapt this method for skewed flows, Gould proposed a manual adjustment to transform normal flows into Gamma-distributed flows. Vogel and McMahon (1995) [12] suggested using the Wilson and Hilferty (1931) [13] transformation as a simpler alternative to the complex Gould procedure for handling skewed flows. They also derived an adjustment for storage to account for auto-correlation identical to that of Phatarfod (1986) [14] but was achieved through an entirely independent approach. An additional variation of the Gould–Dincer approach has emerged, accommodating lognormal inflows. To differentiate between the three variations of the Gould–Dincer approach, distinct labels are applied to each: Gould–Dincer Normal (G-DN), Gould–Dincer Gamma (G-DG) and Gould–Dincer Lognormal (G-DLN) [8,15].

Recently, storage–risk–yield (S-R-Y) relationships have been derived using data produced by various streamflow models [2]. After developing a preliminary design for a water resources project involving one or more reservoirs, it is essential to have an effective method for evaluating the relationship between reservoir capacity, yield, and reliability (S-Y-R). The reservoir storage capacity can only be estimated by using the mean flow and its validity in countries where hydrologic records are lacking or are limited. According to McMahon et al. (2007) [2], the Gould–Dincer (G-D) suite of equations is the only technique that can be applied universally using annual streamflows across the full range of global hydrology. It is important to note that while there are preliminary techniques for specific hydrologic conditions [16] or for estimating reservoir capacity for no-failure yield [17,18], these methods cannot simultaneously estimate capacity and steady-state reliability. The G-D suite of techniques (including normal, lognormal, and Gamma distributions) was specifically developed to estimate the storage-yield-reliability (S-Y-R) relationship for reservoirs. It is the only known method available in a straightforward formula that utilizes annual streamflow statistics to calculate the S-Y-R relationship for a single storage capacity, applicable across the spectrum of global annual streamflow patterns. The G-D technique is the only suitable candidate to examine the estimation capacity-steady state reliability relationships [2]. A comprehensive analysis of the Gould–Dincer suite, including a comparison with extended deficit analysis, behavior analysis, and the sequent peak algorithm, can be found in [2]. They conclude that the Gould–Dincer equation is particularly useful for regional studies of reservoir reliability under climate change scenarios, as it relies on regional statistics rather than time series data, which may not be readily available. The resulting model may also prove useful in other regional planning studies to evaluate the net benefits which result from the broad use of rainwater harvesting systems (RWH) to meet water supply requirements. For all the above reasons, the G-DN technique has been chosen in the current study, as it is the only known method available in the form of a simple formula.

In the technique mentioned above, we work with measured data, and the Gould–Dincer formula is grounded in probability theory, which introduces a degree of uncertainty. Uncertainty can vary from a slight lack of certainty to a complete absence of knowledge or conviction, and it arises from various sources. One source is randomness, such as the unpredictability of a coin toss. Vagueness, resulting from imprecise information, can also lead to uncertainty. Probability theory has long been used to measure and manage uncertainty stemming from randomness. Zadeh (1965) [19] introduced the fuzzy set theory to address uncertainty caused by vagueness. Although fuzzy set theory handles a different type of uncertainty than probability theory, it does not contradict the core principles of probability. Zadeh (1978) [20] also introduced possibility theory as an extension of fuzzy sets, suggesting that fuzzy set theory could assess the possibility of events rather than their probability. The mathematical framework of fuzzy set theory serves as a natural foundation for possibility theory, playing a role similar to that of evidence theory [21] in relation to probability theory. From this perspective, a fuzzy restriction can be understood as a possibility distribution, where the membership function acts as the possibility distribution function. A fuzzy variable is linked to a possibility distribution in a way analogous to how a random variable is tied to a probability distribution. In many cases, a variable can be associated with both a possibility and a probability distribution, with the connection between the two governed by the possibility/probability consistency principle.

The relationship between probability and possibility has been explored by many researchers. Most of these studies have focused on identifying principles that must be satisfied for transformations between the two, and they have developed equations that meet these criteria. Dubois and Prade made significant contributions to this area, first proposing the idea of linking fuzzy sets to nested confidence sets through a probability–possibility transformation [22–29]. According to Oussalah (2000) [30], probability theory and possibility theory (or fuzzy set theory in general) are not as different as they may seem based on their distinct languages or the philosophical debates surrounding them. In fact, probabilistic and possibilistic data can arise at different stages or even at the same stage of problem-solving. They are also connected in the context of imprecise probability, as introduced by Walley (1991) [31], where imprecision may be represented by intervals of real numbers, fuzzy numbers, or fuzzy intervals; by embedding fuzzy sets into random sets (as done by Goodman (1984) [32]); through non-additive probabilities; or through non-monotonic reasoning [33]. According to Nguyen (2005) [34], random elements in probability theory include random sets, which are equivalent to fuzzy sets. In other words, probability theory and possibility theory can be used independently or together, effectively enriching and complementing each other.

Transforming probabilistic data into possibilistic data is advantageous when limited information renders probability data unreliable. This transformation can also facilitate the benefits of possibility theory during the combination phase or simplify the solution process by utilizing possibility values instead of probability values. Conversely, converting possibilistic data into probabilistic data can be valuable in decision-making contexts where a precise outcome is desired. In these situations, decision-makers are typically more interested in understanding what is likely to happen in the future rather than simply what is possible in the future.

In this paper, we present a method for estimating reservoir storage capacity using the G-DN, enhanced by possibility theory. The approach assumes normally distributed and independent annual flows (mean μ and variance σ^2), and considers consecutive n-year inflows, accounting for the effect of auto-correlation on reservoir capacity. To adjust for the auto-correlation effect, the reservoir capacity can be modified by the factor $(1 + \rho)/(1 - \rho)$, where ρ is the lag-one serial correlation coefficient. This makes the proposed formula a function of three variables [μ, σ2, $(1 + \rho)/(1 - \rho)$], which are fuzzy, with their individual probability distributions known, but the overall probability distribution of the formula itself is unknown. To address this, we first conduct a fuzzy estimation of the mean, variance, and auto-correlation coefficient. Then, applying Nguyen's proposition [35] and Mylonas's conjecture [36], we derive a fuzzy estimator for reservoir capacity with confidence intervals. The probability of the α-cuts ($\alpha = 0.05$) is calculated to be at least 95%. This outcome allows the hydraulic engineers working on water resource projects (e.g., irrigation and drainage networks) to make informed decisions for rational and efficient engineering studies.

2. Materials and Methods

2.1. Crisp Model—Gould–Dincer's Normal Approach (G-DN)

Assuming normally distributed and independent annual inflows with mean μ and standard deviation σ, Gould–Dicer's normal approach for the reservoir capacity is [2]:

$$C = \frac{z_p^2}{4(1-\mathrm{D})} C_V^2 \mu \tag{1}$$

and the period for the reservoir to fully drain from an initially full state is:

$$\mathrm{CP} = \frac{z_p^2}{4(1-\mathrm{D})^2} C_V^2 \tag{2}$$

where

$$C_V = \frac{\sigma}{\mu} \tag{3}$$

In Equations (1)–(3), D is the constant draft as ratio of mean annual inflows, C_V is the coefficient of variation of the annual inflows to the reservoir, and p is the annual probability of failure, indicating that the reservoir's inflows will be adequate to meet the target draft with a reliability of $1 - p$.

$$z_p = \Phi^{-1}(1-p) \quad (4)$$

Equation (4) yields the standardized normal variate at p (Φ^{-1} is the inverse distribution function of the standard normal distribution).

To account for the auto-correlation effect on reservoir capacity, one can use a first-order autoregressive model and adjust the reservoir capacity computed from Equation (1) by $(1+\rho)/(1-\rho)$ as follows [12,14]:

$$C = \frac{z_p^2}{4(1-\mathrm{D})} C_V^2 \mu \frac{1+\rho}{1-\rho} \quad (5)$$

$$\mathrm{CP} = \frac{z_p^2}{4(1-\mathrm{D})^2} C_V^2 \frac{1+\rho}{1-\rho} \quad (6)$$

where ρ is the lag-one serial correlation coefficient defined as:

$$\rho = \frac{\frac{1}{n-1}\sum_{i=1}^{n-1} X_i X_{i+1} - \frac{1}{(n-1)^2}\sum_{i=1}^{n-1} X_i \sum_{i=1}^{n-1} X_{i+1}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} X_i^2 - \frac{1}{(n-1)^2}\left(\sum_{i=1}^{n-1} X_i\right)^2}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} X_{i+1}^2 - \frac{1}{(n-1)^2}\left(\sum_{i=1}^{n-1} X_{i+1}\right)^2}} \quad (7)$$

where $\mathrm{X_i}$ is the annual inflows.

2.2. Fuzzy Framework and Definitions

2.2.1. Fuzzy Theory

For readers who may not be familiar with fuzzy theory, we provide definitions covering key concepts in fuzzy theory and the basics of possibility theory.

Definition 1. *A fuzzy number is a fuzzy set* $\tilde{u} : R^1 \to [0,\ 1]$ *with the following properties:*

i. $\tilde{u}$ *is upper semi-continuous;*

ii. $\tilde{u}(x) = 0$*, outside of some interval [c, d];*

iii. *There are real numbers a and b,* $c \le a \le b \le d$ *such that* $\tilde{u}$*is increasing (non decreasing) on [c, a], decreasing (non-increasing) on [b, d], and* $\tilde{u}(x) = 1$ *for each* $x \in [a, b]$*;*

iv. $(\tilde{u}(\lambda x + (1+\lambda)x) \ge min\{\tilde{u}(\lambda x), \tilde{u}((1+\lambda)x)\}, \lambda \in [0,\ 1]$*, and* $\tilde{u}$ *is convex;*

iv. *This fuzzy number has a membership function, denoting the degree of set membership. The membership function of a fuzzy set* $\tilde{u}$ *is denoted by* $\mu_{\tilde{u}}(x)$ *or by* $\tilde{u}(x)$.

Definition 2. *Define* $[\tilde{u}]_\alpha$ *by the following:*

$$[\tilde{u}]_\alpha = \begin{cases} \{(x,\alpha) | \tilde{u}(x) \ge \alpha\}\ if\ 0 < \alpha \le 1, \\ ----- \\ [\tilde{u}]_0\ \ if\ \alpha = 0, \end{cases}$$

where $[\tilde{u}]_0$ *denotes the closure of the support of* $\tilde{u}$*. Then it is easily established that* $\tilde{u}$ *is a fuzzy number if and only if:*

i. $[\tilde{u}]_\alpha$ *is a closed and bounded interval for each* $\alpha | \alpha \in [0,\ 1]$*;*

ii. $[\tilde{u}]_{\alpha=1} \ne 0$.

The $[\tilde{u}]_\alpha$ *is a crisp set and is called α-cut or α-level set of* $\tilde{u}$.

Definition 3. *Let* $\widetilde{u} \in \mathcal{F}\ (R)$, *where* $\mathcal{F}\ (R)$ *is the fuzzy space of all nonempty compact convex subsets of a Banach space. The* α*-cuts of* $\widetilde{u}$ *are* $\left[\widetilde{u}\right]_\alpha = [u_\alpha^-, u_\alpha^+]$. *According to the representation theorem [37] and the theorem of [38], the membership function and the* α*-cut form of a fuzzy number* $\widetilde{u}$ *are equivalent and in particular the* α*-cuts* $\left[\widetilde{u}\right]_\alpha = [u_\alpha^-, u_\alpha^+]$ *uniquely represent* $\widetilde{u}$ *provided that the two functions are monotonic (*u_α^- *increasing,* u_α^+ *decreasing) and* $u_{\alpha=1}^- \le u_{\alpha=1}^+$.

The α-cuts arithmetic operators follow the same rules as those for classical interval numbers [39,40]. These operations maintain properties like associativity and commutativity. But distributivity may not always apply, which can result in the widening of intervals after distribution. This failure occurs because two identical interval numbers are treated as independent. To address this and avoid interval widening, alternative methods such as the vertex method [41] or the reduced transformation method [42,43] can be applied for practical purposes.

2.2.2. Possibility Theory

Below we provide critical definitions regarding the possibility theory that was implemented in the current work.

Definition 4. *Assume that* $\widetilde{F}$ *is a fuzzy subset of referential set* Ω, *representing the set of admissible and mutually exclusive values of a variable x. Now, consider* $\widetilde{A}$ *as another subset of* Ω; *one can assess the degree to which* $\widetilde{A}$ *intersects with* $\widetilde{F}$ *(representing the possibility of event* $\widetilde{A}$*) and the extent to which* $\widetilde{A}$ *encompasses* $\widetilde{F}$ *(representing the certainty of event* $\widetilde{A}$*). This can be expressed by introduction of possibility measure* Π *and necessity measure Nec:*

i. *Possibility of* $\widetilde{A}$: $\Pi(\widetilde{A}) = sup\{\alpha \left| \widetilde{A} \cap \widetilde{F}_\alpha \neq 0 \right\}$;

ii. *Necessity (certainty) of* $\widetilde{A}$: $Nec(\widetilde{A}) = 1 - sup\{\alpha \left| \overline{\widetilde{A}} \cap \widetilde{F}_\alpha \neq 0 \right\}, \overline{\widetilde{A}} = compl.\ \widetilde{A}$.

The above relation (ii) means that Nec $(\widetilde{A})$ *=1-*Π $(\overline{\widetilde{A}})$, i.e., *the certainty of* $\widetilde{A}$ *reflects the impossibility of its complement* $\overline{\widetilde{A}}$.

Definition 5. *Possibility measures are set functions similar to probability measures, but they rely on an axiom which only involves the operation "supremum." A possibility measure* Π *on a set X (e.g., the set of reals) is characterized by a possibility distribution* π*: X* $\rightarrow$ *[0, 1] and is defined by:*

$$\forall A \in X, \Pi(A) = sup\{\pi(x), x \in A\}$$

On finite sets this definition reduces to:

$$\forall A \in X, \Pi(A) = max\{\pi(x), x \in A\}$$

Definition 6. *The possibility distribution function* $\pi(\left[\widetilde{A}\right]_\alpha)$ *of* $\Pi(\left[\widetilde{A}\right]_\alpha)$ *is equal to membership function* $\mu_{\widetilde{A}}$ *of* $\widetilde{A}$ *[20]:*

$$\pi(\left[\widetilde{A}\right]_\alpha) \equiv \mu_{\widetilde{A}}.$$

Definition 7. *The two possibility distributions* π_x, π_x' *are consistent with the probability distribution* p_x . *The* π_x *distribution is more specific than* π_x' *if* $\pi_x < \pi_x'$. *A possibility distribution*

π_x consistent with the probability distribution p_x is called maximal specificity if it is more specific than each other possibility distribution:

$$\pi_x : \pi_x < \pi'_x, \forall x$$

Definition 8. *The principle of possibility/probability consistency (PPCP) establishes conditions under which probability and possibility distributions are considered consistent with each other. Initially the PPCP introduced by [20], the principle proposes a set of statements that define the relationship between possibility and probability, allowing for the transformation of possibilistic data into probabilistic data and vice versa. According to Dubois and Prade (2001) [26], this principle is based on the idea that possibility represents a less strict concept than probability. This leads to*

$$\forall A \text{ measurable}, Nec\left(\left[\tilde{A}\right]_\alpha\right) \leq P\left(\left[\tilde{A}\right]_\alpha\right) \leq \Pi\left(\left[\tilde{A}\right]_\alpha\right)$$

Definition 9. *For a variable Y with a known continuous probability distribution function p, the fuzzy number $\tilde{Y}$, which has a possibility measure $\Pi\left(\left[\tilde{Y}\right]_\alpha\right) = \mu_{\tilde{Y}}$, is the fuzzy estimator of Y. This fuzzy number satisfies the consistency principle and verifies $P\left(\left[\tilde{Y}\right]_\alpha\right) \geq Nec\left(\left[\tilde{Y}\right]_\alpha\right) = 1 - \alpha$, so that the probability of $\left[\tilde{Y}\right]_\alpha$ is greater than equal to $1-\alpha$. The α-cuts are the confidence intervals of P, and the confidence level is α.*

Definition 10. *For a fuzzy function $\tilde{F}(\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_N)$ with unknown probability distribution, while $\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_N$ are fuzzy numbers with known probability distribution, the α-cut $\left[\tilde{F}\right]_\alpha = [F_\alpha^-, F_\alpha^+]$ following Nguyen (1978) [35] is $\left[\tilde{F}\right]_\alpha = \left[\tilde{F}([X_1]_\alpha, [X_2]_\alpha, \ldots\ldots, [X_N]_\alpha)\right]_\alpha$.*

Definition 11. *A The probability of α-cuts of $\tilde{F}(\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_N)$ is greater than or equal to $1 - \alpha$ and the α-cuts are the intervals of confidence of $\tilde{F}(\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_N)$ [36].*

2.2.3. Fuzzy Model of the Gould–Dincer Normal (G-DN) Approach

The Gould–Dincer normal approach model in fuzzy case is written as follows:

(a) Case ρ included

$$\tilde{C} = \frac{z_p^2}{4(1-D)}\tilde{C}_V^2\tilde{\mu}\tilde{G}, \;\; C_V = \frac{\sigma}{\mu}, \;\; G = \frac{1+\rho}{1-\rho} \tag{8}$$

$$\tilde{CP} = \frac{z_p^2}{4(1-D)^2}\tilde{C}_V^2\tilde{G}, \;\; C_V = \frac{\sigma}{\mu}, \;\; G = \frac{1+\rho}{1-\rho} \tag{9}$$

For practical reasons, the following is posed:

$$f = \frac{z_p^2}{4(1-D)}, \;\; \tilde{X}_1 = \tilde{\sigma}^2, \;\; \tilde{X}_2 = \tilde{G}, \;\; \tilde{X}_3 = \tilde{\mu} \tag{10}$$

Equations (8) and (9) take the following form:

$$\tilde{C} = f\frac{\tilde{X}_1\tilde{X}_2}{\tilde{X}_3} \tag{11}$$

$$\widetilde{CP} = f_1 \frac{\widetilde{X}_1 \widetilde{X}_2}{\widetilde{X}_3^2} \tag{12}$$

According to Nguyen (1978) [35] (Definition 9), if

$$x_1 \in \widetilde{X}_1,\ \ x_2 \in \widetilde{X}_2,\ \ x_3 \in \widetilde{X}_3,\ \ \widetilde{C} : \widetilde{X}_1 \times \widetilde{X}_2 \times \widetilde{X}_3 \to \widetilde{Z} \tag{13}$$

thus, a necessary and sufficient condition for achieving the following equalities,

$$\left[\widetilde{C}\right]_\alpha = f \frac{\left[\widetilde{X}_1\right]_\alpha \cdot \left[\widetilde{X}_2\right]_\alpha}{[X_3]_\alpha} \tag{14}$$

$$\left[\widetilde{CP}\right]_\alpha = f_1 \frac{\left[\widetilde{X}_1\right]_\alpha \cdot \left[\widetilde{X}_2\right]_\alpha}{\left[\widetilde{X}_3^2\right]_\alpha} \tag{15}$$

ensuring the function's continuity, the following relation is obtained:

$$\forall z \in \widetilde{Z}, sup_{(x_1,x_2,x_3) \in \widetilde{C}^{-1}(z)} \left\{ \mu_{\widetilde{X}_1}(x_1) \wedge \mu_{\widetilde{X}_2}(x_2) \wedge \mu_{\widetilde{X}_3}(x_3) \right\} \tag{16}$$

(b) Case ρ is avoided

$$\widetilde{C} = \frac{z_p^2}{4(1-\mathrm{D})} \widetilde{C}_V^2 \widetilde{\mu} = \frac{z_p^2}{4(1-\mathrm{D})} \frac{\widetilde{\sigma}^2}{\widetilde{\mu}} \tag{17}$$

$$\widetilde{CP} = \frac{z_p^2}{4(1-\mathrm{D})^2} \frac{\widetilde{\sigma}^2}{\widetilde{\mu}^2} \tag{18}$$

For practical reasons, the following is posed:

$$f = \frac{z_p^2}{4(1-\mathrm{D})},\ \ \widetilde{X}_1 = \widetilde{\sigma}^2,\ \ \widetilde{X}_2 = \widetilde{\mu} \tag{19}$$

Equations (8) and (9) take the following form:

$$\widetilde{C} = f \frac{\widetilde{X}_1}{\widetilde{X}_2} \tag{20}$$

$$\widetilde{CP} = f_1 \frac{\widetilde{X}_1}{\widetilde{X}_2^2} \tag{21}$$

According to Nguyen (1978) (Definition 9) [35] if

$$x_1 \in \widetilde{X}_1,\ \ x_2 \in \widetilde{X}_2,\ \ \widetilde{C} : \widetilde{X}_1 \times \widetilde{X}_{23} \to \widetilde{Z} \tag{22}$$

thus, a necessary and sufficient condition for achieving the following equalities,

$$\left[\widetilde{C}\right]_\alpha = f \frac{\left[\widetilde{X}_1\right]_\alpha}{[X_2]_\alpha} \tag{23}$$

$$[\widetilde{CP}]_{\alpha} = f_1 \frac{\left[\widetilde{X}_1\right]_{\alpha}}{\left[\widetilde{X}_2^{2}\right]_{\alpha}} \quad (24)$$

ensuring the function's continuity, the following relation is obtained:

$$\forall z \in \widetilde{Z}, sup_{(x_1,x_2)\in \widetilde{C}^{-1}(z)} \left\{ \mu_{\widetilde{X}_1}(x_1) \wedge \mu_{\widetilde{X}_2}(x_2) \right\} \quad (25)$$

The arithmetic operation for interval numbers in Equations (14) and (15) is avoided, for practical reasons and in order to prevent widening; for these reasons, the reduced transformation method [42–44] is used.

2.2.4. Transformation Method

Decomposition of fuzzy numbers

Two forms of the transformation method are available, the general and the reduced forms [42–44]. The reduced form is applied when dealing with functions that have n independent parameters, which are assumed to be uncertain. Additionally, the function must be monotonic with respect to each variable, without any local extrema. For more complex, non-monotonic problems, the general transformation method is used. In this study, the reduced form was employed for practical reasons, as the functions in Equations (11) and (12) are monotonic, nonlinear, and involve three fuzzy variables. Each fuzzy number is divided into m intervals, j = 1, 2, . . ., m, as defined by the α-cuts at the a-levels μ_j.

The fuzzy numbers of Equations (11) and (12) (n = 3 in this case) can be broken down into a set of m intervals, $j = 1, \ldots, m$, of the form (decomposition principle) [20]:

$$X_i^{(j)} = \left\{ X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)} \right\} \quad (26)$$

with $X_i^{(j)} = \left[a_i^{(j)}, b_i^{(j)}\right]$, $a_i^{(j)} \le b_i^{(j)}$, $j = 1,\ 2,\ \ldots,\ m$.

Notation: All the fuzzy parameters $X_i^{(1)}, X_i^{(2)}, \ldots, X_i^{(m)}$ can be regarded as the coordinates of points located on the n-dimensional hypersurface $X_i^{(1)} \times X_i^{(2)}, \ldots, \times X_i^{(m)}$, nested according to their level of membership. In the case of the reduced transformation form, only the 2^n vertex points of the n-dimensional cuboids are considered for the evaluation of the problem. Figure 1 illustrates the case n = 3, which is a cube with 2^3 vertices and α = 0, 1/3, 2/3, 1. The cuboid for the membership level μ = 1 is degenerated to one single point.

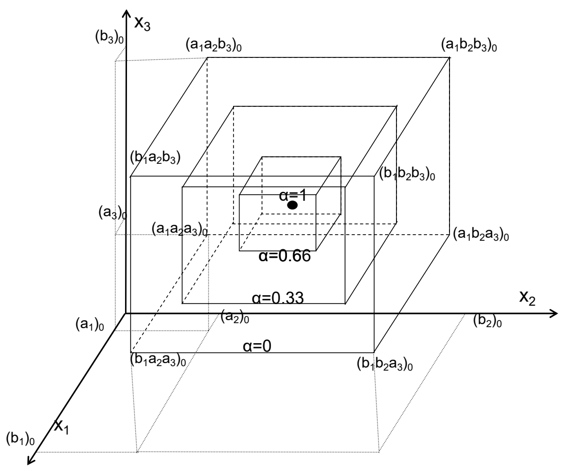


Figure 1. Geometric interpretation of the transformation scheme for n = 3.

Transformation of the intervals

The intervals $X_i^{(j)}, i = 1, 2, \ldots n$ could be transformed into arrays (based on the reduced transformation method) of the following form:

$$\hat{X}_i^{(j)} = \overbrace{((a_i^{(j)}, b_i^{(j)}), (a_i^{(j)}, b_i^{(j)}), \ldots, (a_i^{(j)}, b_i^{(j)})}^{2^{i-1} couples} \tag{27}$$

$$a_i^{(j)} = \underbrace{(\alpha_i^{(j)}, \ldots, \alpha_i^{(j)})}_{2^{n-i} elements}, b_i^{(j)} = \underbrace{(\beta_i^{(j)}, \ldots, \beta_i^{(j)})}_{2^{n-i} elements} \tag{28}$$

(a) In the present case ($\widetilde{C}$, $\widetilde{CP}$, n = 3 variables, ρ included):

$$\begin{bmatrix} \hat{X}_1^j \\ \hat{X}_2^j \\ \hat{X}_3^j \end{bmatrix} = \begin{bmatrix} \alpha_1^{(j)}, \alpha_1^{(j)}, \alpha_1^{(j)}, \alpha_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)} \\ \alpha_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)}, \beta_2^{(j)}, \alpha_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)}, \beta_2^{(j)} \\ \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)} \end{bmatrix} = [3 \times 8] \tag{29}$$

The above matrix can now be written as follows:

$$\begin{bmatrix} \hat{X}_1^j \\ \hat{X}_2^j \\ \hat{X}_3^j \end{bmatrix} = \begin{bmatrix} \alpha_{1,1}^{(j)}, \alpha_{1,2}^{(j)}, \alpha_{1,3}^{(j)}, \alpha_{1,4}^{(j)}, \alpha_{1,5}^{(j)}, \alpha_{1,6}^{(j)}, \alpha_{1,7}^{(j)}, \alpha_{1,8}^{(j)} \\ \alpha_{2,1}^{(j)}, \alpha_{2,2}^{(j)}, \alpha_{2,3}^{(j)}, \alpha_{2,4}^{(j)}, \alpha_{2,5}^{(j)}, \alpha_{2,6}^{(j)}, \alpha_{2,7}^{(j)}, \alpha_{2,8}^{(j)} \\ \alpha_{3,1}^{(j)}, \alpha_{3,2}^{(j)}, \alpha_{3,3}^{(j)}, \alpha_{3,4}^{(j)}, \alpha_{3,5}^{(j)}, \alpha_{3,6}^{(j)}, \alpha_{3,7}^{(j)}, \alpha_{3,8}^{(j)} \end{bmatrix} \equiv \begin{bmatrix} \alpha_1^{(j)}, \alpha_1^{(j)}, \alpha_1^{(j)}, \alpha_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)} \\ \alpha_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)}, \beta_2^{(j)}, \alpha_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)}, \beta_2^{(j)} \\ \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)}, \alpha_3^{(j)}, \beta_3^{(j)} \end{bmatrix} \tag{30}$$

According to transformation method for each column (i):

$$w_i^{(j)} = f\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}},\ w_i^{(j)} = f\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}},\ \text{i} = 1,\ 2,\ \ldots,\ 8,\ (\text{for } \big[\widetilde{C}]_\alpha) \tag{31}$$

$$w_i^{(j)} = f_1\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{(\alpha_{3,i}^{(j)})^2},\ \text{i} = 1,\ 2,\ \ldots,\ 8,\ (\text{for } [\widetilde{CP}]_\alpha) \tag{32}$$

Thus, for each α-cut = i we have:

$$\big[\widetilde{C}]_{\alpha=i} = \left[\left(min\left\{f\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}}\right\}\right)^-, \left(max\left\{f\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}}\right\}\right)^+\right],\ j = 1, 2, \ldots, m. \tag{33}$$

$$\big[\widetilde{CP}]_{\alpha=i} = \left[\left(min\left\{f_1\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}}\right\}\right)^-, \left(max\left\{f_1\frac{\alpha_{1,i}^{(j)}\alpha_{2,i}^{(j)}}{\alpha_{3,i}^{(j)}}\right\}\right)^+\right],\ j = 1, 2, \ldots, m. \tag{34}$$

(b) $\widetilde{C}$, $\widetilde{CP}$, n = 2 variables, ρ is avoided:

$$\begin{bmatrix} \hat{X}_1^j \\ \hat{X}_2^j \end{bmatrix} = \begin{bmatrix} \alpha_1^{(j)}, \alpha_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)} \\ \alpha_2^{(j)}, \beta_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)} \end{bmatrix} = [2 \times 4]$$

$$\begin{bmatrix} \hat{X}_1^j \\ \hat{X}_2^j \end{bmatrix} = \begin{bmatrix} \alpha_{1,1}^{(j)}, \alpha_{1,2}^{(j)}, \alpha_{1,3}^{(j)}, \alpha_{1,4}^{(j)} \\ \alpha_{2,1}^{(j)}, \alpha_{2,2}^{(j)}, \alpha_{2,3}^{(j)}, \alpha_{2,4}^{(j)} \end{bmatrix} \equiv \begin{bmatrix} \alpha_1^{(j)}, \alpha_1^{(j)}, \beta_1^{(j)}, \beta_1^{(j)} \\ \alpha_2^{(j)}, \beta_2^{(j)}, \alpha_2^{(j)}, \beta_2^{(j)} \end{bmatrix} \tag{35}$$

$$w_i^{(j)} = f\frac{\alpha_{1,i}^{(j)}}{\alpha_{2,i}^{(j)}},\ \text{i} = 1,\ 2,\ \ldots,\ 4,\ (\text{for } \big[\widetilde{C}]_\alpha) \tag{36}$$

$$w_i^{(j)} = f_1 \frac{\alpha_{1,i}^{(j)}}{(\alpha_{2,i}^{(j)})^2},\ \text{i} = 1,\ 2,\ \ldots,\ 4,\ (\text{for } \left[\tilde{\text{CP}}\right]_\alpha) \tag{37}$$

Thus, for each α-cut = i we have:

$$\left[\tilde{\text{C}}\right]_{\alpha=i} = \left[\left(min\left\{f\frac{\alpha_{1,i}^{(j)}}{\alpha_{2,i}^{(j)}}\right\}\right)^-, \left(max\left\{f\frac{\alpha_{1,i}^{(j)}}{\alpha_{2,i}^{(j)}}\right\}\right)^+\right], j = 1, 2, \ldots, m. \tag{38}$$

$$\left[\tilde{CP}\right]_{\alpha=i} = \left[\left(min\left\{f_1\frac{\alpha_{1,i}^{(j)}}{\alpha_{2,i}^{(j)}}\right\}\right)^-, \left(max\left\{f_1\frac{\alpha_{1,i}^{(j)}}{\alpha_{2,i}^{(j)}}\right\}\right)^+\right], j = 1, 2, \ldots, m. \tag{39}$$

The final solution is:

$$P(\left[\tilde{\text{C}}\right]_\alpha) \geq Nec(\left[\tilde{\text{C}}\right]_\alpha) = 1 - \alpha \tag{40}$$

$$P(\left[\tilde{CP}\right]_\alpha) \geq Nec(\left[\tilde{CP}\right]_\alpha) = 1 - \alpha \tag{41}$$

According to possibility theory, $\left[\tilde{\text{C}}\right]_\alpha$, $\left[\tilde{CP}\right]_\alpha$ are called confidence intervals and the confidence level, i.e., the probability of these intervals $P(\left[\tilde{\text{C}}\right]_\alpha)$, $P(\left[\tilde{CP}\right]_\alpha)$, is greater than or equal to $1 - \alpha$. The risk level is α, i.e., the probability that the real value falls outside the interval. Normally 95%, 99% values of $(1 - \alpha)$ often used in the measurement area [27].

3. Results and Discussion

The storage capacity and the length of the critical drawdown period of Mitta Mitta River at Tallandoon are estimated using the inflow data for a period of 34 years [45].

3.1. Crisp Estimation of C and CP

For the crisp estimations of the storage capacity required and the length of the critical drawdown period, the values of μ and σ are given for a period of n = 34 years as follows:

$$\overline{x} = 1274 \times 10^6 \text{ m}^3 \text{ and } \sigma = 731 \times 10^6 \text{ m}^3$$

Using the above values, the estimation of the coefficient of variation is

$$C_V = \frac{\sigma}{\overline{x}} = \frac{731 \times 10^6}{1274 \times 10^6} = 0.57$$

Considering now D = 0.75 and ρ = 5%, for which $z_\rho = \Phi^{-1}(1 - 0.05) = 1.65$, we obtain from Equations (1) and (2)

$$C = \frac{z_p^2}{4(1-\text{D})} C_V^2 \overline{x} = 1127 \times 10^6 \text{m}^3$$

and

$$\text{CP} = \frac{z_p^2}{4(1-\text{D})^2} C_V^2 = 3.53 \text{ years}$$

For D = 0.5, we have

$$\text{C} = 563.5 \times 10^6 \text{ m}^3$$

and

$$\text{CP} = 0.884 \text{ years}$$

The annual serial correlation of these annual flows is found to be ρ = 0.06, so the adjustment factor is

$$\frac{1+\rho}{1-\rho} = 1.1276$$

Therefore, the adjusted crisp estimation of the storage capacity and the length of the critical drawdown period for D = 0.75 is

$$C = 1.13 \times 1127 \times 10^6 \approx 1271 \times 10^6 \text{ m}^3$$

$$\text{CP} = 1.13 \times 3.53 \approx 3.99 \text{ years}$$

and for D = 0.50

$$C \approx 635.51 \times 10^6 \text{m}^3$$

$$\text{CP} \approx 0.997 \text{ years.}$$

3.2. Fuzzy Estimation of C and CP

The fuzzy forms of Equations (1) and (2) are Equations (8) and (9):

$$\tilde{C} = \frac{z_p^2}{4(1-\text{D})} \frac{\tilde{\sigma}^2}{\tilde{\mu}} \tilde{G}, \quad G = \frac{1+\rho}{1-\rho}, \ \widetilde{\text{CP}} = \frac{z_p^2}{4(1-\text{D})^2} \frac{\tilde{\sigma}^2}{\tilde{\mu}^2} \tilde{G}$$

We cannot infer confidence intervals of C and CP, since we cannot derive probability density function of the statistics $\frac{\sigma^2}{\overline{x}} \frac{1+\rho}{1-\rho}$ and $\frac{\sigma^2}{\overline{x}^2} \frac{1+\rho}{1-\rho}$. But it is possible to use Nguyen proposition [35] and find the following form [Equations (14) and (15)] for α-cuts:

$$\left[\tilde{C}\right]_\alpha = f \frac{\left[\tilde{\sigma}^2\right]_\alpha}{\left[\tilde{\mu}\right]_\alpha} \left[\tilde{G}\right]_\alpha, \ G = \frac{1+\rho}{1-\rho}, \ \left[\widetilde{\text{CP}}\right]_\alpha = f_1 \frac{\left[\tilde{\sigma}^2\right]_\beta}{\left[\tilde{\mu}^2\right]_\alpha} \left[\tilde{G}\right]\alpha \tag{42}$$

3.2.1. Estimation of the Mean μ of a Random Variable from a Large Sample $(\tilde{X}_1)$

If the random variable X follows any distribution, then the (1 − β)% confidence interval of the mean μ of X derived from a random sample of observations of X of large size n (n > 30) with sample mean and variance $\overline{x}$ and s^2 is

$$\left[\overline{x} - z_{\frac{\beta}{2}} \frac{s}{\sqrt{n}}, \overline{x} + z_{\frac{\beta}{2}} \frac{s}{\sqrt{n}}\right] \tag{43}$$

Notation: In statistics, the letter α is used instead of β. But here the letter β is used, because the letter α is used for α-cuts.

Therefore, the α-cuts of the membership function of the fuzzy estimator $\tilde{\mu}$ for the mean μ of X are

$$\mu_{\tilde{\mu}}[\alpha] = \left[\overline{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right], 0.001 < \alpha \le 1 \tag{44}$$

where

$$z_{\frac{\alpha}{2}} = \Phi^{-1}(1 - \frac{\alpha}{2}) \tag{45}$$

3.2.2. Estimation of the Serial Correlation Coefficient $(\widetilde{X}_2)$

The point estimator of the serial correlation coefficient ρ of a sequence of n measurements xi of the random variable X defined in Equation (7) is

$$r = \frac{\frac{1}{n-1}\sum_{i=1}^{n-1} x_i x_{i+1} - \frac{1}{n-1}\sum_{i=1}^{n-1} x_i \frac{1}{n-1}\sum_{i=1}^{n-1} x_{i+1}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} x_i^2 - \frac{1}{(n-1)^2}(\sum_{i=1}^{n-1} x_i)^2}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1} x_{i+1}^2 - \frac{1}{(n-1)^2}(\sum_{i=1}^{n-1} x_{i+1})^2}} \tag{46}$$

As proved in Fisher (1915) [46], the transformed random variable

$$Z_r = \frac{1}{2} ln \frac{1+r}{1-r} \tag{47}$$

follows normal distribution with mean

$$E(Z_r) = \frac{1}{2} ln \frac{1+\rho}{1-\rho} \tag{48}$$

where ρ is the serial correlation coefficient and standard deviation

$$\sigma_r = \sqrt{\frac{1}{n-3}} \tag{49}$$

Therefore,

$$\begin{aligned} & Prob\left\{-z_{1-\frac{\beta}{2}} \leq \frac{Z_r - E(Z_r)}{\sigma_r} \leq z_{1-\frac{\beta}{2}}\right\} = 1-\beta \\ \Leftrightarrow\ & Prob\left\{Z_r - z_{1-\frac{\beta}{2}}\sigma_r \leq E(Z_r) \leq Z_r + z_{1-\frac{\beta}{2}}\sigma_r\right\} = 1-\beta \\ \Leftrightarrow\ & Prob\left\{Z_r - z_{1-\frac{\beta}{2}}\sigma_r \leq \frac{1}{2} ln\frac{1+\rho}{1-\rho} \leq Z_r + z_{1-\frac{\beta}{2}}\sigma_r\right\} = 1-\beta \\ \Leftrightarrow\ & Prob\left\{e^{2(Z_r - z_{1-\frac{\beta}{2}}\sigma_r)} \leq \frac{1+\rho}{1-\rho} \leq e^{2(Z_r + z_{1-\frac{\beta}{2}}\sigma_r)}\right\} = 1-\beta \end{aligned} \tag{50}$$

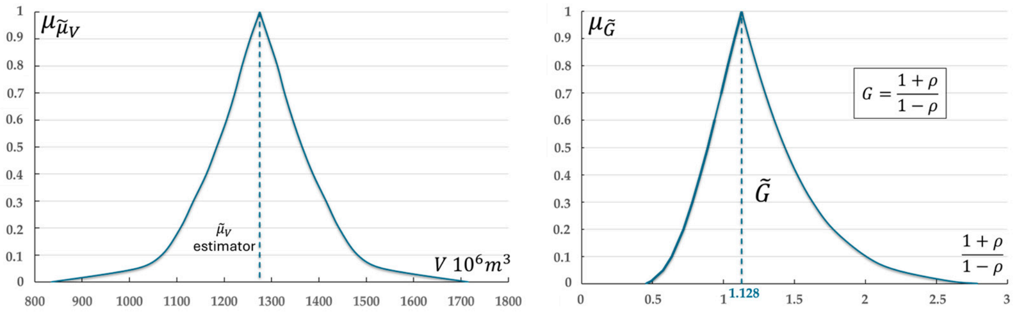
Therefore, the $(1-\beta)$% confidence interval of $\frac{1+\rho}{1-\rho}$ derived from a sample of n observations is

$$\left[e^{2(Z_r - z_{1-\frac{\beta}{2}}\sigma_r)} \leq \frac{1+\rho}{1-\rho} \leq e^{2(Z_r + z_{1-\frac{\beta}{2}}\sigma_r)}\right] \tag{51}$$

where z_r is the sample value of Z_r. So, the α-cuts of the possibility distribution associated with the fuzzy function $\widetilde{G}$ of $G = (1+\rho)/(1-\rho)$ are

$$[\widetilde{G}]_\alpha = \left[e^{2(Z_r - z_{1-\frac{\alpha}{2}}\sigma_r)} \leq \frac{1+\rho}{1-\rho} \leq e^{2(Z_r + z_{1-\frac{\alpha}{2}}\sigma_r)}\right], 0.001 \leq \alpha \leq 1 \tag{52}$$

Figure 2 illustrates the fuzzy estimators or possibility distributions of $\widetilde{\mu}, \widetilde{G}$ and $G = \frac{1+\rho}{1-\rho}$ for which the probability distribution functions are known and for that reason the corresponding possibility distribution functions are similar [47].



(**a**) (**b**)

Figure 2. The possibility distribution of the fuzzy estimator of (**a**) the mean μ of the annual inflows V and (**b**) $\tilde{G}$ of the annual inflows X.

3.2.3. Estimation of the Variance σ^2 of a Normal Variable $(\tilde{X}_3)$

If the random variable X is normally distributed, the $(1-\beta)$% confidence interval for the variance σ^2 of X can be calculated using a random sample size n and a sample variance s^2, as specified in [47]:

$$\left[\frac{(n-1)s^2}{\chi^2_{R,\frac{\beta}{2};n-1}}, \frac{(n-1)s^2}{\chi^2_{L,\frac{\beta}{2};n-1}}\right] \tag{53}$$

where χ^2_{n-1} is the value of the chi-squared distribution with k = n − 1 degrees of freedom and F^{-1}_{n-1} is the inverse distribution function of the χ^2_{n-1} distribution:

$$\begin{aligned} \chi^2_{L,\beta;n-1} &= \chi^2_{1-\beta;n-1} = F^{-1}_{n-1}(\beta), \\ \chi^2_{R,\beta;n-1} &= \chi^2_{\beta;n-1} = F^{-1}_{n-1}(1-\beta). \end{aligned} \tag{54}$$

Therefore, the α-cuts of the possibility distribution for the fuzzy estimator of the variance σ² are as follows:

$$\left[\tilde{\sigma}^2\right]_\alpha = \left[\frac{(n-1)s^2}{\chi^2_{R,\frac{\alpha}{2};n-1}}, \frac{(n-1)s^2}{\chi^2_{L,\frac{\alpha}{2};n-1}}\right] \tag{55}$$

Figure 3 illustrates the fuzzy estimators or possibility distributions of $\tilde{\sigma}^2$ for which the probability distribution functions are known and for that reason the corresponding possibility distribution functions are similar.

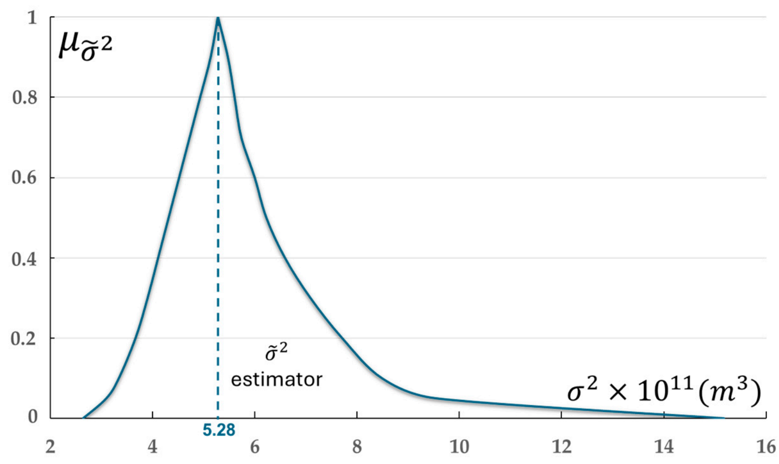


Figure 3. The possibility distribution of the fuzzy estimator of the variance σ^2 of the annual inflows X.

3.3. Estimation of the α-Cuts of $\widetilde{C}$ and $\widetilde{CP}$

After having estimated now the estimators $\widetilde{X}_1$, $\widetilde{X}_2$, $\widetilde{X}_3$, it is easy to find the α-cuts of $\widetilde{C}$ and $\widetilde{CP}$ by application of Equations (24) and (25), in which $f = 2.7225$, $f_1 = 10.89$ for D = 0.75, and $f = 1.36125$, $f_1 = 2.7225$ for D = 0.5. The results are shown in Figures 4–7.

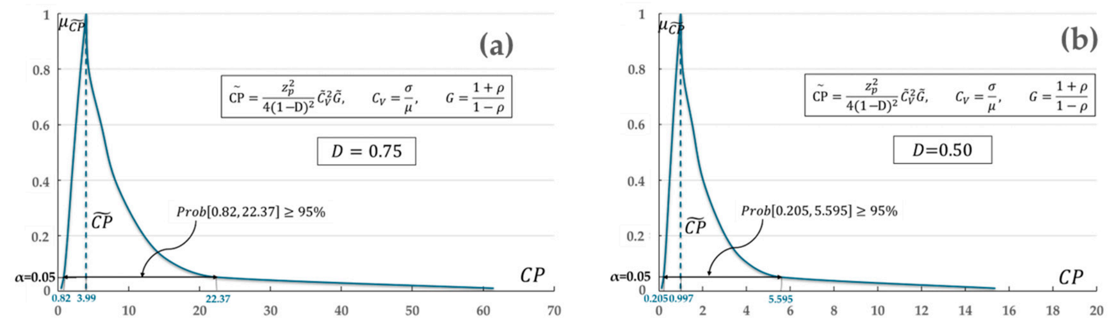


Figure 4. Fuzzy estimator of the length of the critical drawdown period for (**a**) D = 0.75 and (**b**) D = 0.50.

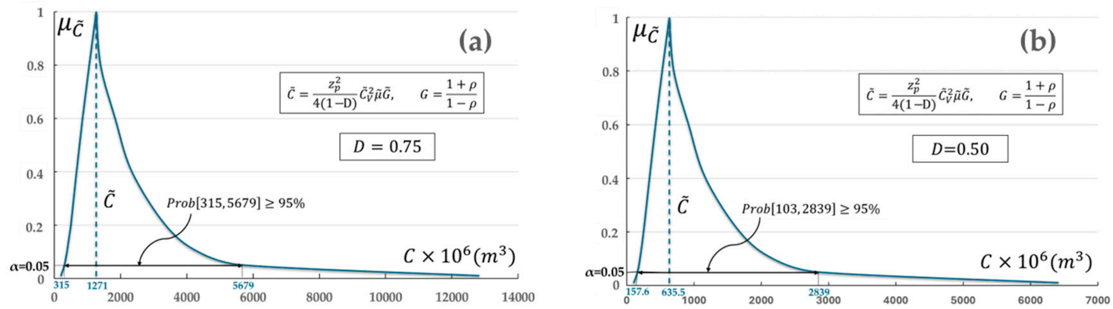


Figure 5. Fuzzy estimator of the storage capacity required for (**a**) D = 0.75(z_p = 1.65) and (**b**) D = 0.5(z_p = 1.65).

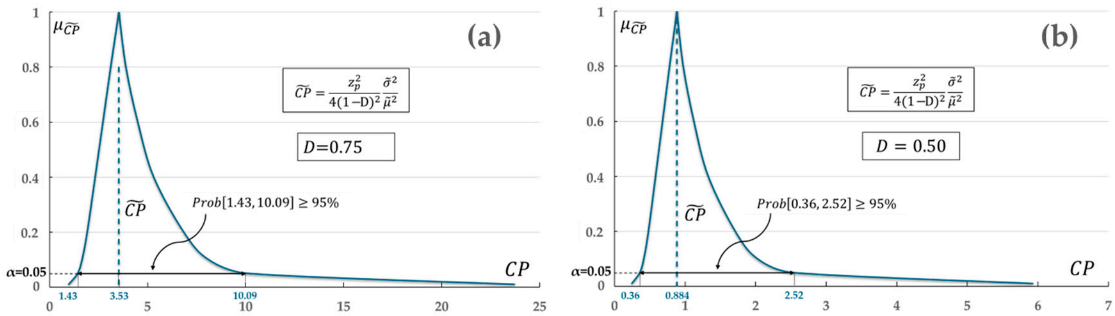


Figure 6. Fuzzy estimator of the length of the critical drawdown period for (**a**) D = 0.75 and (**b**) D = 0.50.

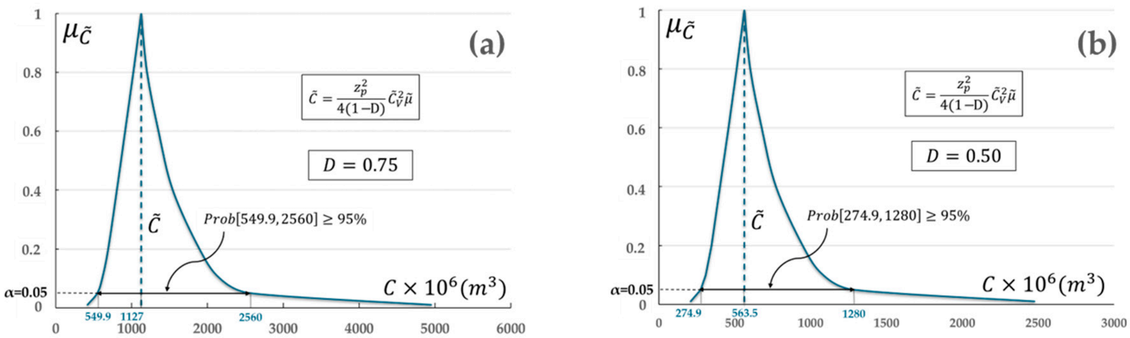


Figure 7. Fuzzy estimator of the storage capacity required for (**a**) D = 0.75 and (**b**) D = 0.50.

Specifically, Figures 4 and 5 illustrate the fuzzy estimators of reservoir capacity $\widetilde{C}$ and the critical drawdown period $\widetilde{CP}$ adjusting $(1+\rho)/(1-\rho)$, with ρ being the lag-one serial correlation coefficient, for a constant draft D equal to 0.75 and 0.5.

Figures 6 and 7 illustrate also the fuzzy estimators of reservoir capacity $\widetilde{C}$ and the critical drawdown period $\widetilde{CP}$ without adjusting the ratio $(1+\rho)/(1-\rho)$, but with the same constant draft D equal to 0.75 and 0.50.

In all figures of fuzzy estimators or possibility distributions, the nodal values of $\pi\,(x^m)$ coincide with the crisp values; e.g., crisp estimation for C in case of D = 0.75 was found to be 1271×10^6 m^3. The nodal value x^m in Figure 5a illustrating the fuzzy estimator of the storage capacity (D = 0.75) has exactly the same value.

Moreover, and in order to draw reliable conclusions, it was considered necessary to calculate the confidence intervals for the reservoir capacity $\widetilde{C}$ for a constant draft D equal to 0.75 and 0.5 and for the α-cuts 0.01, 0.05, 1, 2, 4, 6, 8, and 1 (Table 1).

Table 1. Confidence intervals for the reservoir capacity $\widetilde{C}$ for a constant draft D equal to 0.75 and 0.5 and for the α-cuts 0.01, 0.05, 1, 2, 4, 6, 8, and 1, $(1+\rho)/(1-\rho)$ adjusting.

α-cut	**$P=1-a$**	**$C \times 10^6$ m^3**			
		***D*=0.75**		***D*=0.50**	
		C^-	**C^+**	**C^-**	**C^+**
0.01	0.99	206	12,821	103	6410
0.05	0.95	315	5679	158	2839
0.1	0.9	389	4341	195	2171
0.2	0.8	497	3360	248	1680
0.4	0.6	675	2346	337	1173
0.6	0.4	857	1842	428	921
0.8	0.2	1055	1400	527	700
1	0	1271	1271	635	635

These α-cuts have a risk level α; i.e., the likelihood of the real value falling outside the interval, e.g., choosing $\alpha = 0.95$ for D = 0.75, one has a probability greater than or equal to be in the interval [315, 5679] and a risk of 5% to be outside this interval.

For convenience, let us take as an example the case of $\alpha = 5\%$ based on Table 1. For this case, the corresponding reservoir storage capacity resulted in 315×10^6 m^3 and by extension to a low construction cost of the dam, with a high probability to be constructed (95%) and with a risk of 5% to not be constructed. On the other hand, if we considered the case of the reservoir storage capacity of 497×10^6 m^3, then the construction cost of the dam will be increased with a lower probability to be constructed equal to 80% and with a risk of 20% to not be constructed.

In addition, it is crucial to highlight that according to Dubois and Prade (1990) [29], there is a link between fuzzy sets and random sets using the concept of the α-cut, and the random sets can be arranged into a nested family of sets with inclusion. From the engineering perspective, the extreme values of the confidence intervals are considered the most important, while the left boundary seems to include the most valuable information because it can provide significantly lower construction costs. In contrast, the right boundary does not seem to provide the same level of information as the left. For a better explanation, let us assume that a hydraulic engineer at the design phase chooses 5679×10^6 m^3 as a construction value, which corresponds to a high probability of construction (95%) with 5% risk. However, in this case the construction cost will be very expensive, while the 5% risk concerns only the water and not the construction cost of the dam. In the end, if the risk will be satisfied, then the water volumes will exceed the design ones, with disastrous consequences for the dam. Thus, it is reasonable to conclude that although the risk seems small regarding the design water volumes, the financial costs are very big and the construction health of the dam cannot be guaranteed.

Summarizing the results, in the present crisp case with D = 0.75, the value of the capacity C was found to be 1271 $\times 10^6$ m^3, and for D = 0.5, it was 634.5 $\times 10^6$ m^3. In the fuzzy case using the possibility theory, the value of the capacity for D = 0.75 is the internal $[315, 5679] \times 10^6$ m^3 with a probability of $\geq$95% and a risk level α = 5%, and for D = 0.5, the value of the capacity C is interval $[158, 2839] \times 10^6$ m^3 with a probability of $\geq$95% and a risk level α = 5%.

From all the above, we could state that the general problem of the design and construction of a hydraulic project such as a dam, is multicriteria while except the storage capacity depends also on other crucial factors like the construction costs, environmental issues, soil characteristics, policies, etc.

In general, the proposed approach successfully captures the complex interplay between reservoir capacity C or the period for the reservoir to empty CP and the mean annual inflows μ, the standard deviation of annual inflows σ, and the lag-one serial correlation coefficient, allowing for reliable predictions of reservoir capacity and period for the reservoir to empty. The capacity to quantify uncertainties and validate predictions significantly boosts the model's reliability in informing decision-making processes concerning water resource management. These insights are particularly relevant for managing water resources in semi-arid regions, offering accurate estimations of reservoir capacity. The validated model acts as a crucial decision support tool, facilitating the formulation of effective water management strategies.

4. Conclusions

This work underscores the advantages of integrating fuzzy logic into hydrological modeling. By offering a framework for calculating fuzzy intervals, it provides a more comprehensive tool for managing uncertainty in reservoir storage capacity and discharge.

The main innovation of this work is that it handles the fuzzy logic theory in such a way that allows us to estimate the fuzzy intervals for both reservoir capacity (C) and time required for a reservoir to empty (CP) by leveraging the possibility distributions of fuzzy estimators for the mean and variance, coupled with interval arithmetic. The proposed methodology is particularly valuable in scenarios where weak sources of information make probability-based data unrealistic or insufficient. Moreover, it reduces computational complexity in certain cases by using possibility values instead of probability values. Thus, the proposed approach not only enhances the accuracy of estimations but also provides a more detailed understanding of uncertainty, supporting more reliable and informed decision-making for sustainable water resource management. This framework offers engineers a practical tool for tackling real-world hydrological challenges, particularly when conventional probabilistic methods may fall short. The method is applicable to a wide range of engineering problems where uncertainty and limited data are significant concerns, making it a valuable addition to the existing toolkit for water resources and other environmental engineering projects.

In general, it should be highlighted that for practical applications, such as in water resource projects, it is crucial for engineers to make well-informed decisions by considering the deviations between crisp values and fuzzy intervals. This ensures that decisions are based on robust probability estimates with minimal risk, ultimately leading to more resilient and adaptive infrastructure designs.

In conclusion, probability theory and possibility theory can be utilized independently or in combination, each enhancing and complementing the other in a meaningful way. In the future, this method should be extended in cases of multicriteria analysis in order to include more complex engineering problems. This expansion would enable its use in broader range of scenarios, including those that require balancing multiple conflicting objectives, thereby enhancing its utility in decision-making processes across various engineering disciplines.

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