

Introduction to μ SR

Muon Spin Rotation/Relaxation

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- The polarized muon as a magnetic micro-probe
- Generation of polarized muon beams
- Time evolution of muon spin polarization: depolarization and relaxation
- Some typical examples
- Magnetism
- Superconductivity
- Studies in thin films, heterostructures
- Not treated: many things: Muonium (semiconductors), level crossing techniques (chemistry, soft matter), dynamical and critical phenomena (magnetism), resonance...

<http://people.web.psi.ch/morenzoni>

Script of lecture ETH-Z/Uni ZH: Physics with muons

BOOKS

•A. Yaouanc, P. Dalmás de Réotier, MUON SPIN ROTATION, RELAXATION and RESONANCE (Oxford University Press, 2010)

•A. Schenck, MUON SPIN ROTATION SPECTROSCOPY, (Adam Hilger, Bristol 1985)

Literature

•E. Karlsson, SOLID STATE PHENOMENA, As Seen by Muons, Protons, and Excited Nuclei, (Clarendon, Oxford 1995)

•S.L. Lee, S.H. Kilcoyne, R. Cywinski eds, MUON SCIENCE: MUONS IN PHYSICS; CHEMISTRY AND MATERIALS, (IOP Publishing, Bristol and Philadelphia, 1999)

INTRODUCTORY ARTICLES

•S.J. Blundell, SPIN-POLARIZED MUONS IN CONDENSED MATTER PHYSICS, Contemporary Physics 40, 175 (1999)

•P. Bakule, E. Morenzoni, GENERATION AND APPLICATIONN OF SLOW POLARIZED MUONS, Contemporary Physics 45, 203-225 (2004).

REVIEW ARTICLES, APPLICATIONS

•P. Dalmás de Réotier and A. Yaouanc, MUON SPIN ROTATION AND RELAXATION IN MAGNETIC MATERIALS, J. Phys. Condens. Matter 9 (1997) pp. 9113-9166

•A. Schenck and F.N. Gygax, MAGNETIC MATERIALS STUDIED BY MUON SPIN ROTATION SPECTROSCOPY, In: Handbook of Magnetic Materials, edited by K.H.J. Buschow, Vol. 9 (Elsevier, Amsterdam 1995) pp. 57-302

•B.D. Patterson, MUONIUM STATES IN SEMICONDUCTORS, Rev. Mod. Phys. 60 (1988) pp. 69-159

•A. Amato, HEAVY-FERMION SYSTEMS STUDIED BY μ SR TECHNIQUES, Rev. Mod. Phys., 69, 1119 (1997)

•V. Storchak, N. Prokovev, QUANTUM DIFFUSION OF MUONS AND MUONIUM ATOMS IN SOLIDS, Rev. Mod. Physics, 70, 929 (1998)

•J. Sonier, J. Brewer, R. Kiefl, μ SR STUDIES OF VORTEX STATE IN TYPE-II SUPERCONDUCTORS, Rev. Mod. Physics, 72, 769 (2000)

•E. Roduner, THE POSITIVE MUON AS A PROBE IN FREE RADICAL CHEMISTRY, Lecture Notes in Chemistry No. 49 (Springer Verlag, Berlin 1988)

Muon properties

Properties of polarized (positive) muons make them sensitive magnetic microprobes of matter.

Mass:	$m_\mu = 105.658 \text{ MeV}/c^2 \approx 207 m_e \approx 1/9 m_p$	
Charge:	$+e, (-e)$	interstitial position (generally), local probe
Spin :	$s = 1/2$	
Magnetic moment:	$\mu_\mu = g_\mu \frac{e\hbar}{2m_\mu} s$ $\mu_\mu = 3.18 \mu_p$	$(g_\mu \cong 2.00116592069(60))$ very sensitive magnetic probe $10^{-3} - 10^{-4} \mu_B$ (no quadrupolar effects)
Gyromagnetic ratio:	$\gamma_\mu = \frac{\mu_\mu}{\hbar s} = g_\mu \frac{e}{2m_\mu}$	851.615 MHz/T
Life time:	$\tau_\mu = 2.19714 \mu\text{s}$	Fluctuation time window $10^{-5} < t < 10^{-11} \text{ s}$
Bound state:	μ^+e^-	Muonium, H-Isotop

μ SR: Muon Spin Rotation/Relaxation

Method:

- Implant and thermalize ~100% polarized muons in matter (stopping time in solid ~ 10 ps, no initial loss of polarization, stop site: generally interstitial).

$$P(0) \cong 1$$

- Magnetic moment of muon interacts with local magnetic fields (moments, currents, spins) $\rightarrow P(t)$

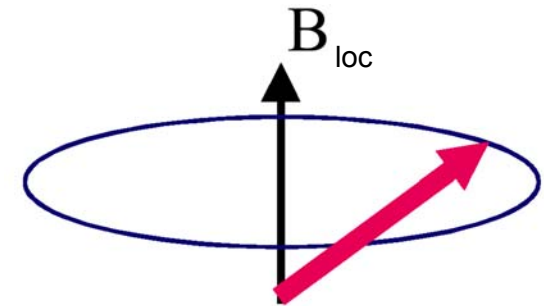
- $P(t)$ is characterized by precession and/or depolarization/relaxation.

- Observe time evolution of the polarization $P(t)$ of the muon ensemble via asymmetric muon decay: (positrons preferentially emitted along muon spin).

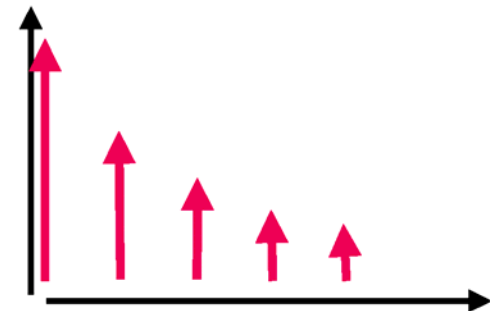
- $P(t)$ contains information about static and dynamic properties of local environment (fields, moments,...)

$$\frac{d\vec{\mu}_\mu}{dt} = \gamma_\mu (\vec{\mu}_\mu \times \vec{B}(t)) \quad \vec{P} = \frac{\langle \vec{s} \rangle}{\frac{1}{2}\hbar}$$

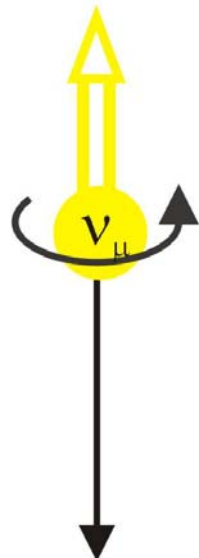
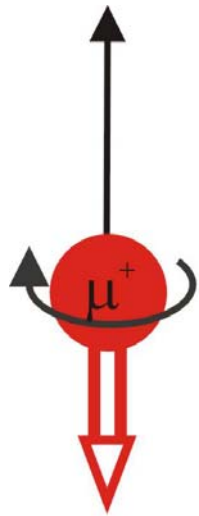
$$\frac{d\vec{P}}{dt} = \gamma_\mu (\vec{P} \times \vec{B}(t))$$



$$\omega_L = \gamma_\mu B_{loc}$$



Production of polarized muons



26 ns

Parity violation in pion decay allows production of polarized muon beams.

Only “left handed” neutrinos \rightarrow

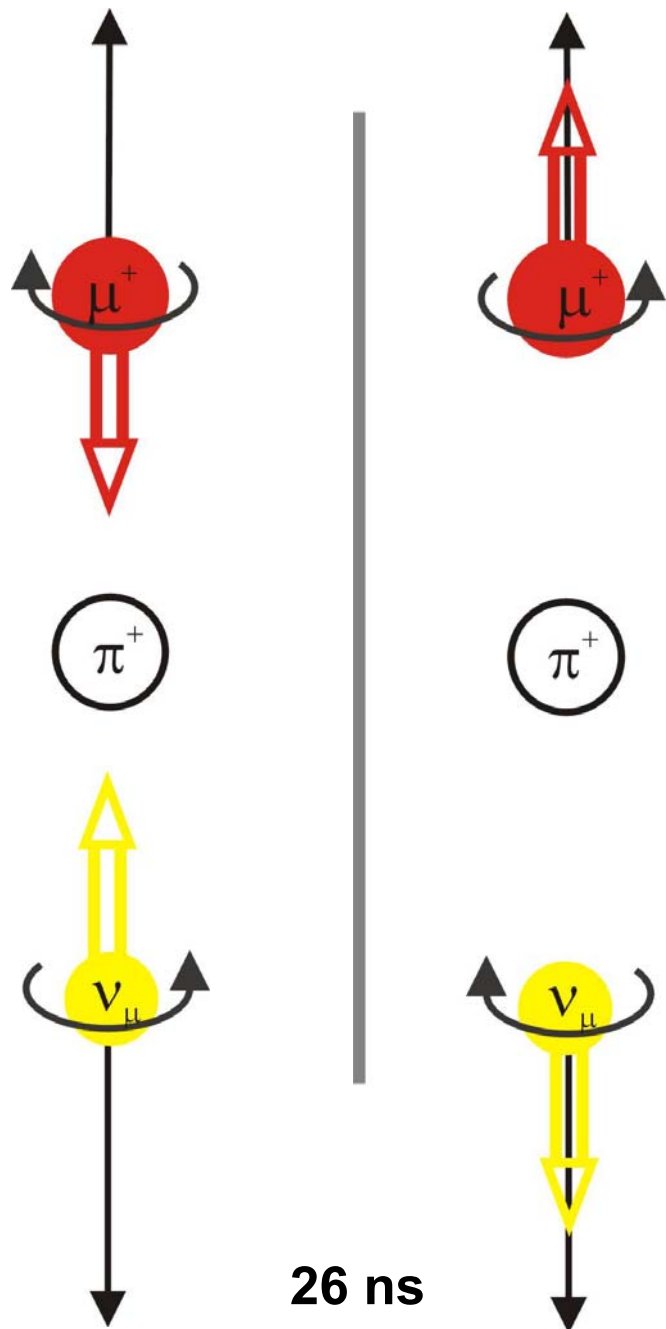
in pion rest frame muon spin antiparallel to momentum.

Kinematics of pion decay at rest;
from energy and momentum conservation:

Momentum: $p_\mu = 29.79 \text{ MeV}/c$

Kinetic energy: $E_\mu = 4.12 \text{ MeV}$

Production of polarized muons



Parity violation in pion decay allows production of polarized muon beams.

Only “left handed” neutrinos →

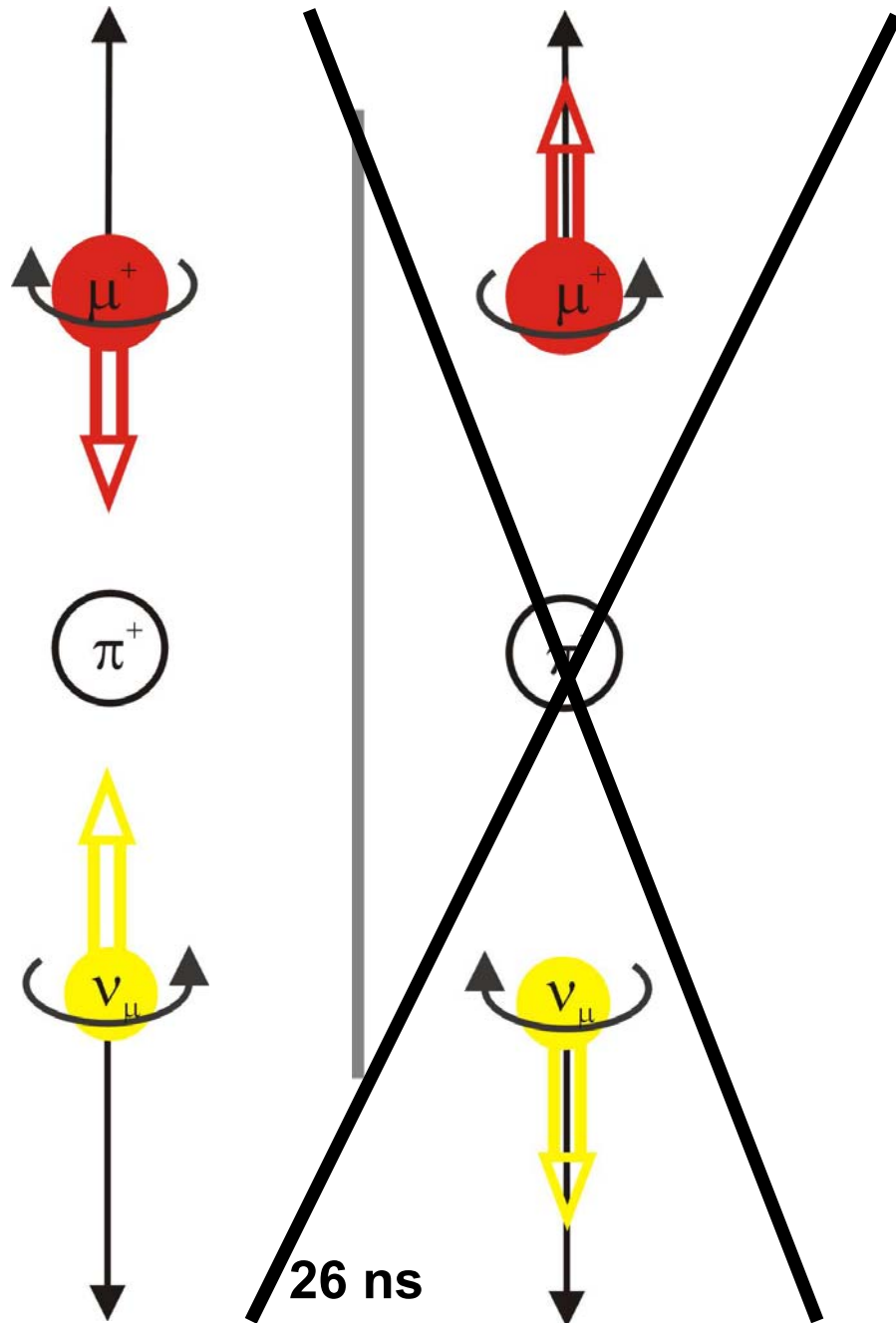
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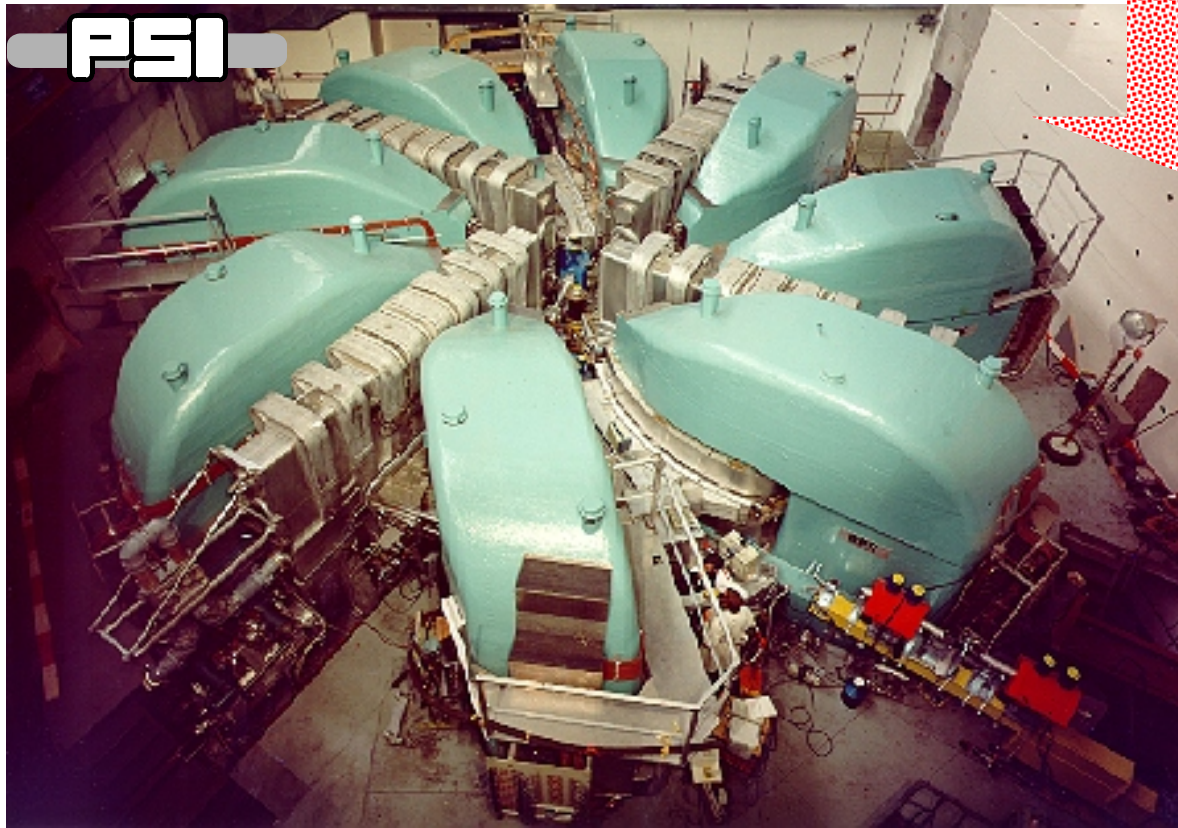
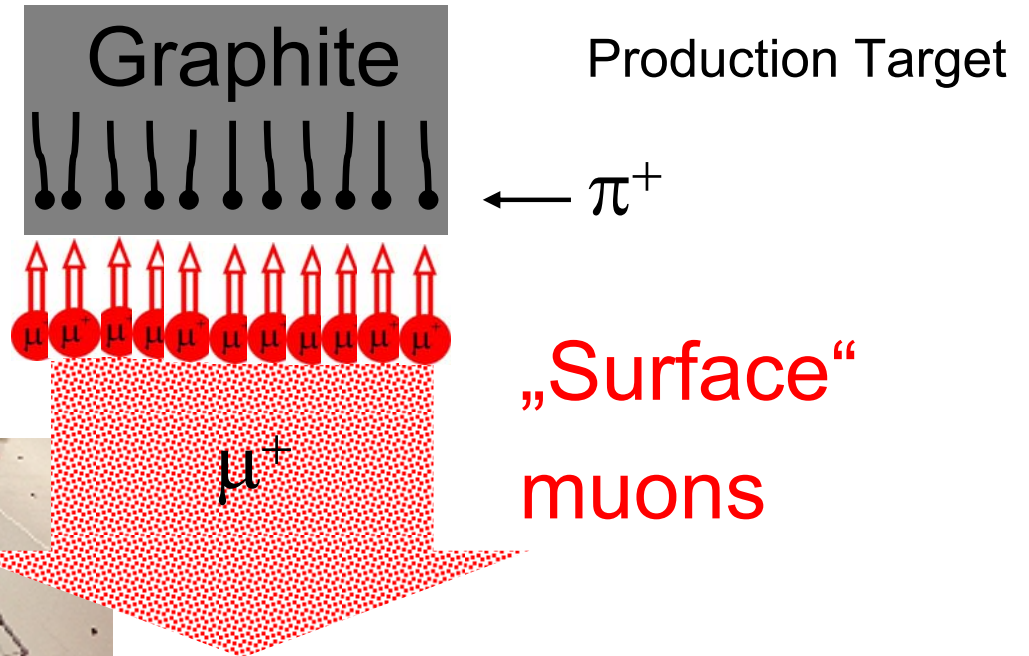
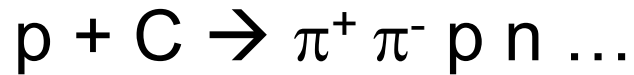
Kinematics of pion decay at rest;
from energy and momentum conservation:

Momentum: $p_\mu = 29.79 \text{ MeV}/c$

Kinetic energy: $E_\mu = 4.12 \text{ MeV}$

Generation of polarized muons (μ^+)

2.2 mA \cong $1.4 \cdot 10^{16}$ Protons/sec
with 600 MeV



$\sim 10^7 - 10^8 \mu^+/\text{sec}$

100 % pol.

$\sim 4 \text{ MeV}$

generally used for “bulk”
condensed matter studies

For thin film studies: eV-30 keV

S μ S: The Swiss Muon Source

High Field μ SR, 9.5 T, 20 mK

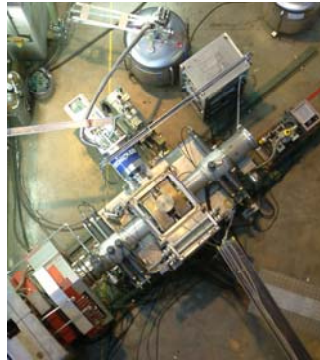
Until 2011 Avoided
Level crossings
instrument ALC



GPS

General Purpose Surface
Muon Instrument
Muon energy: 4.2 MeV (μ^+)

0.6 T, 1.8 K

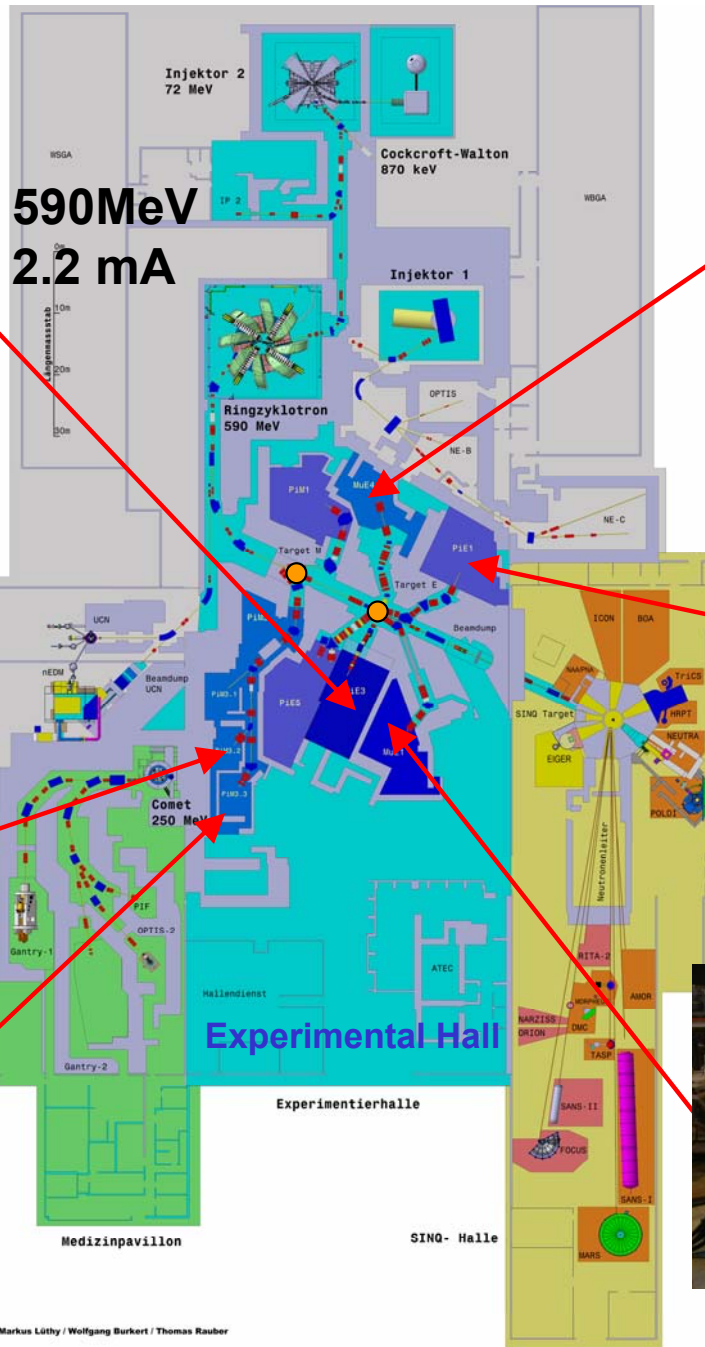


Shared Beam Surface Muon Facility (Muon On REquest)

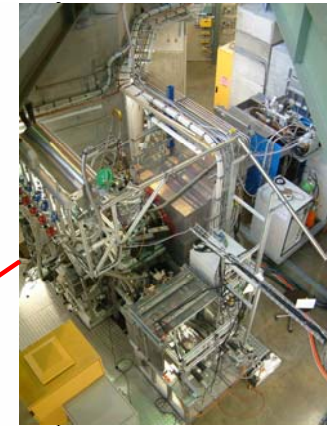
LTF

Low Temperature Facility
Muon energy: 4.2 MeV (μ^+)

3 T,
20 mK- 4 K



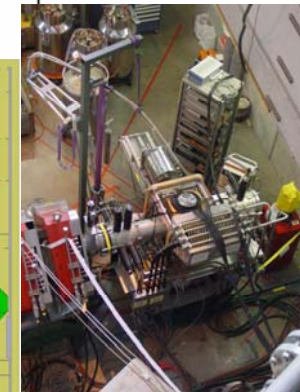
590MeV
2.2 mA



LEM

Low-energy muon beam
and instrument, tunable
energy (0.5-30 keV, μ^+),
thin-film, near-surface
and multi-layer studies
(1-300 nm)

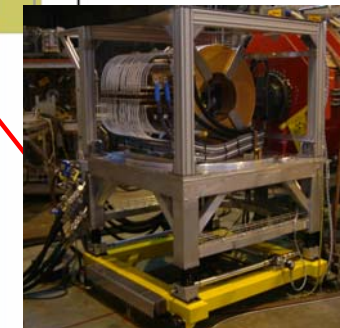
0.3 T,
2.5 K



DOLLY

General Purpose
Surface Muon Instrument
Muon energy: 4.2 MeV (μ^+)

0.5 T
2 K (0.25K)

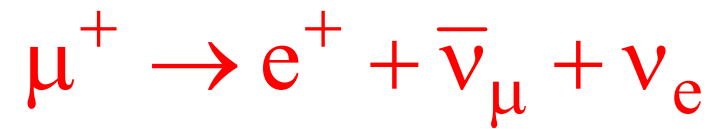


GPD

General Purpose Decay
Channel Instrument
Muon energy: 5 - 60 MeV
(μ^+ or μ^-)

0.5 T,
300 mK
2.8 GPa

Measuring P(t): Muon Decay



- Muon decay (life time 2.2. μs) violates parity conservation
→ asymmetric decay
- Positrons preferentially emitted along muon spin (along polarization vector of muon ensemble)

$$\frac{dN_{e^+}(\theta)}{d\Omega} \propto \left(1 + \frac{1}{3} P \cos \theta\right) = \left(1 + \frac{1}{3} \vec{P} \cdot \vec{n}\right)$$

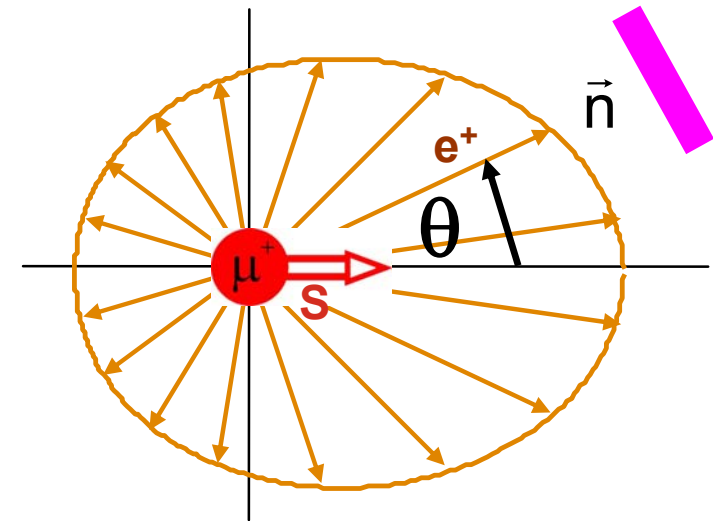
\vec{n} : direction of observation (detector position)

- Measuring positrons allows to observe time evolution of the polarization P(t) of the muon ensemble
- Positron intensity as a function of time after implantation:

$$N_{e^+}(t) = N_0 \left[1 + A_0 P(t)\right] e^{-\frac{t}{\tau_\mu}} \quad P(t) = \vec{P}(t) \cdot \vec{n}$$

- A_0 : Maximum observable asymmetry
theoretically: $A_0 = 1/3$
practically it depends on setup (average over solid angle, absorption in materials): $A_0 = 0.25 - 0.30$
- $A_0 P(t)$ is called asymmetry: A(t)

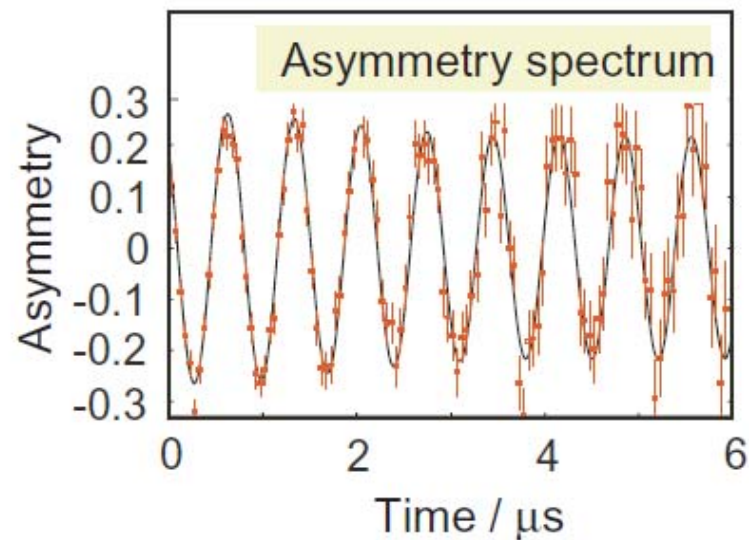
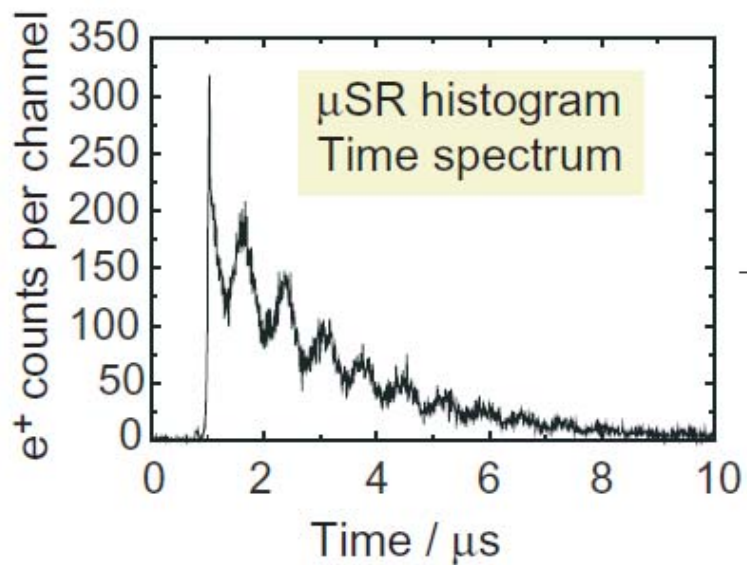
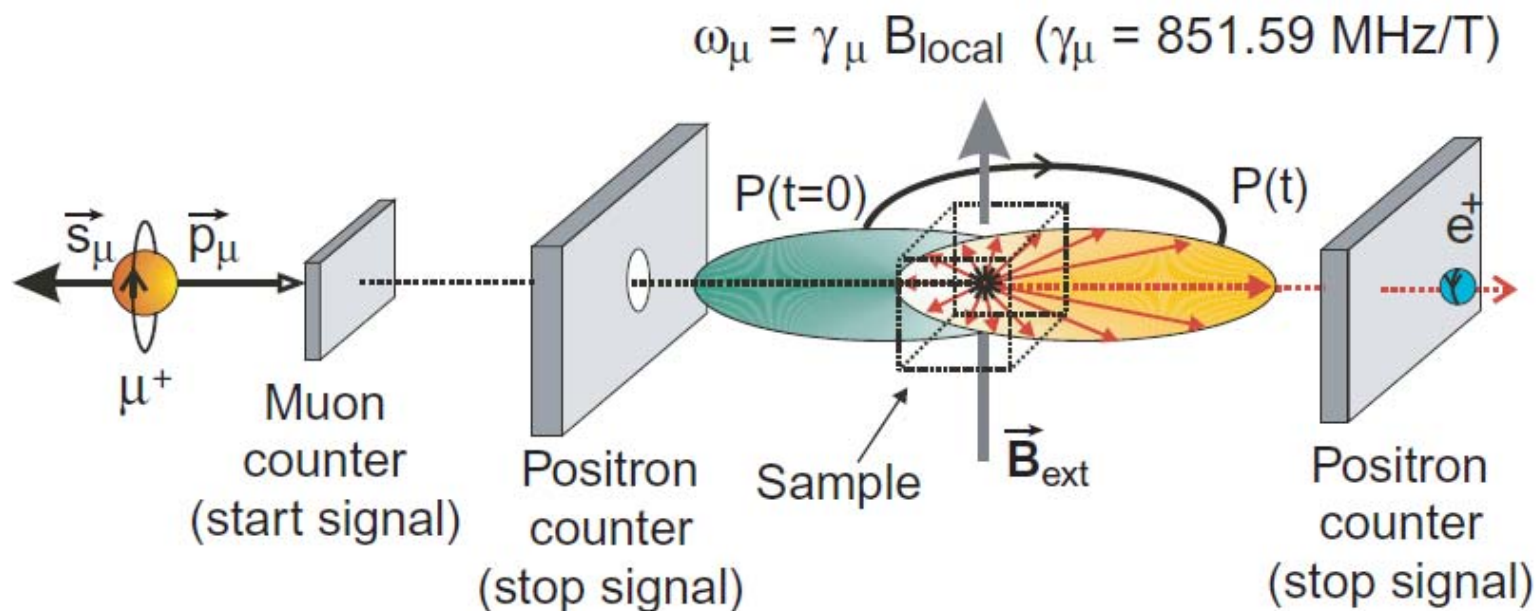
For $P = 1$:



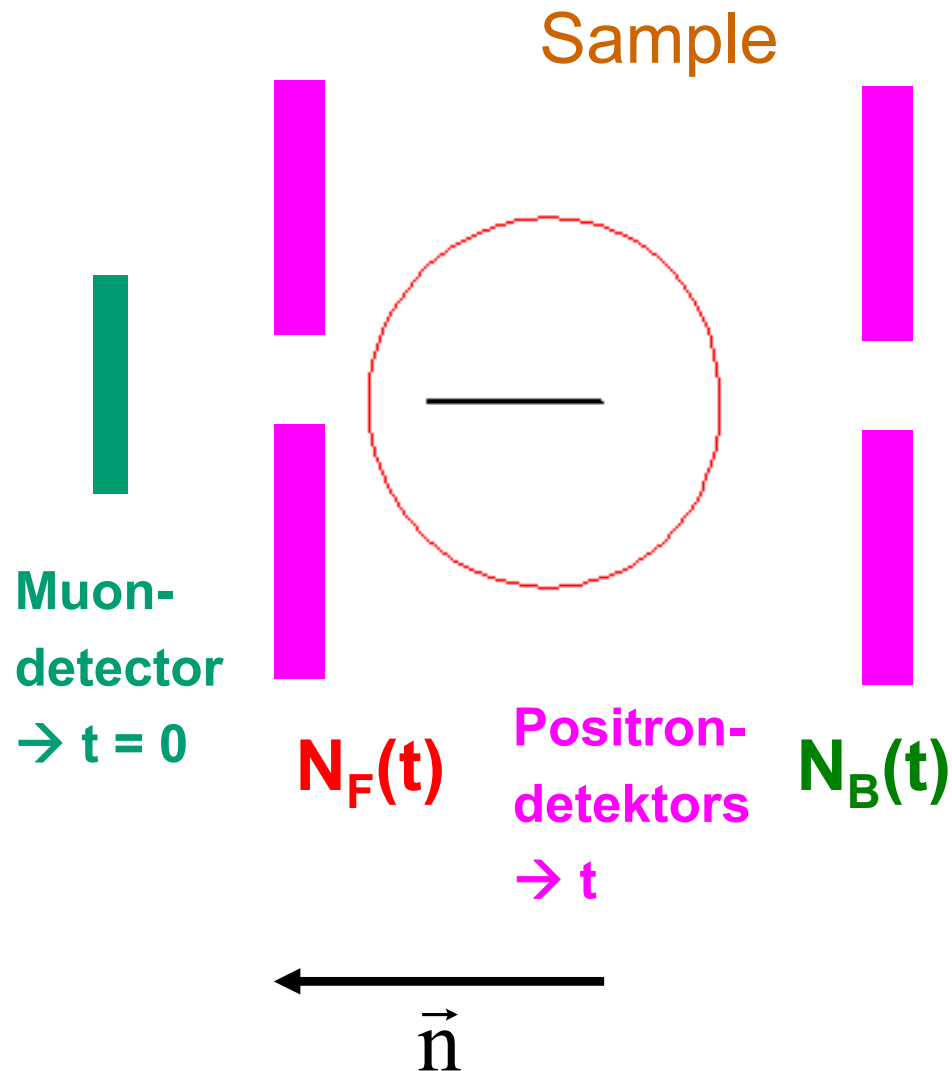
$$\frac{dN_{e^+}(\theta)}{d\Omega} \propto \left(1 + \frac{1}{3} P \cos \theta\right)$$

θ : angle between spin (polarization) and positron direction

Principle of a μ SR experiment



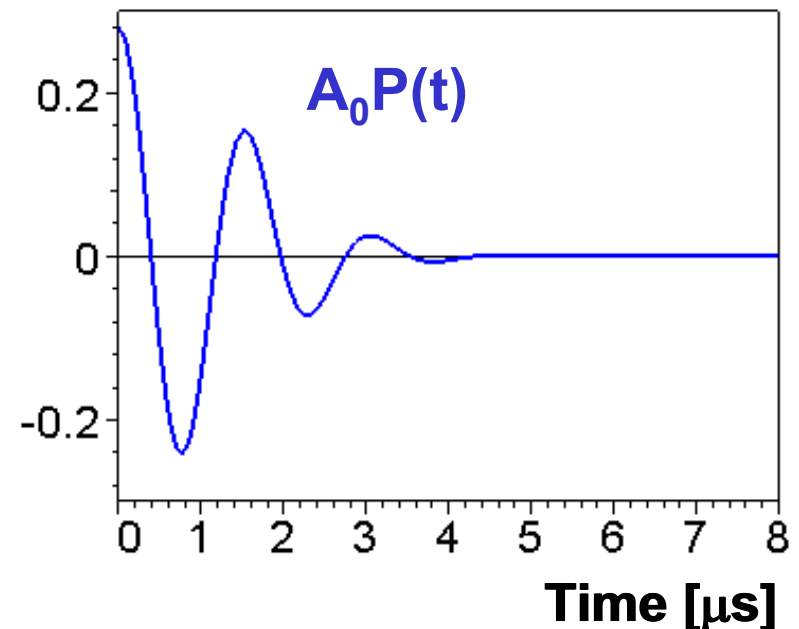
μ SR: Muon Spin Rotation/Relaxation

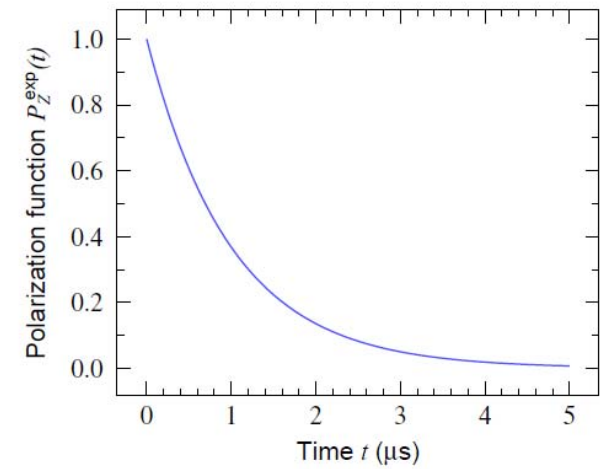
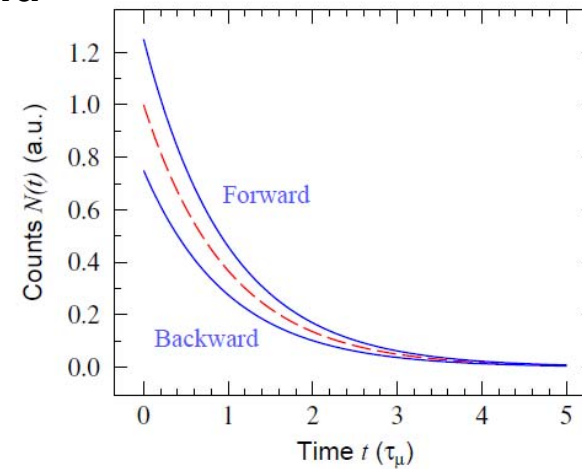
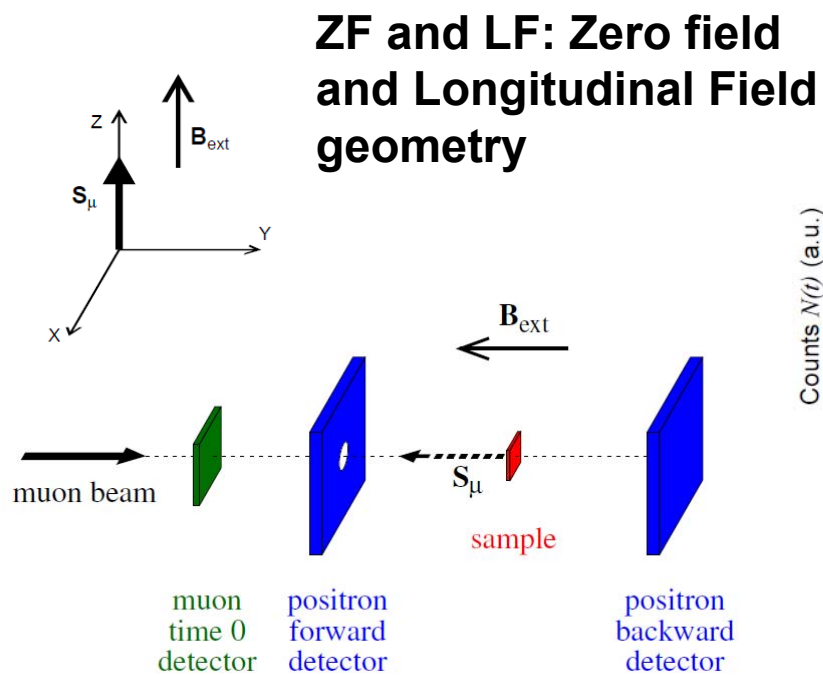
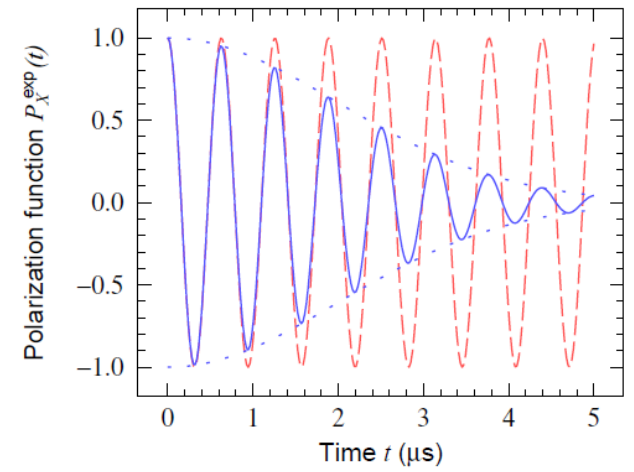
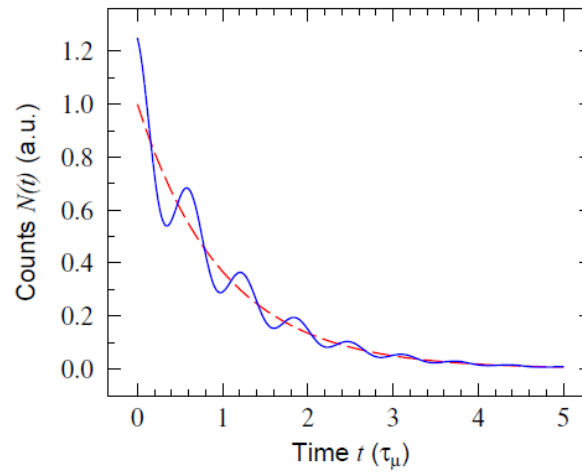
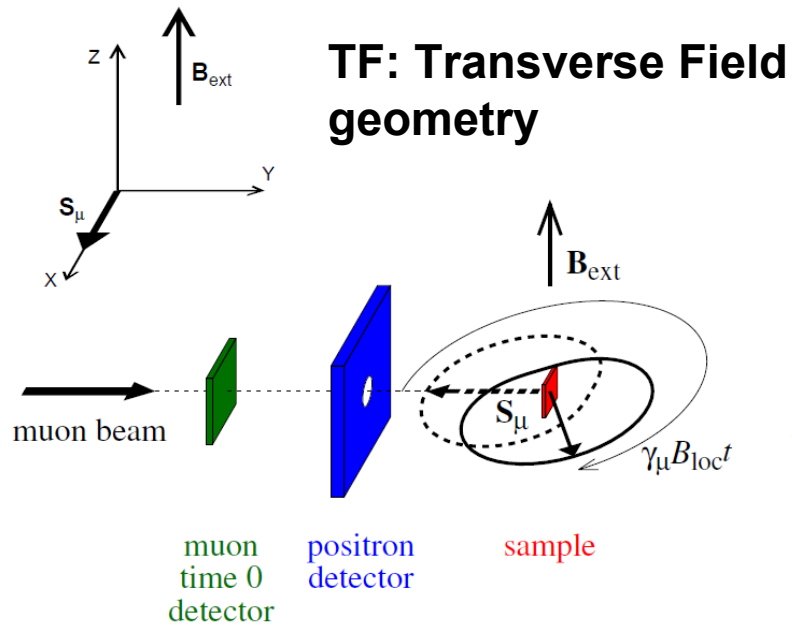


$$N_F(t) = N_0 \left[1 + A_0 \vec{P}(t) \cdot \vec{n} \right] e^{-\frac{t}{\tau_\mu}}$$

$$N_B(t) = N_0 \left[1 - A_0 \vec{P}(t) \cdot \vec{n} \right] e^{-\frac{t}{\tau_\mu}}$$

$$\frac{N_F(t) - N_B(t)}{N_F(t) + N_B(t)} = AP(t) \quad (P(t) = \vec{P}(t) \cdot \vec{n})$$





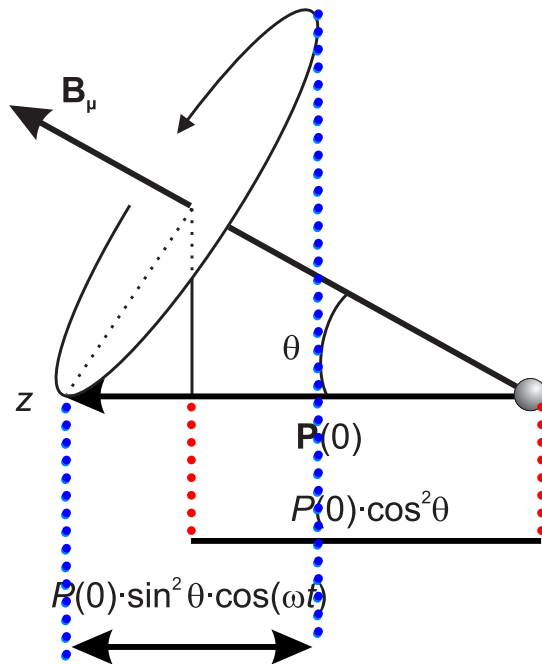
P(t): time evolution of polarization

$$\frac{d\vec{P}}{dt} = \gamma_{\mu} (\vec{P} \times \vec{B}(t)) \quad \vec{B} \text{ is the total field at muon site (i.e. including applied field)}$$

Simplest case:

All muons in the sample experience the same static field $\vec{B} = (B_x, B_y, B_z)$

Static means: \vec{B} does not change over observation time (5-10 τ_{μ}): $\frac{B(t)}{dB(t)} \gg \tau_{\mu}$



$\vec{P}(0) \parallel \hat{z} \equiv \hat{n}$ (Direction of observation)

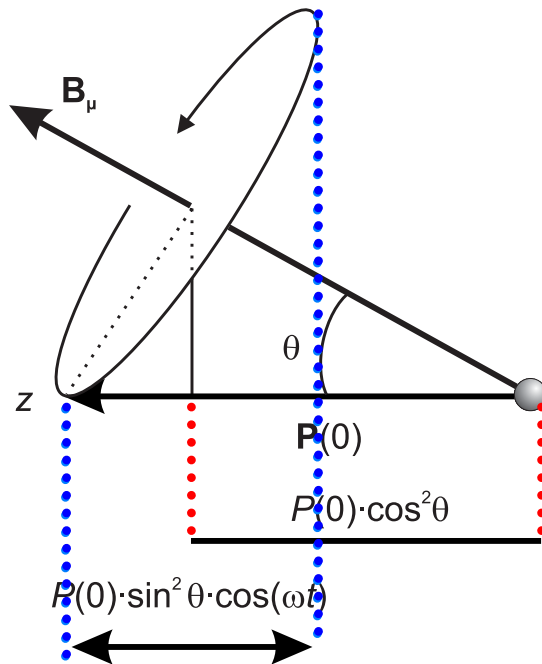
$$P_{\vec{B}}(t) = \cos^2 \theta + \sin^2 \theta \cos(\omega_L t) = \frac{B_z^2}{B^2} + \frac{B_x^2 + B_y^2}{B^2} \cos(\gamma_{\mu} B t)$$

$\omega_L \equiv \gamma_{\mu} B$ Larmor Frequency (Spin precession frequency)

P(t): time evolution of polarization

$$\frac{d\vec{P}}{dt} = \gamma_{\mu} (\vec{P} \times \vec{B}(t)) \quad \vec{B} \text{ is the total field at muon site (i.e. including applied field)}$$

In case the muons experience a field distribution $p(\vec{B})$

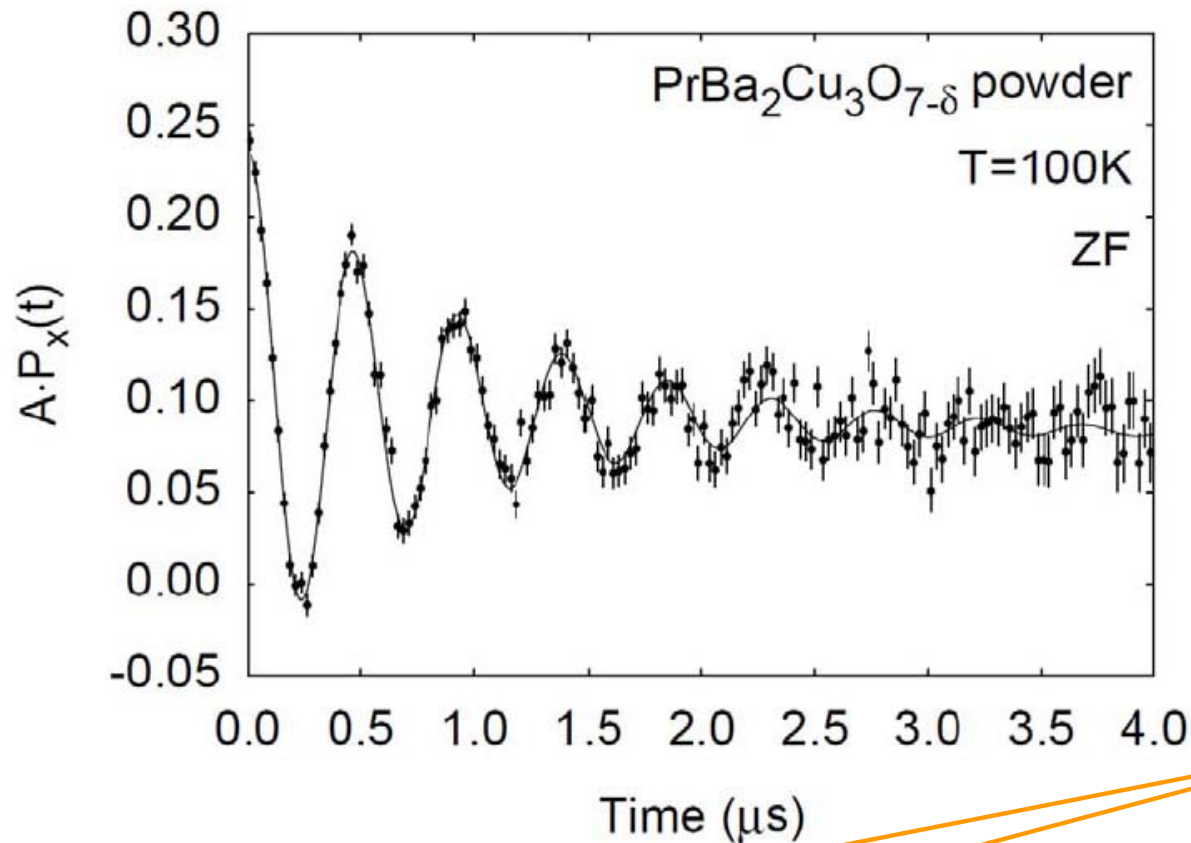


$$P(t) = \int p(\vec{B}) P_{\vec{B}}(t) d^3B = \int p(\vec{B}) \left[\frac{B_z^2}{B^2} + \frac{B_x^2 + B_y^2}{B^2} \cos(\gamma_{\mu} B t) \right] d^3B$$

Magnetism: polycrystalline sample

$|\vec{B}|$ equal all over the sample, isotropic direction:

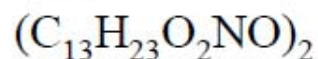
$$P(t) = \frac{1}{4\pi} \int \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu |\vec{B}| t) d\phi d(\cos \theta) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu |\vec{B}| t)$$



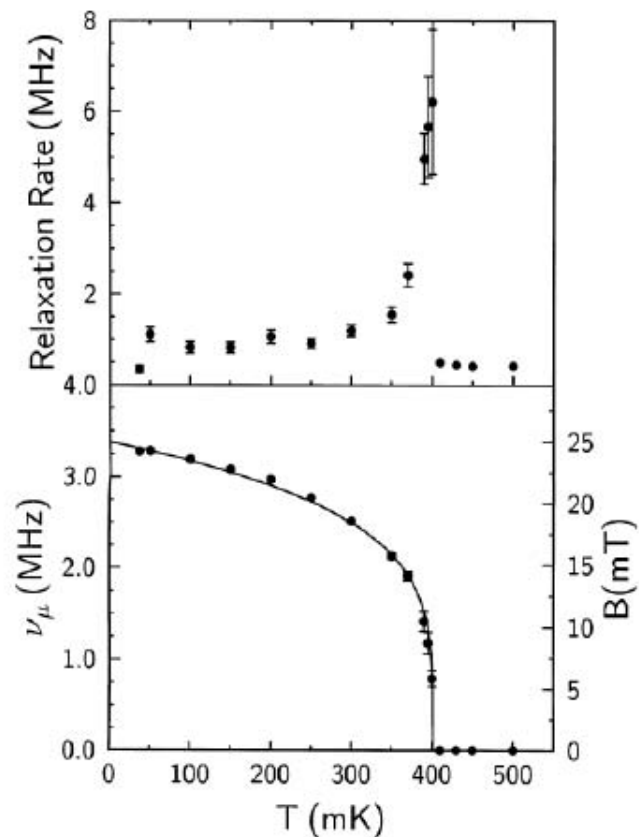
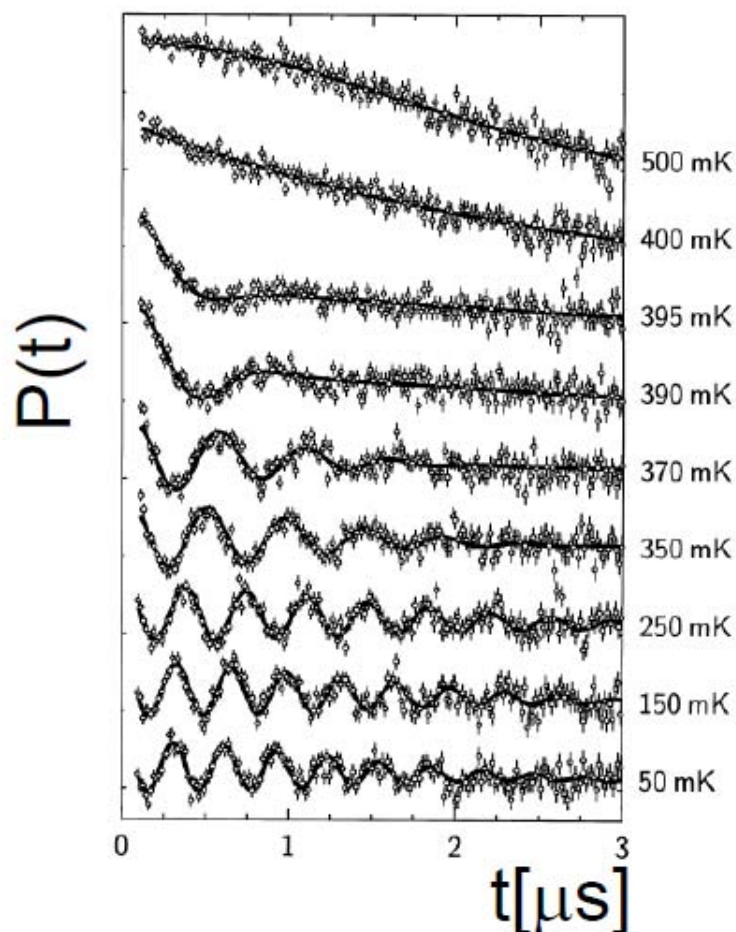
$$A_0 P(t) = A_0 \left[e^{-\lambda_L t} \frac{1}{3} + e^{-\lambda_T t} \frac{2}{3} \cos(\gamma_\mu |\vec{B}| t) \right]$$

Microscopic magnetometry

$$B_{\text{ext}} = 0$$



Organic antiferromagnet



$$P(t) = a_L(t) + a_T e^{-\lambda t} \cos(2\pi\nu_\mu t)$$

$$\lambda = \frac{1}{T} \quad \text{relaxation rate, } [\mu\text{s}^{-1}] \text{ or } [\text{MHz}]$$

S. Blundell et al., Physica B (2000)

Local field in magnetic materials

Internal field : generally sum of dipolar :

$$\vec{B}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3(\vec{\mu}_i \cdot \vec{r}_{i\mu}) \cdot \vec{r}_{i\mu} - \mu_i r_{i\mu}^2}{r_{i\mu}^5}$$

$$B_{\text{dip}} \approx \frac{\mu_0}{4\pi} \frac{\mu_i}{r_{i\mu}^3} \approx \frac{\mu_i [\mu_B]}{d^3 [\text{\AA}^3]} \text{ T} \approx 0.1 \text{ T}$$

and contact field (spin density at muon site):

$$\vec{B}_{\text{hf}}(\vec{r}_\mu) = \frac{2\mu_0}{3} \mu_B \rho_{\text{spin}}(\vec{r}_\mu) \cong \frac{2\mu_0}{3} \mu_B |\phi(\vec{r}_\mu)|^2 \langle \vec{s} \rangle$$

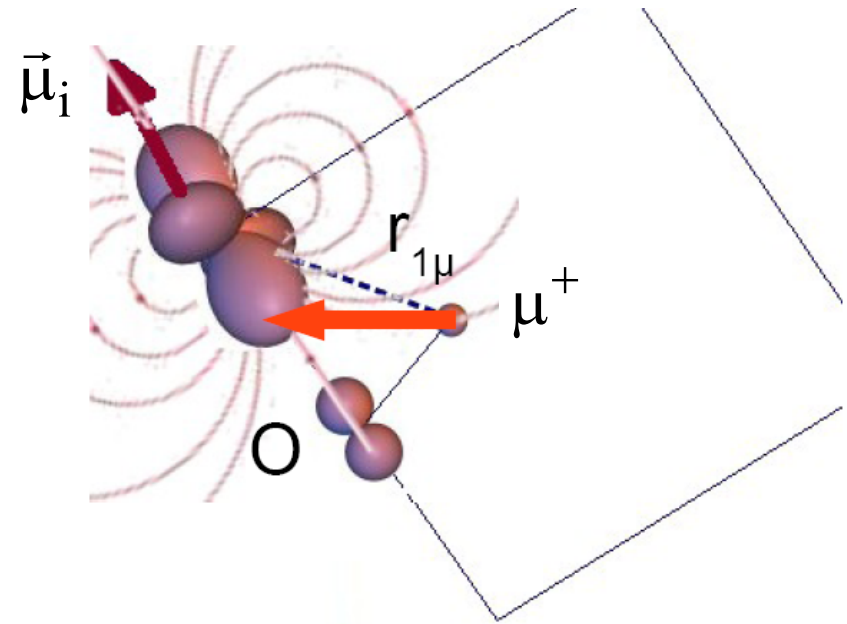
High sensitivity:

μSR time window 10-20 μs

$\rightarrow \nu_\mu \cong 50 \text{ kHz}$ detectable

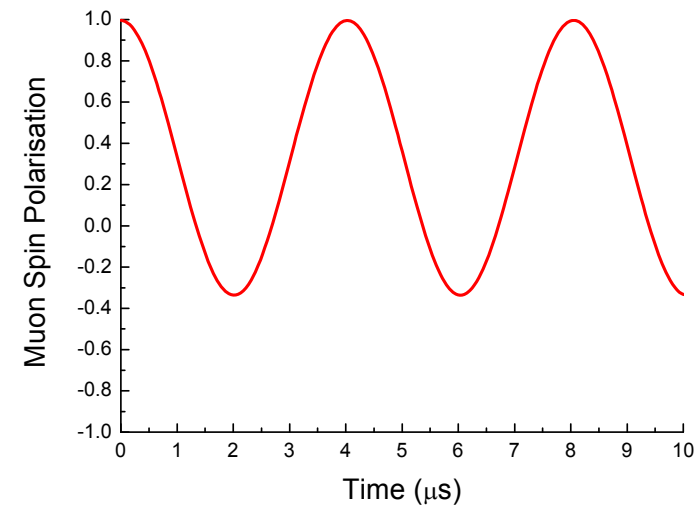
$\rightarrow B = \frac{2\pi}{\gamma_\mu} \nu_\mu \approx 0.1 \text{ mT (Gauss)}$

(corresponds to $0.001 \mu_B$ or nuclear moments μ_n)

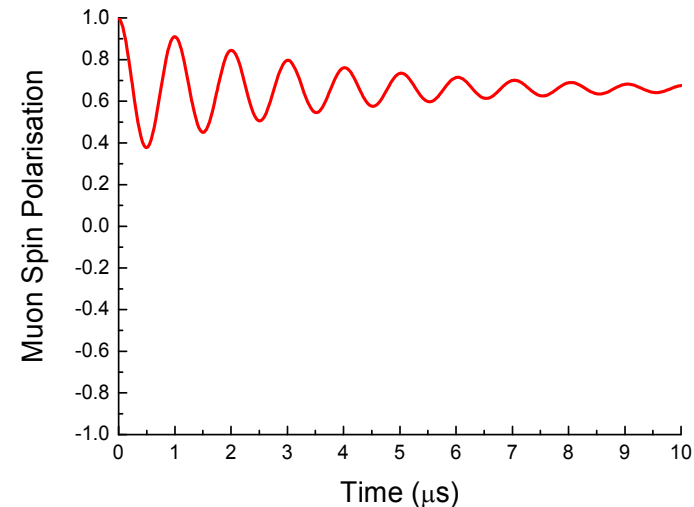
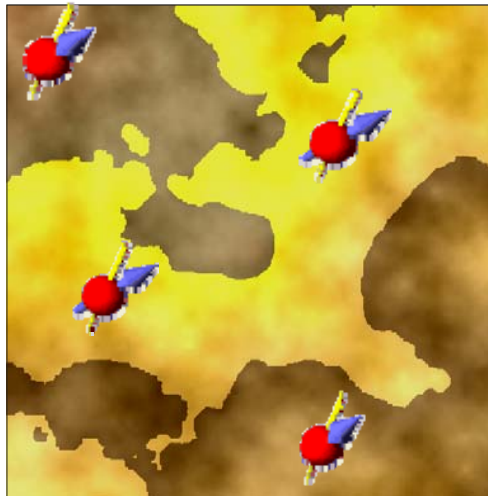


Inhomogeneous materials: determination of volume fraction

Homogeneous:



Inhomogeneous:



Amplitude a = Magnetic volume fraction

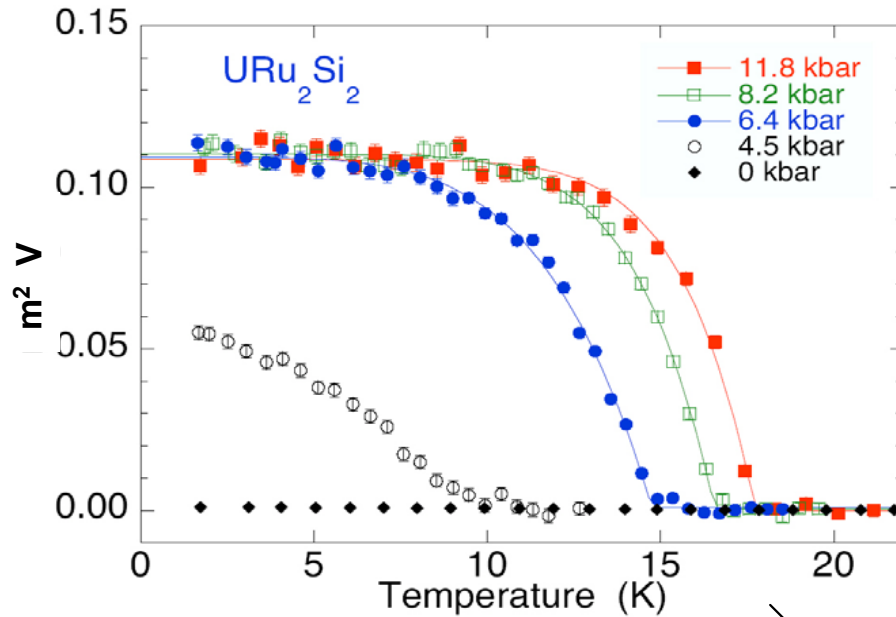
Frequency ω = Local field, size of magnetic moments

Damping λ, σ = inhomogeneity of magnetic regions

Example URu₂Si₂

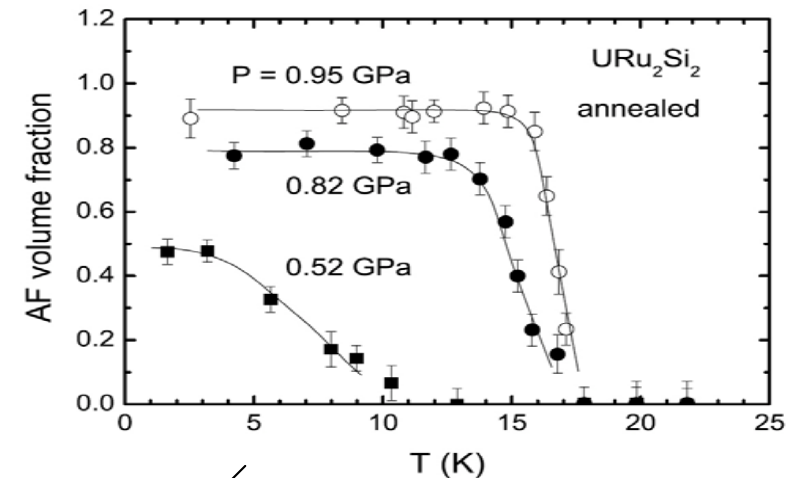
Neutron scattering:

F. Bourdarot et al., condmat/0312206



Muon Spin Rotation:

A. Amato et al., J. Phys.: Condens. Matter **16** (2004) S4403

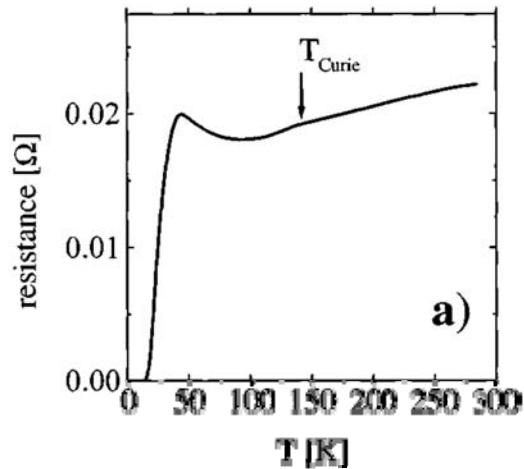


Phase separation in magnetic and non magnetic volumes

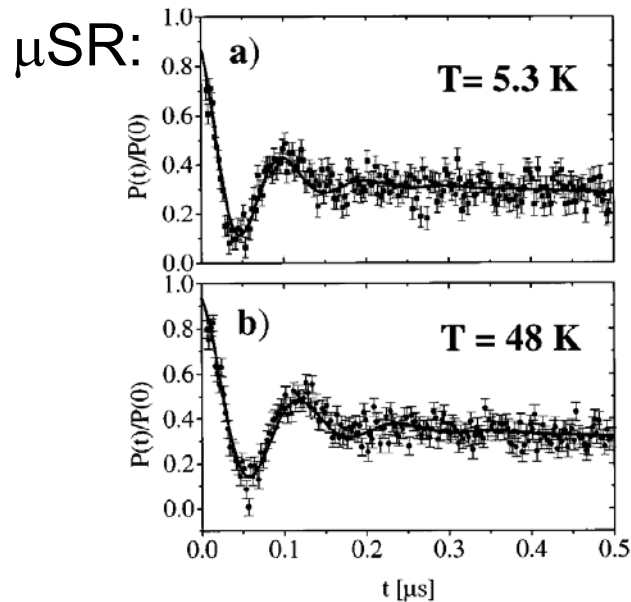
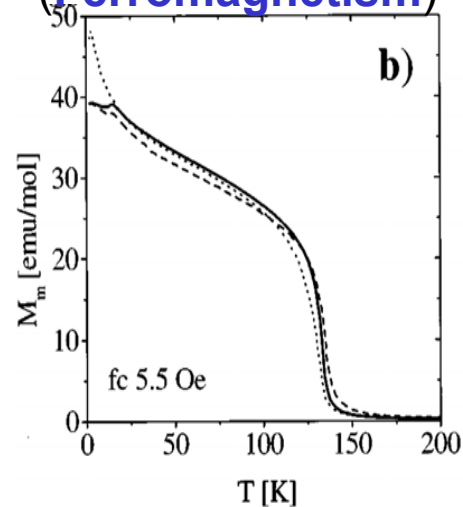
Only the combination of neutron and muon data allows the correct interpretation of the data

Example: $\text{RuSr}_2\text{GdCu}_2\text{O}_8$

Resistivity:
(Superconductivity)

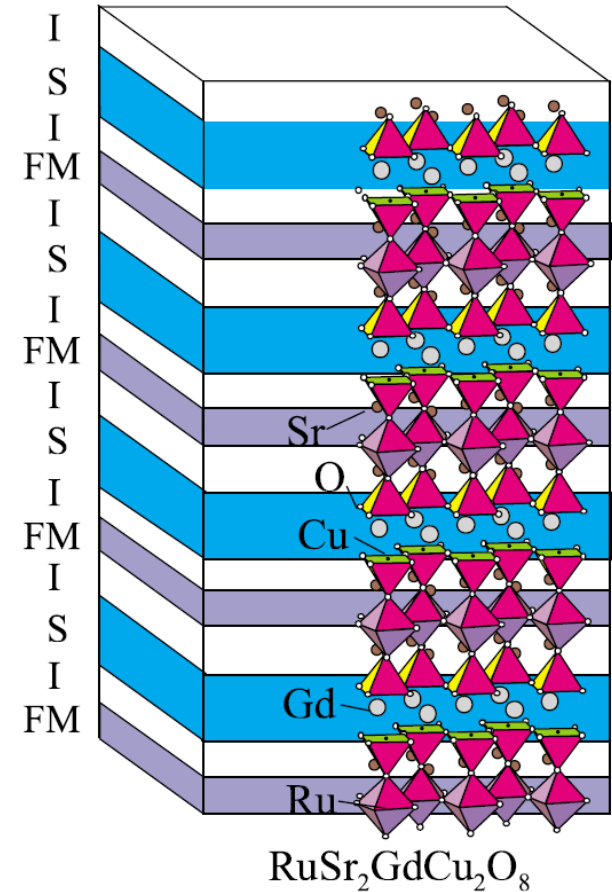


Magnetization:
(Ferromagnetism)



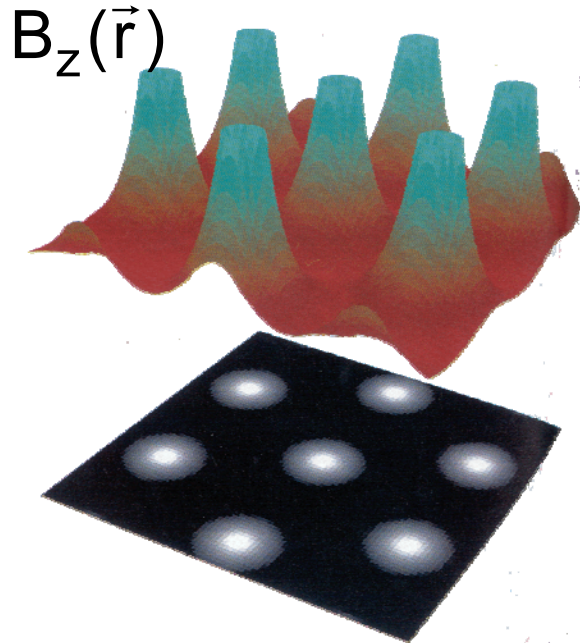
**~100%
magnetic volume**

**Microscopic
coexistence of
superconductivity
and magnetism**



Structure:
T. Nachtrab et al., Phys. Rev. Lett. **92** (2004) 117001

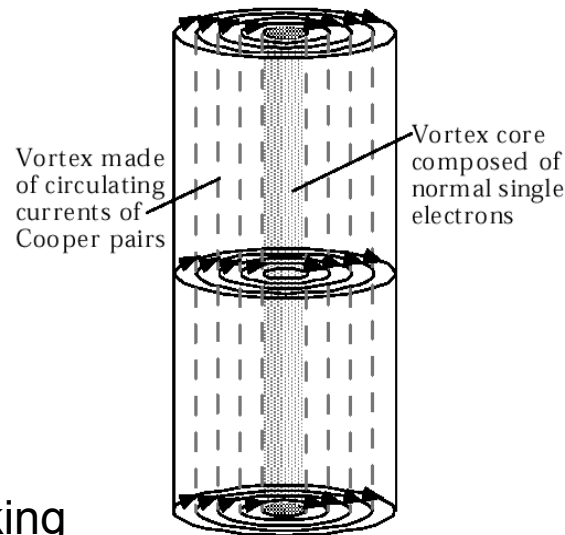
Vortex state of a type II superconductor



1 vortex:

$$\Phi_0 = \frac{h}{2e}$$

Flux generated is one quantum of flux
 $\phi = \phi_0 = h/2e$

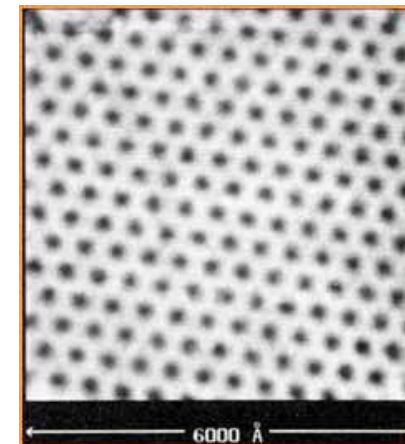
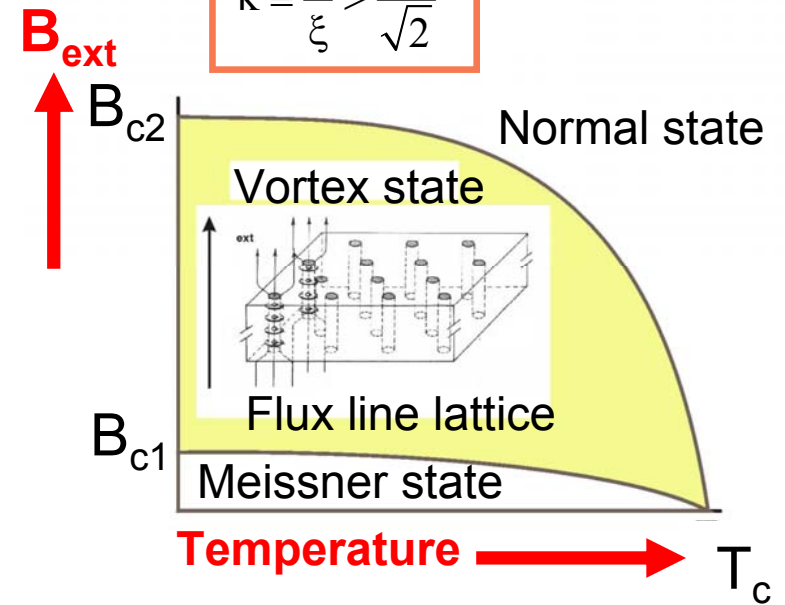


Vortex of supercurrents

$B(r)$ can be obtained from a modified London equation taking into account the flux generated by the regular array of vortices

$$\Delta \vec{B}(\vec{r}) - \frac{\vec{B}(\vec{r})}{\lambda^2} = \frac{\Phi_0}{\lambda^2} \sum_{\vec{R}} \delta(\vec{r} - \vec{R}) \hat{z}$$

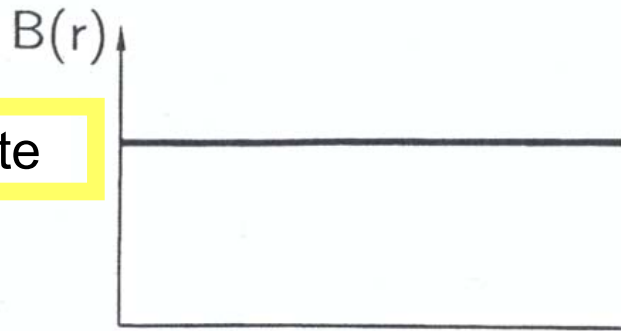
$$\kappa \equiv \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$$



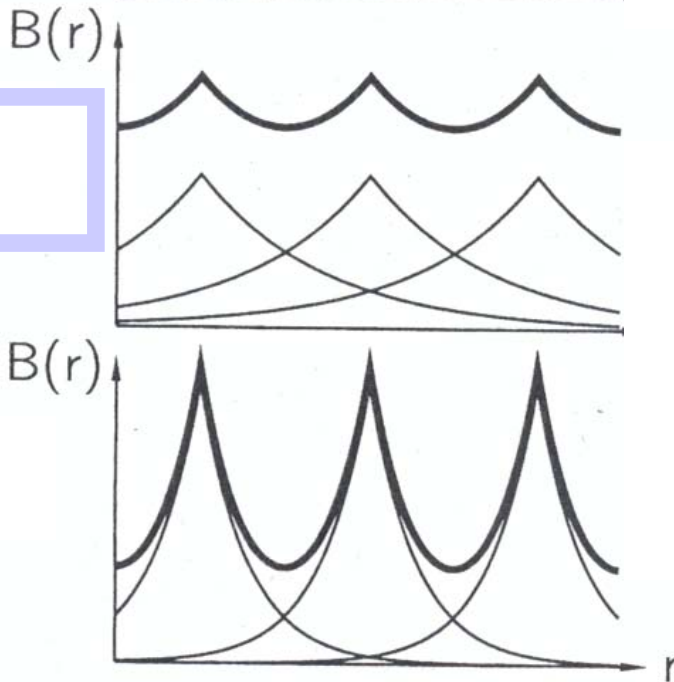
Surface image by Scanning Tunnel Microscopy
 NbSe₂, 1T, 1.8K
H. F. Hess et al. Phys. Rev. Lett. 62, 214 (1989)

μ SR in the vortex state

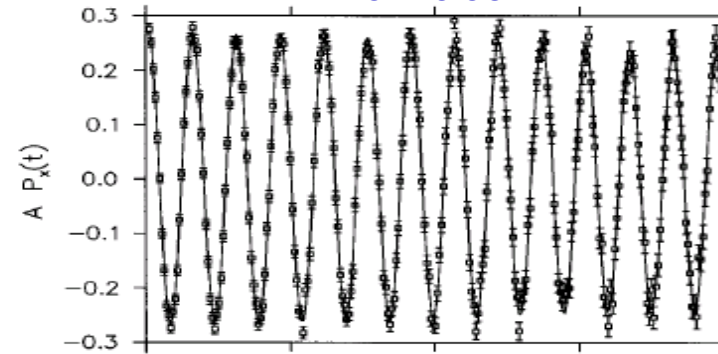
Normal state



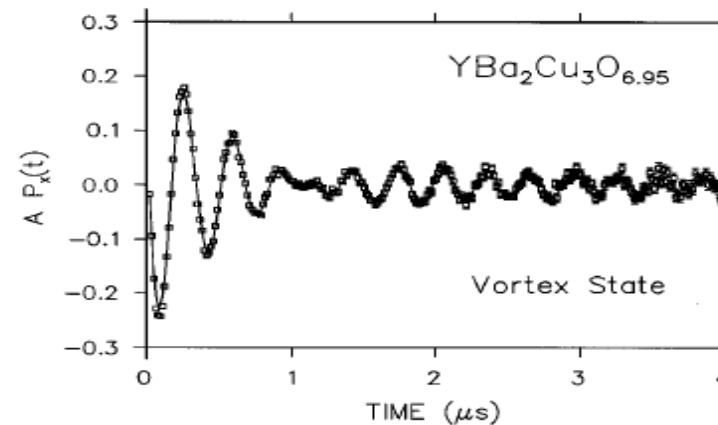
Vortex-state



$\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$



$$AP_x(t) = A \cos(\gamma_\mu B t + \Phi)$$



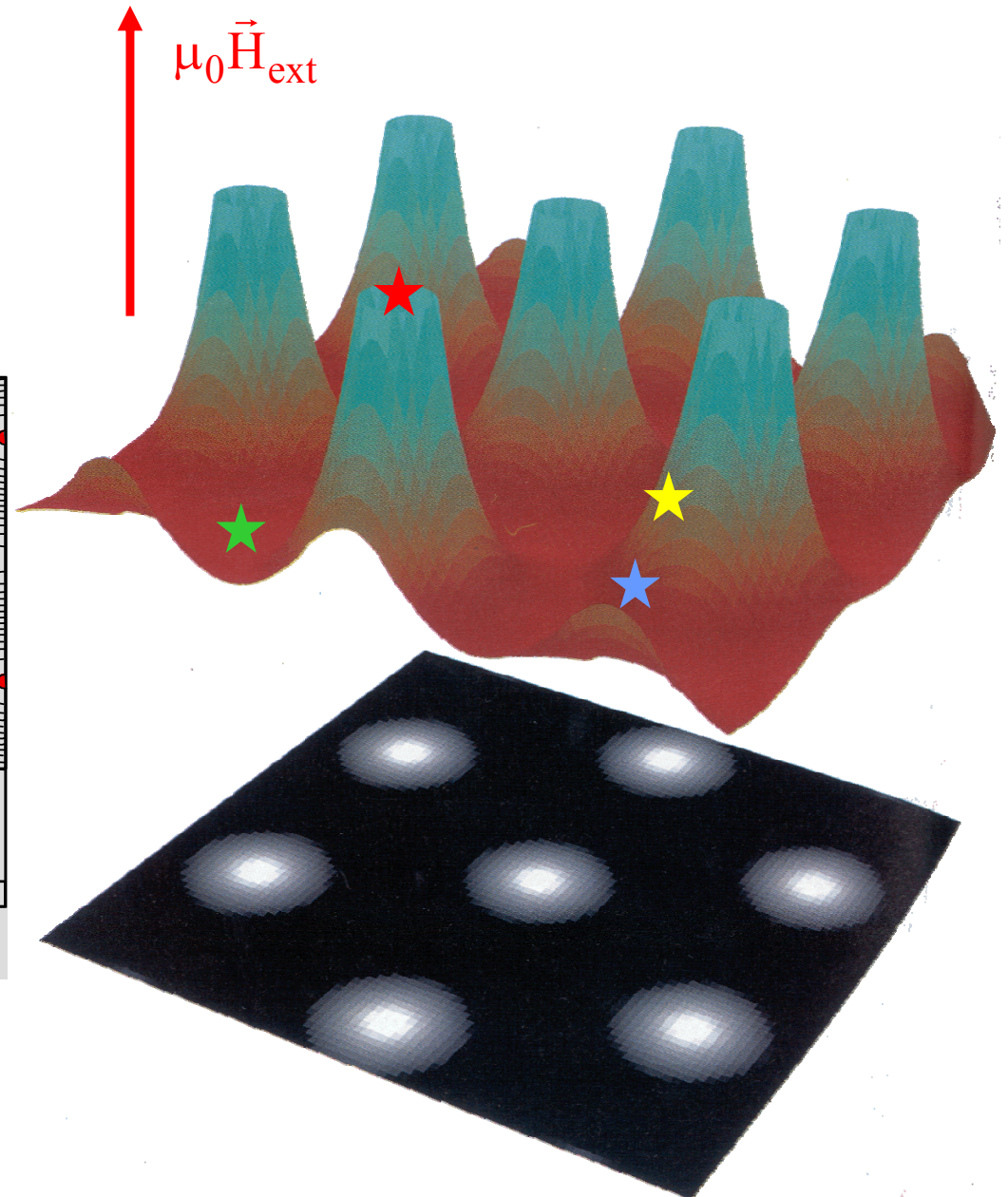
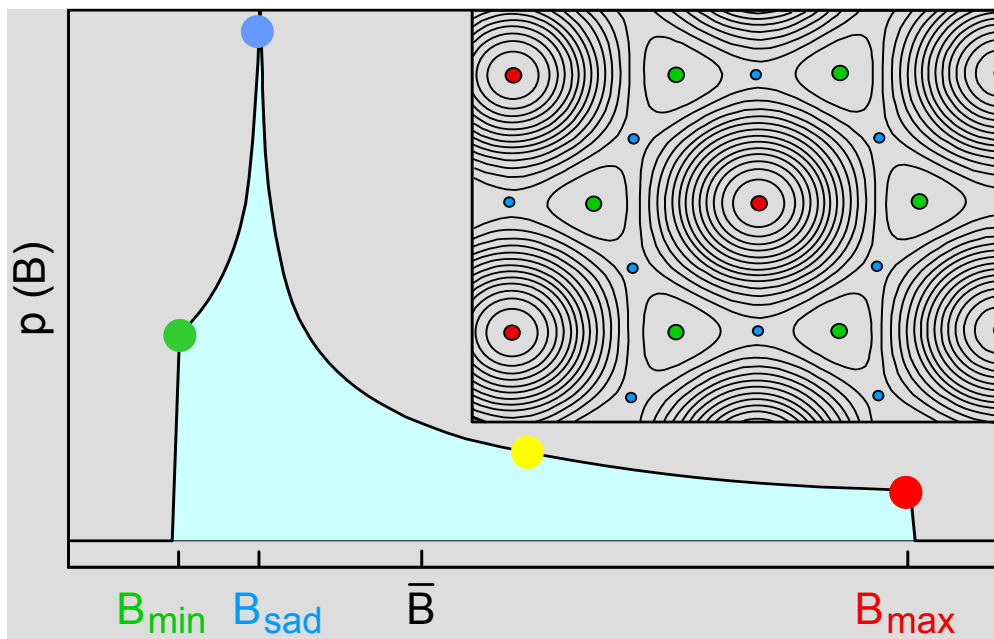
$$P_x(t) = \frac{1}{N} \sum_{i=1}^N \cos(\gamma_\mu B(\vec{r}_i) t + \phi)$$

$$P_x(t) = \int p(B_z) \cos(\gamma_\mu B_z t + \phi) dB_z$$

Field distribution vortex state

$$P_x(t) = \int p(B_z) \cos(\gamma_\mu B_z t + \phi) dB_z$$

$p(B_z)$: field distribution
(field averaged over all muon sites)

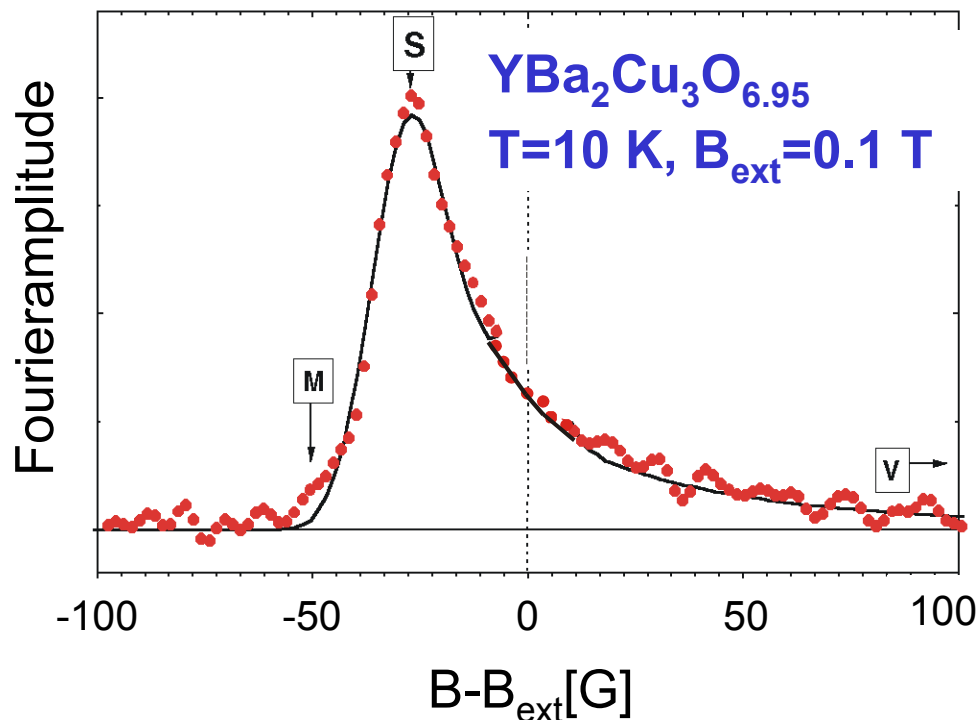


Bishop et al., Scientific American 48 (1993)

Field distribution in vortex state

$$P_x(t) = \int p(B_z) \cos(\gamma_\mu B_z t + \phi) dB_z$$

$p(B_z)$: microscopic magnetic field distribution $p(B_z) \longleftrightarrow B_z(r)$
= Fourier transform of time evolution of polarization $P(t)$



- Structure, symmetry of the Flux line lattice
- Vortex motion
- Characteristic lengths: magnetic penetration depth λ , radius of the vortex core (coherence length)
- Classification scheme of superconductors

Spatial dependence of field and field width

$$\Delta \vec{B}(\vec{r}) - \frac{\vec{B}(\vec{r})}{\lambda^2} = \frac{\Phi_0}{\lambda^2} \sum_{\vec{R}} \delta(\vec{r} - \vec{R}) \hat{z}$$

can be explicitly solved in reciprocal space:

$$B_z(\vec{r}) = \sum_{\vec{k}} \frac{\langle B \rangle}{1 + \lambda^2 k^2} e^{i\vec{k}\vec{r}}$$

and the second moment calculated

$$\langle \Delta B_z^2 \rangle = \langle B_z^2 \rangle - \langle B_z \rangle^2$$

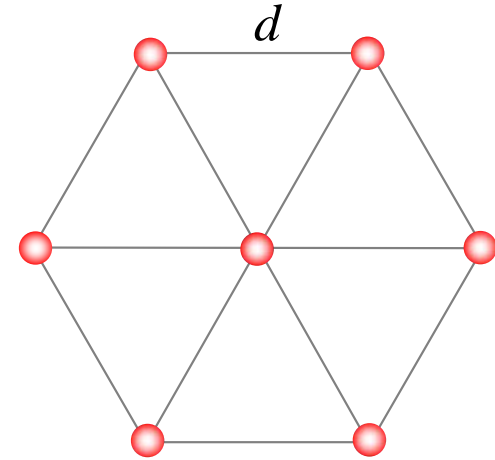
we obtain:

$$\langle \Delta B_z^2 \rangle = 0.00371 \frac{\Phi_0^2}{\lambda^4}$$

A μ SR measurement of the second moment of the field distribution allows to determine the London penetration depth λ .

$$\rightarrow \lambda(T) = \sqrt{\frac{m^*}{\mu_0 e^2 n_s(T)}}$$

n_s : supercarrier density, m^* : effective mass

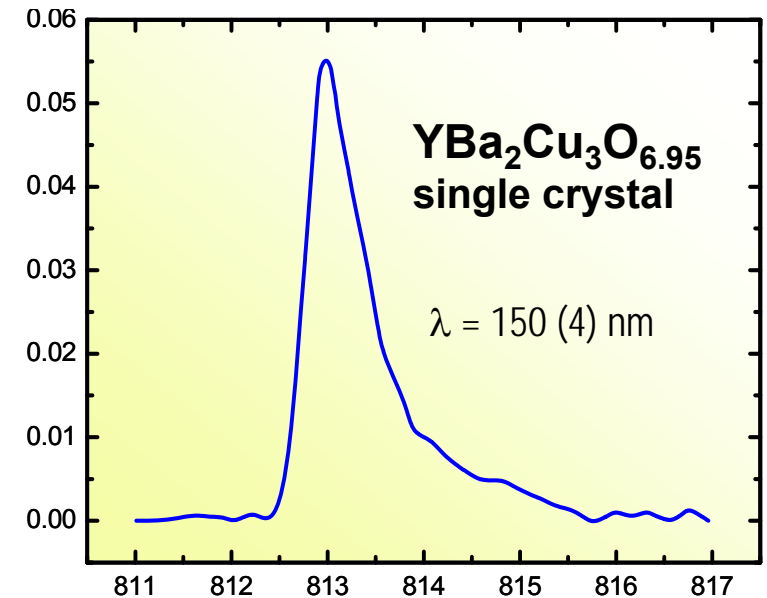


Field distribution in vortex state

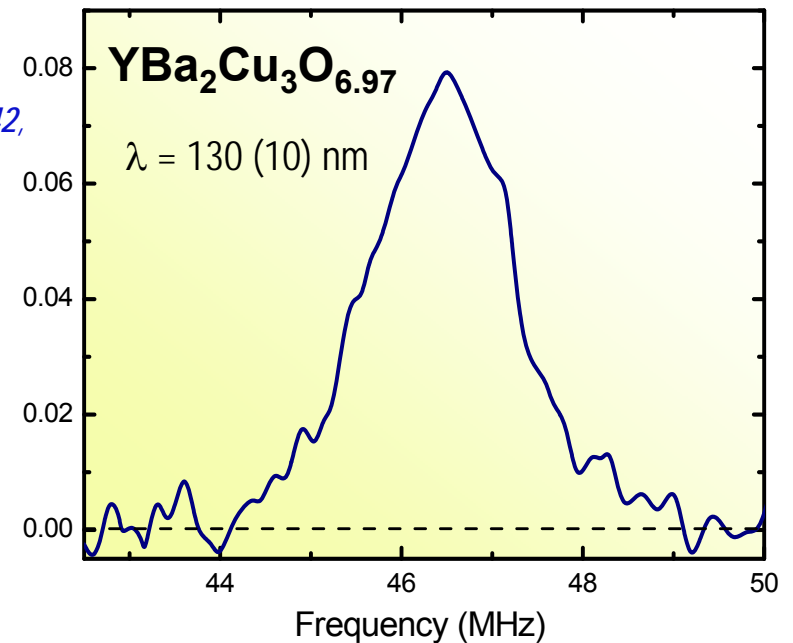
Single crystals:
asymmetric field distribution.
Allow to study anisotropic properties of
high temperature superconductors

Polycrystals or sintered samples:
large density and disorder of pinning
sites → strong smearing of the field
distribution. Can be approximated by
Gauss distribution

*Sonier et al.,
PRL 83, 4156
(1999)*



*Pümpin et al.,
Phys. Rev. B 42,
8019 (1990)*



Gauss field distribution and polarization

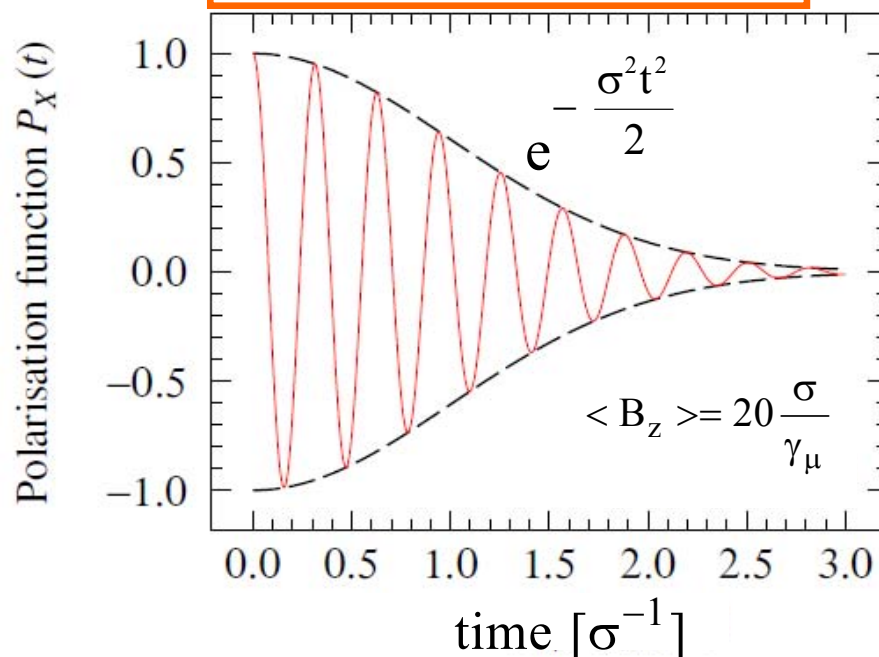
Gaussian damped precession

$$P(t) = e^{-\frac{\sigma^2 t^2}{2}} \cos(\gamma_\mu \langle B_z \rangle t)$$

$$\sigma^2 = \gamma_\mu^2 \langle \Delta B_z^2 \rangle \quad \text{Relaxation rate } [\mu\text{s}^{-1}]$$

$$\sigma = \gamma_\mu \Phi_0 \sqrt{0.00371} \frac{1}{\lambda^2}$$

$$\sigma \text{ } [\mu\text{s}^{-1}] = 0.1074 \frac{1}{\lambda \text{ } [\mu\text{m}]^2}$$

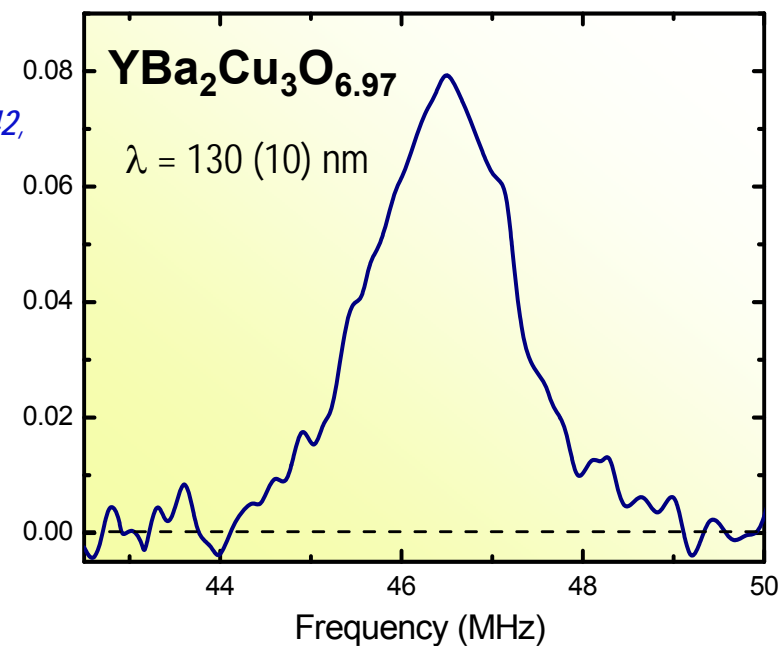


Gaussian field distribution

$$p(B_z) = \frac{\gamma_\mu}{\sqrt{2\pi}\sigma} e^{-\frac{\gamma_\mu^2 (\langle B_z \rangle - B_z)^2}{2\sigma^2}}$$

$$\frac{\sigma^2}{\gamma_\mu^2} = \langle \Delta B_z^2 \rangle$$

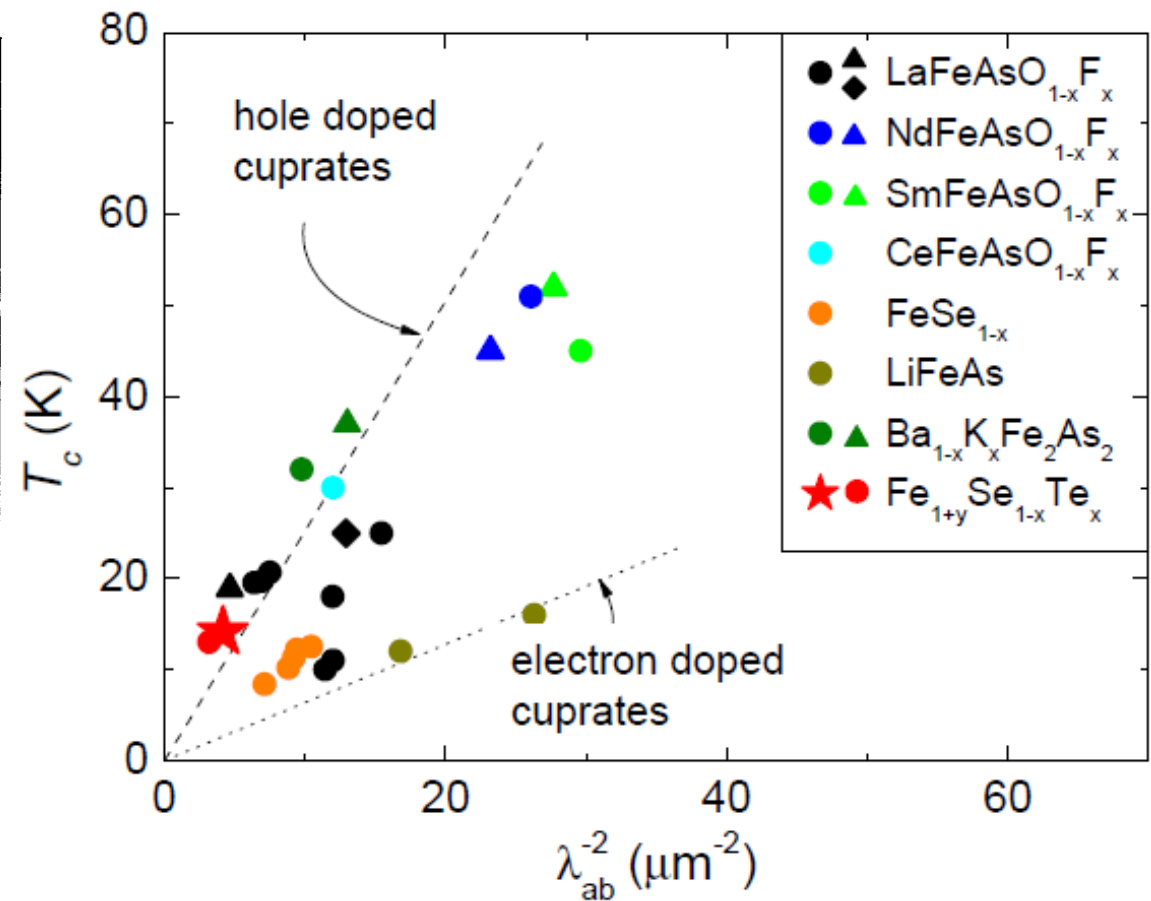
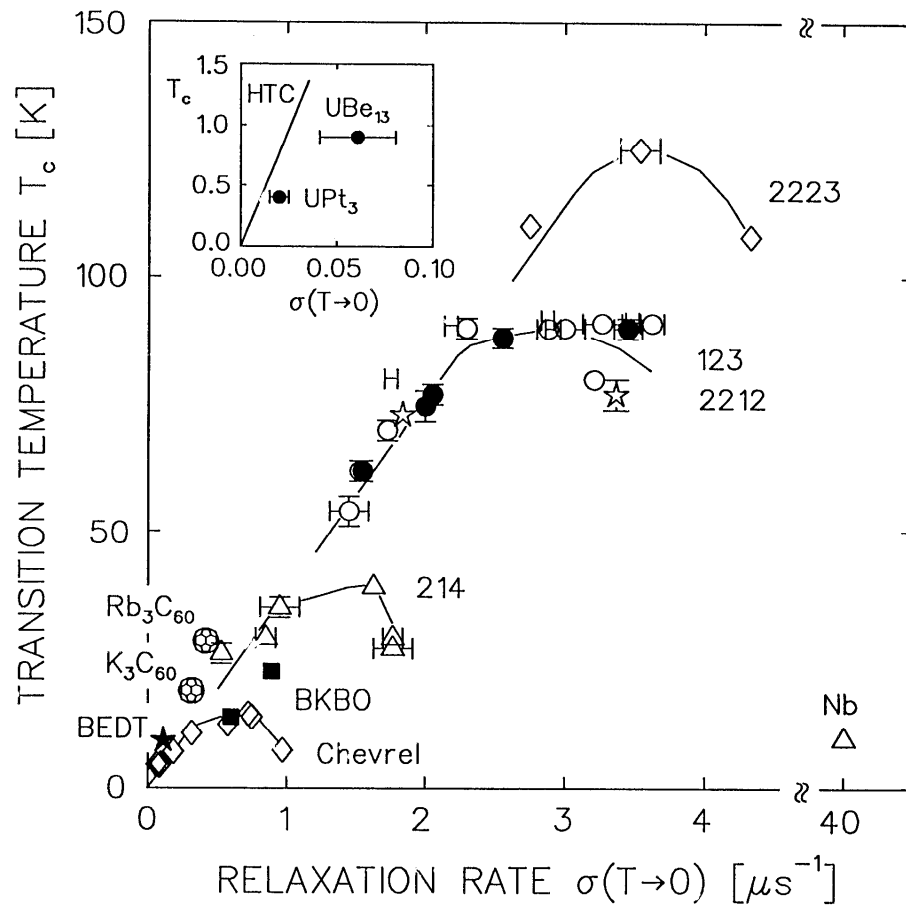
*Pümpin et al.,
Phys. Rev. B 42,
8019 (1990)*



Classification of superconductors

$$T_c \text{ versus } \sigma \propto \frac{1}{\lambda^2} \propto \frac{n_s}{m^*}, \text{ Uemura plot}$$

Y. Uemura et al., Phys. Rev. Lett. 66, 2665 (1991)



T-dependence of sc carrier density and sc gap

From μ SR:

$$\sigma_{\mu} = \gamma_{\mu} \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2}$$

$$\lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}}$$

$$\Rightarrow \sigma_{\mu} \propto \frac{\mu_0 e^2}{m} n_s$$

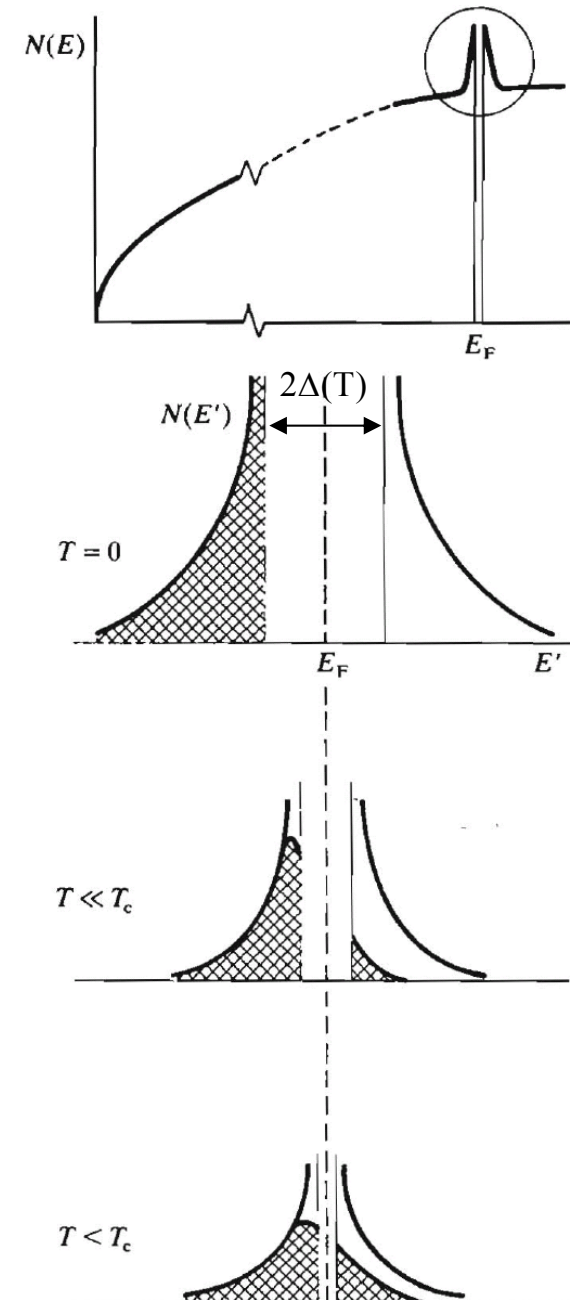
indications on the SC gap

By taking into account the thermal population of the quasiparticles excitations of the Cooper pairs (Bogoliubov quasiparticles):

$$n_s(T) = n_s(0) \left(1 - \frac{2}{k_B T} \int_0^{\infty} f(\epsilon, T) [1 - f(\epsilon, T)] d\epsilon \right)$$

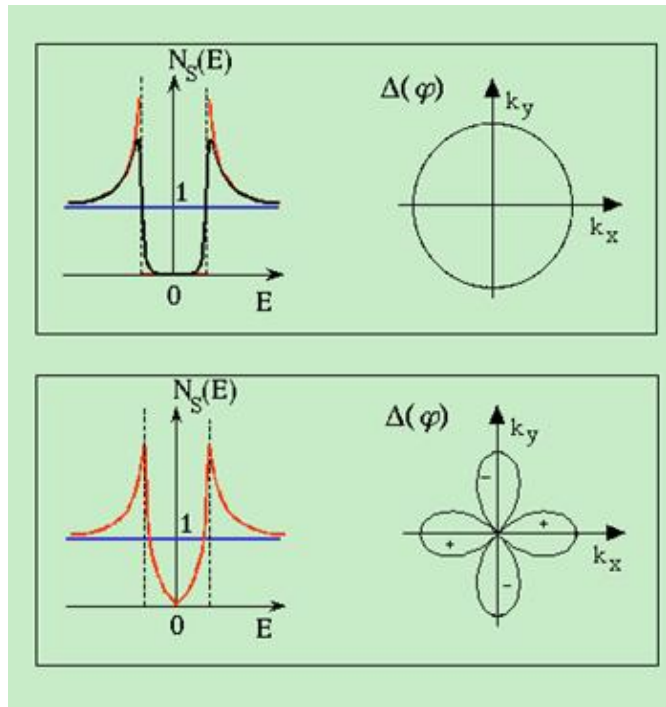
with:

$$f(\epsilon, T) = \left(1 + \exp \left[\sqrt{\epsilon^2 + \Delta(T)^2} / k_B T \right] \right)^{-1}$$



T-dependence of sc carrier density and sc gap

Low temperature dependence of magnetic penetration depth reflects symmetry of superconducting gap function



s – wave gap
 $\Delta(T, \varphi) = \Delta(T)$

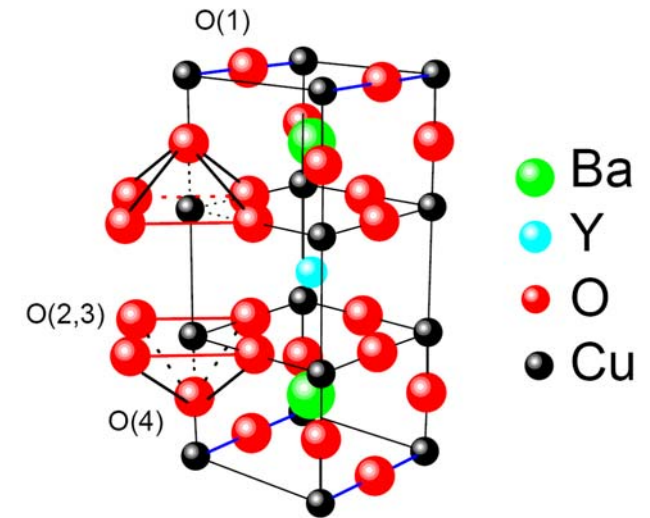
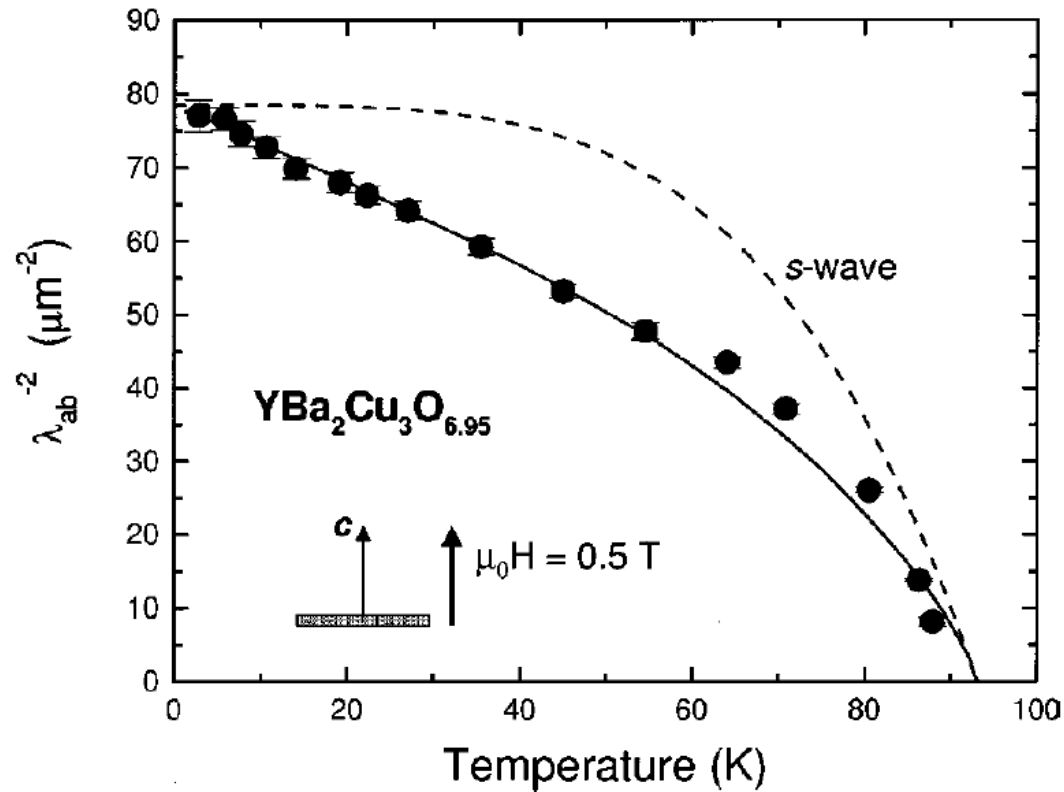
$$\lambda(T) = \lambda(0) \left(1 + \sqrt{\frac{\pi \Delta(0)}{2k_B T}} \exp[-\Delta(0) / k_B T] \right)$$

d – wave gap
 $\Delta(T, \varphi) = \Delta(T) \cos(2\varphi)$

$$\lambda(T) = \lambda(0) \left(1 + \frac{\ln 2}{\Delta(0)} T \right)$$

$$\tan \varphi = \frac{k_y}{k_x}$$

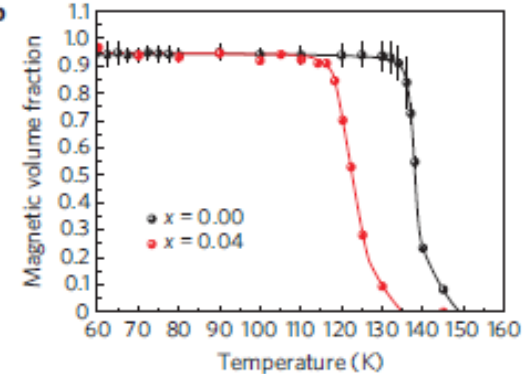
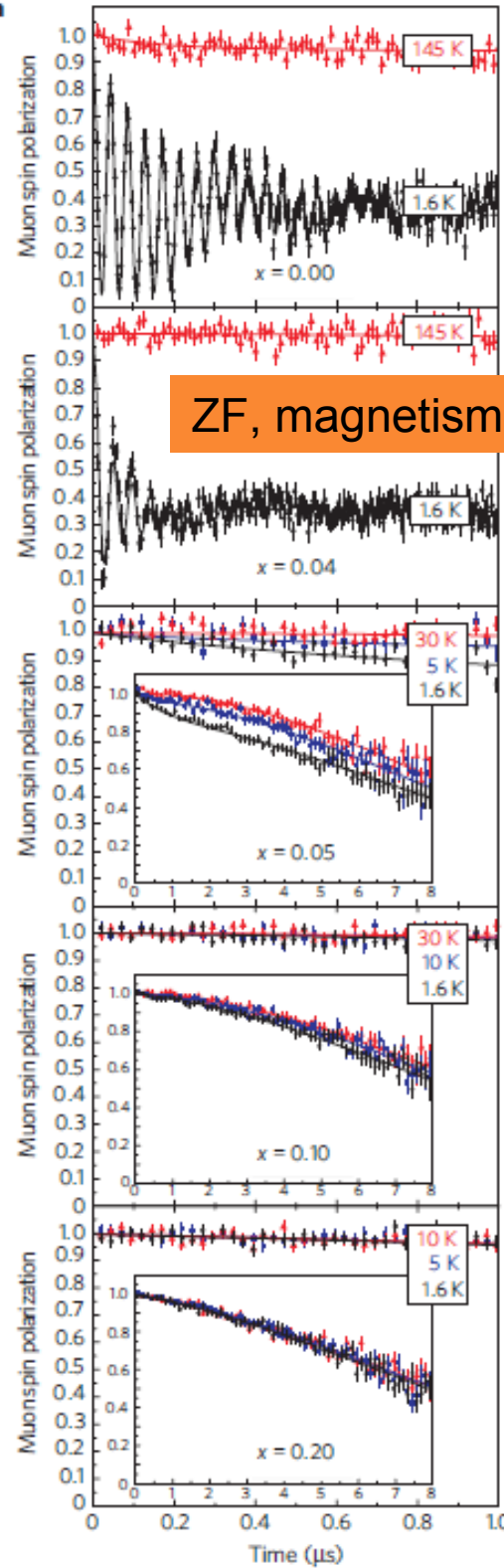
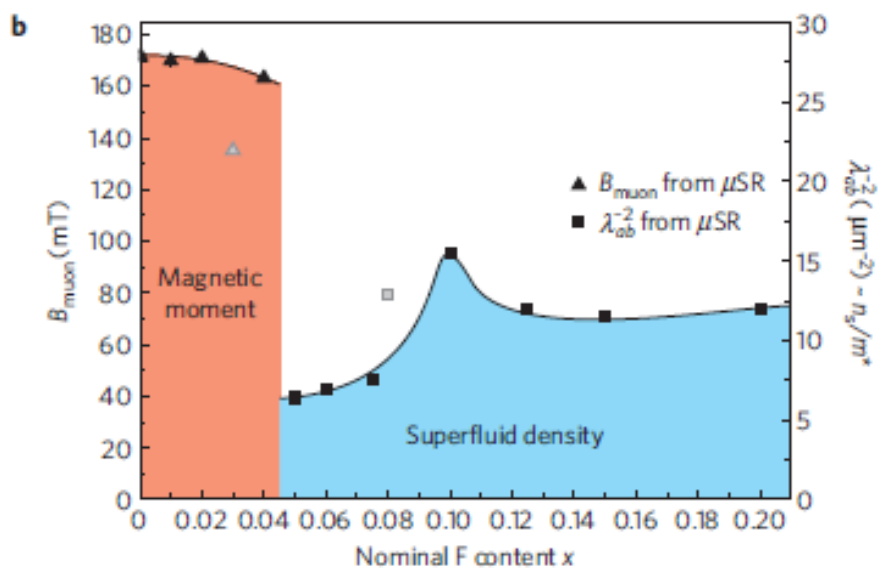
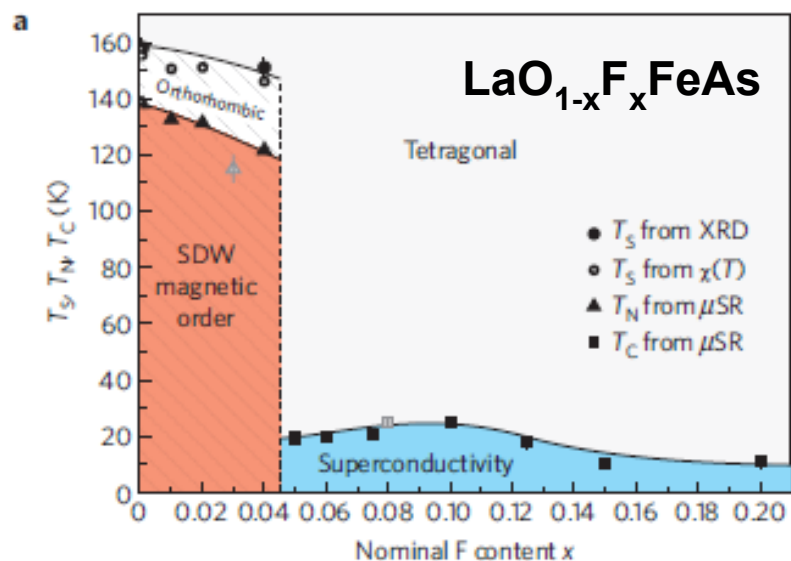
T-dependence of sc carrier density and sc gap



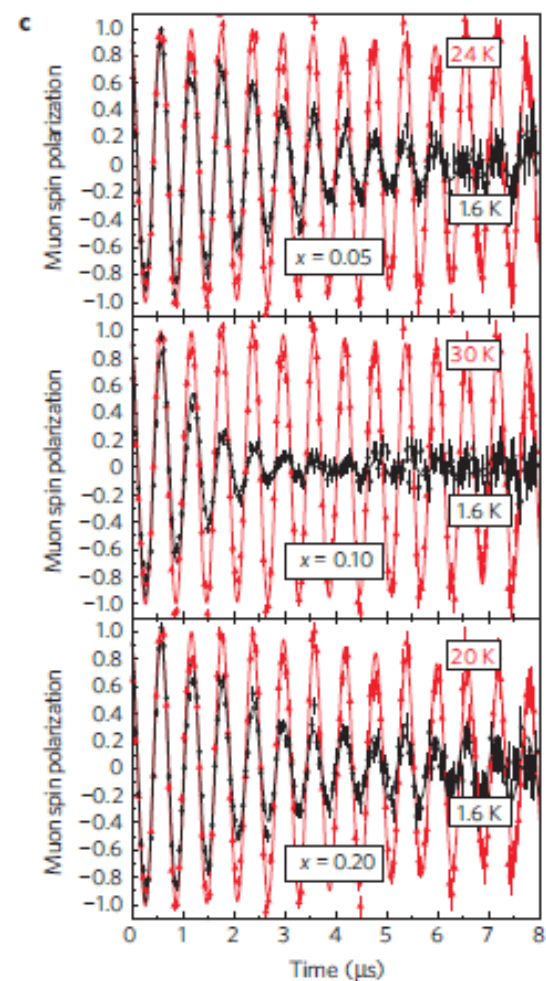
μ SR measurement: *J. Sonier et al., Phys. Rev. Lett., 72, 744 (1994)*

microwave measurement: *W.N. Hardy et al., Phys. Rev. Lett 70, 3999 (1993)*

Phase diagram

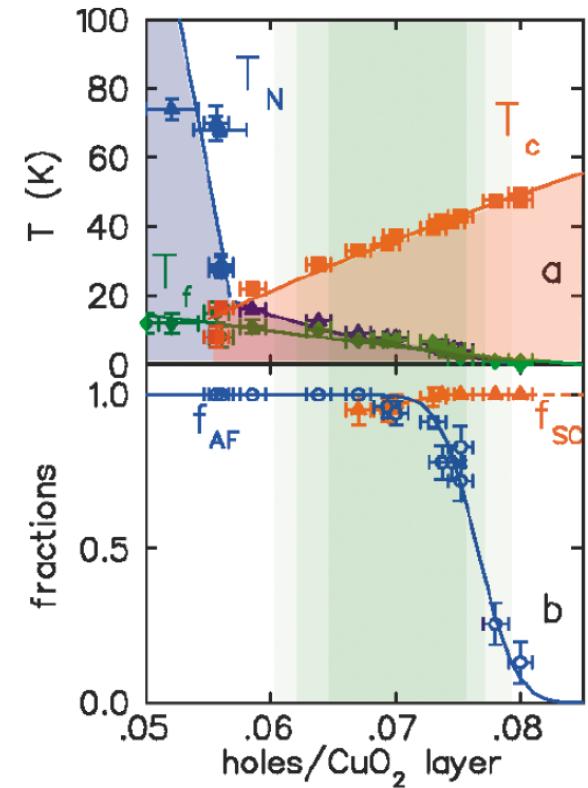
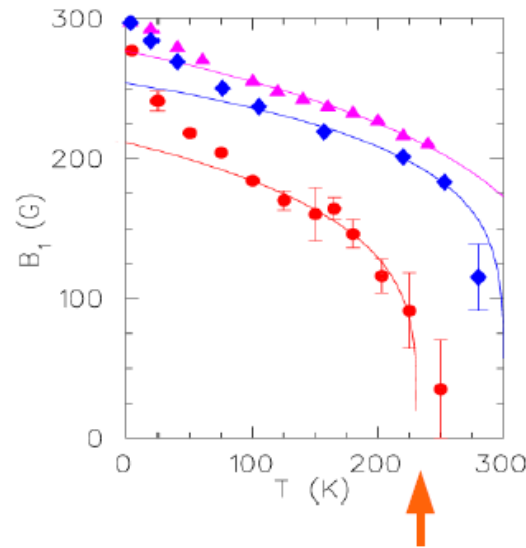
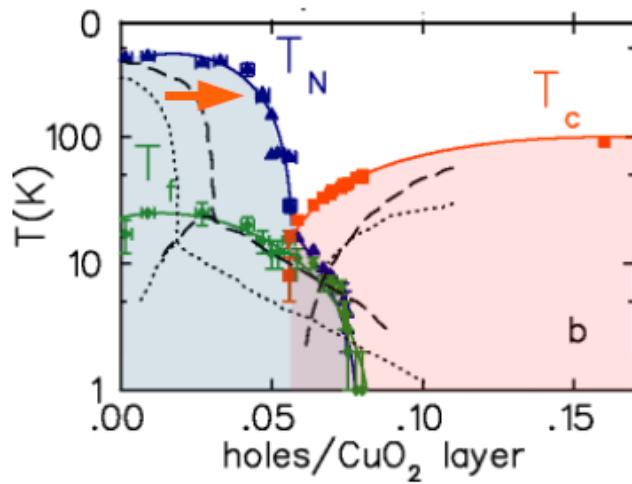


TF, superconductivity



H. Luetkens et al., Nature Materials 8, 305 - 309 (2009)

Coexistence of magnetic and sc order: $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$



Magnetism from ZF measurements:

$$A_z(t) = a_L G_z(t) + a_T G_x(t) \cos(\gamma_\mu |\vec{B}_1| t)$$

$$a_L + a_T = a_{ZF}$$

\vec{B}_1 : local field

for a homogeneous magnetic sample:

$$\frac{a_L}{a_{ZF}} = \frac{1}{3} \quad \text{and} \quad \frac{a_T}{a_{ZF}} = \frac{2}{3}$$

if only part of the sample is magnetic

$$\frac{a_L}{a_{ZF}} > \frac{1}{3} \quad \text{and} \quad \frac{a_T}{a_{ZF}} < \frac{2}{3}$$

Magnetic volume fraction:

$$f_{AF} = \frac{3}{2} \frac{a_T}{a_{ZF}} = \frac{3}{2} \left(1 - \frac{a_L}{a_{ZF}}\right)$$

Superconducting volume fraction :

$$f_{SC} = \frac{a_{TF}}{a_0} \quad \text{where } a_0 \text{ is obtained at } T > T_c$$

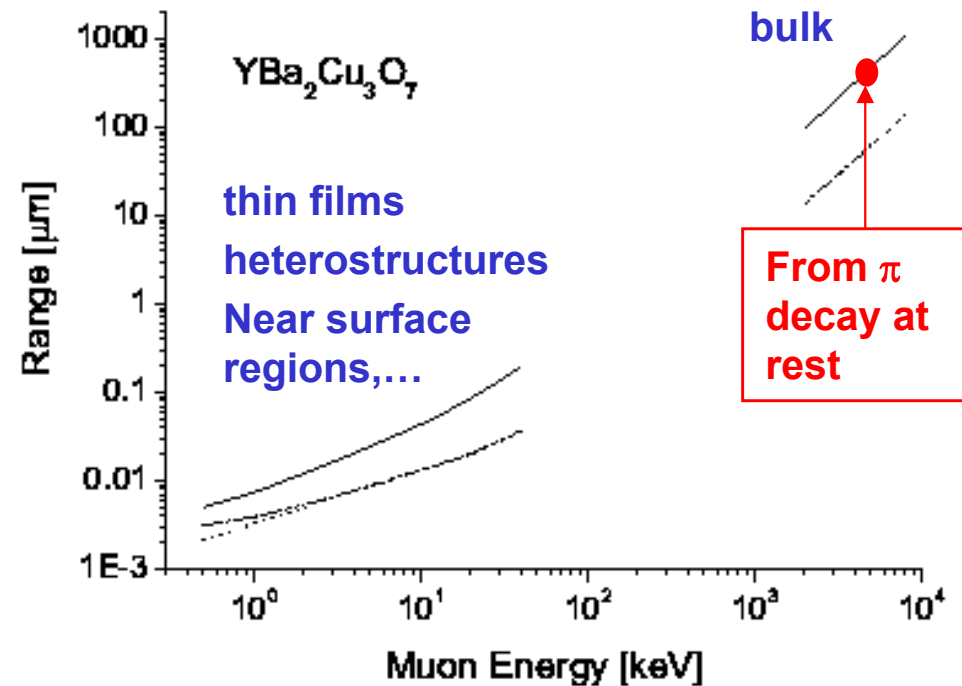
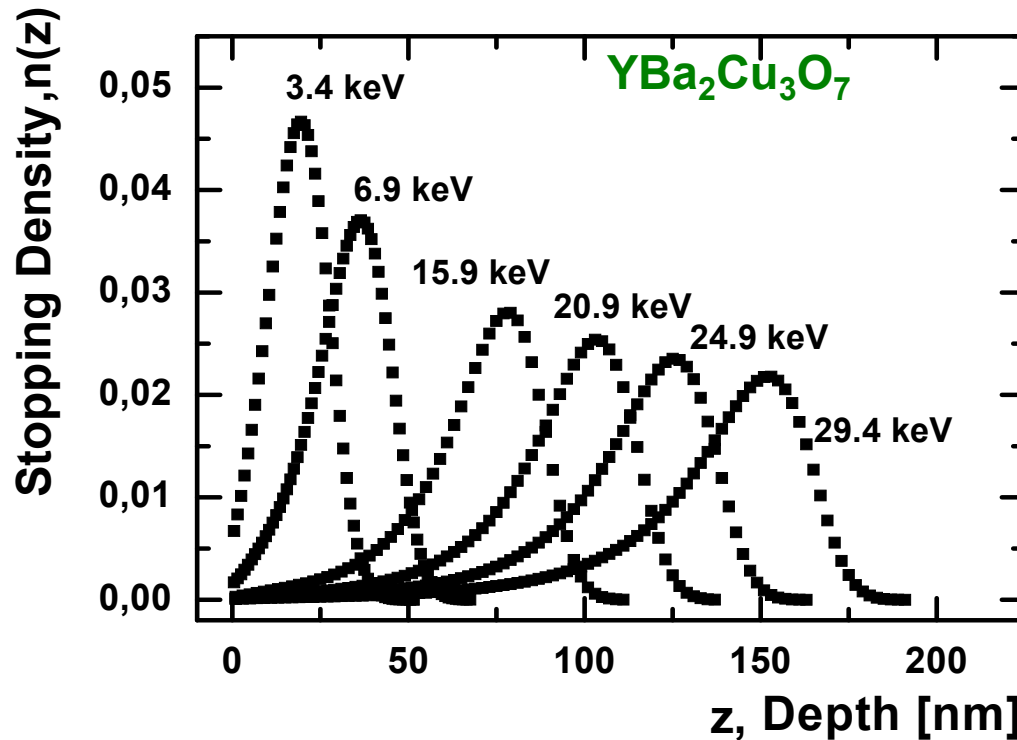
Superconductivity (vortex state) from TF

$$A_x(t) = a_{TF} e^{-\frac{(\sigma_\mu^2 + \sigma_n^2)t^2}{2}} \cos(\gamma_\mu B_\mu t)$$

$$B_\mu = \mu_0 H(1 + \chi) \quad \text{and} \quad \chi < 1$$

S. Sanna et al., *Phys. Rev. Lett.*, **93** 207001 (2004)

Implantation profiles and ranges of muons



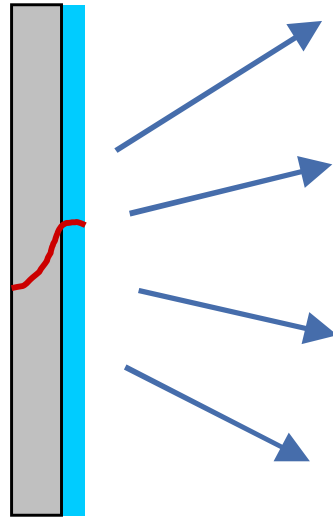
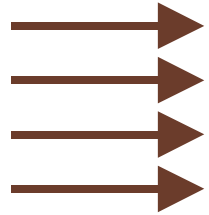
- For thin films studies we need muons with energies in the region of keV rather than MeV
- Tunable energy ($E_\mu < 30$ keV) allows depth-dependent μSR studies ($\sim 1 - 200$ nm)
- Low energy muons are a new magnetic/spin probe for thin films, multilayers, near surface regions, buried layers,...

Stopping profiles calculated with the Monte Carlo code Trim.SP *W. Eckstein, MPI Garching*

Experimentally tested: *E. Morenzoni, H. Glückler, T. Prokscha, R. Khasanov, H. Luetkens, M. Birke, E. M. Forgan, Ch. Niedermayer, M. Pleines, NIM B192, 254 (2002).*

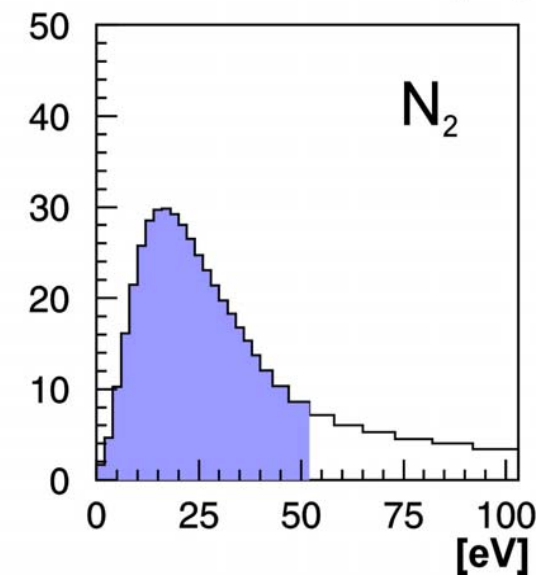
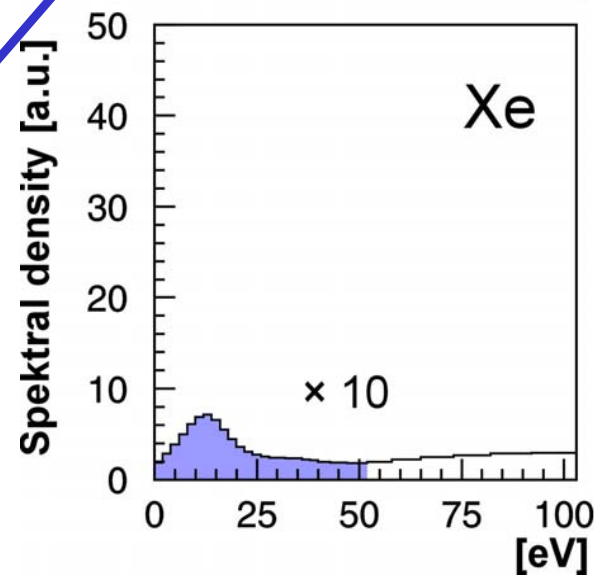
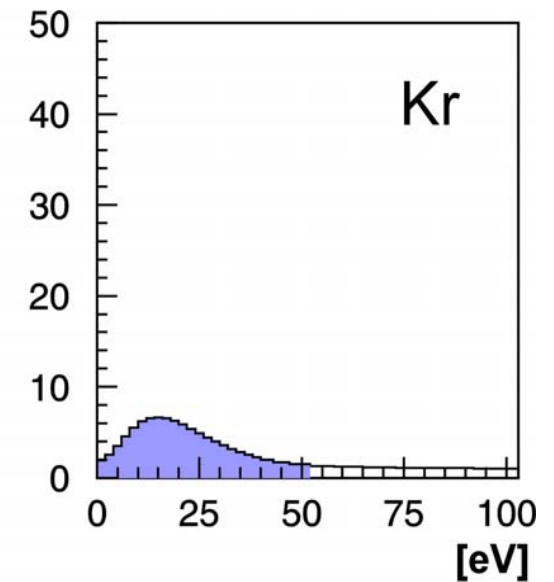
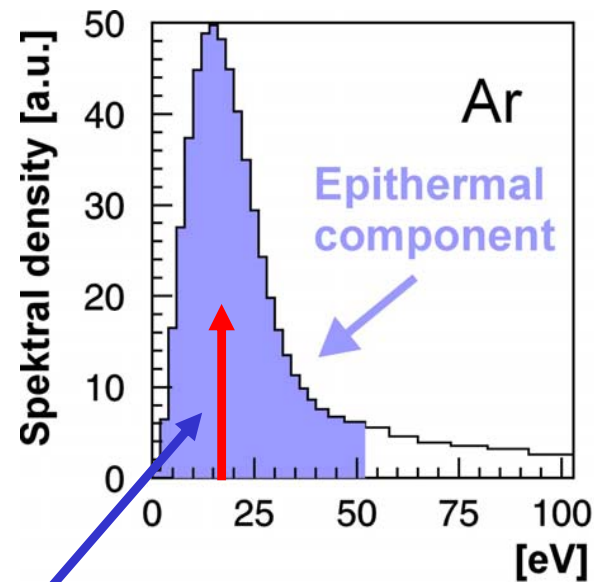
Generation of polarized epithermal muons by moderation

„Surface“
Muons
~ 4 MeV
~ 100% polarized



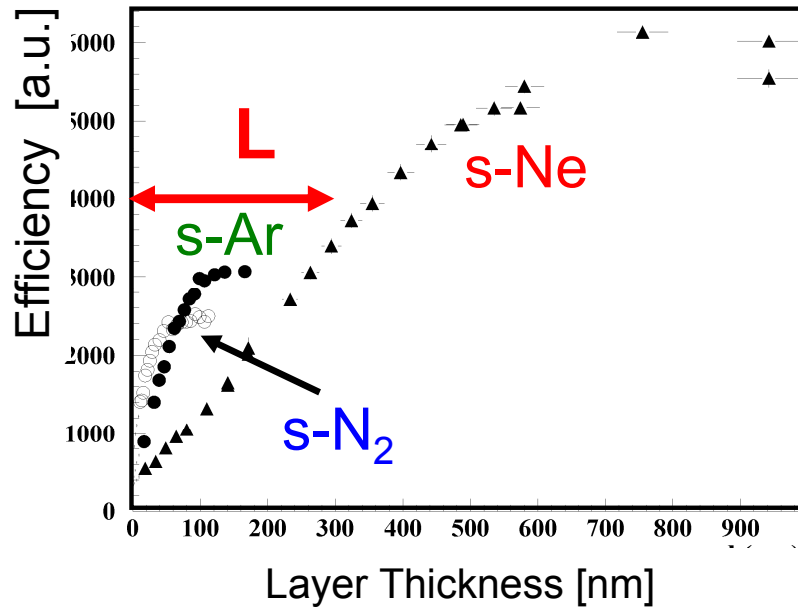
~100 μm Ag ~ 500 nm
6 K s-Ne, Ar,
s-N₂

Our source of low energy muons (E ~ 15 eV)

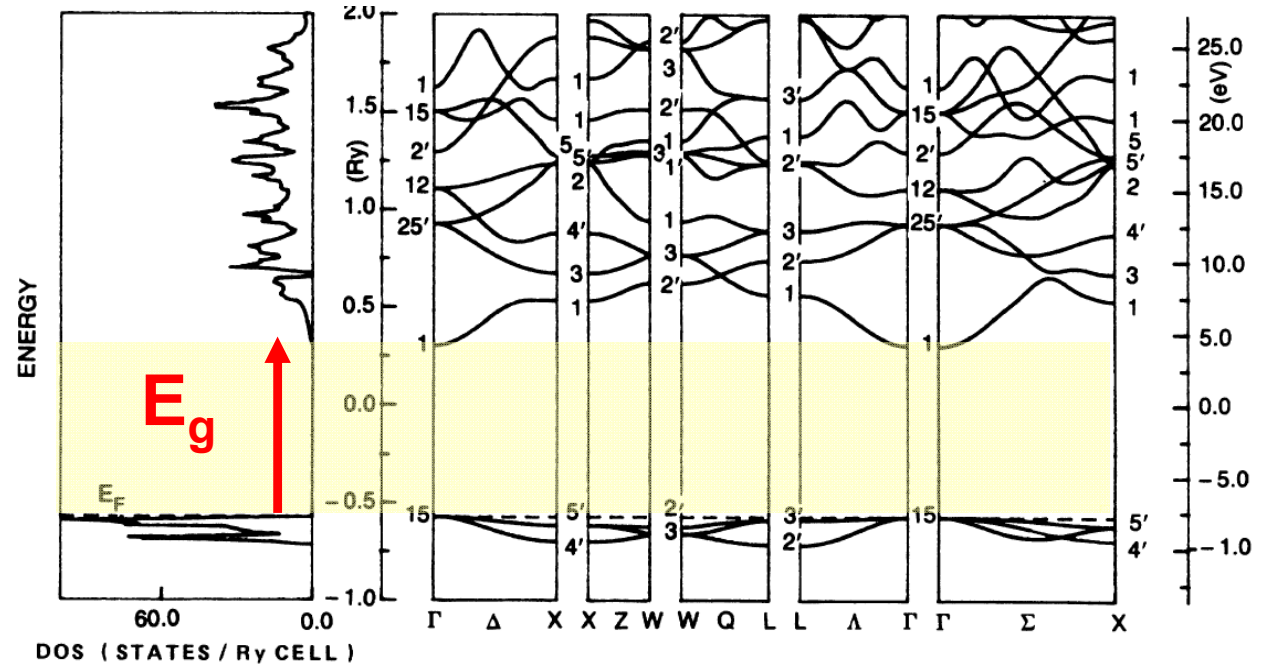


E. Morenzoni et al. J. Appl. Phys. **81**, 3340 (1997).
T. Prokscha et al. Appl. Surf. Sci. (2001)
D. Harshmann et al., Phys. Rev. B36, 8850 (1987)

Characteristics of epithermal muons



s-Ar: Density of states and band structure



Moderation mechanism:

- suppression of electronic energy loss for $E_{\mu} \approx E_g$ (wide band gap insulator)
- escape before thermalization
- large escape depth L (50-250 nm)

Moderation efficiency:

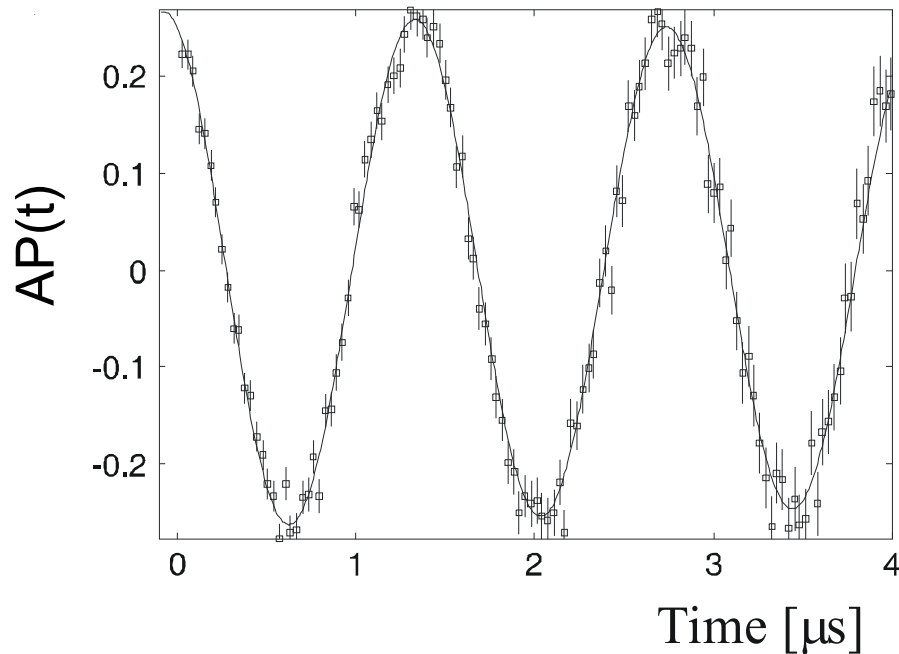
$$\varepsilon_{\mu^+} \equiv \frac{N_{\text{epith}}}{N_{4\text{MeV}}} \approx \frac{(1 - F_{\text{Mu}})L}{\Delta R} \approx 10^{-4} - 10^{-5}$$

ΔR : Range width of surface muons $\approx 100 \mu\text{m}$

Characteristics of epithermal muons

Polarization of epithermal muons is a necessary condition for their use in μ SR

Larmor precession of epithermal muons in an external field.



From the amplitude we conclude:

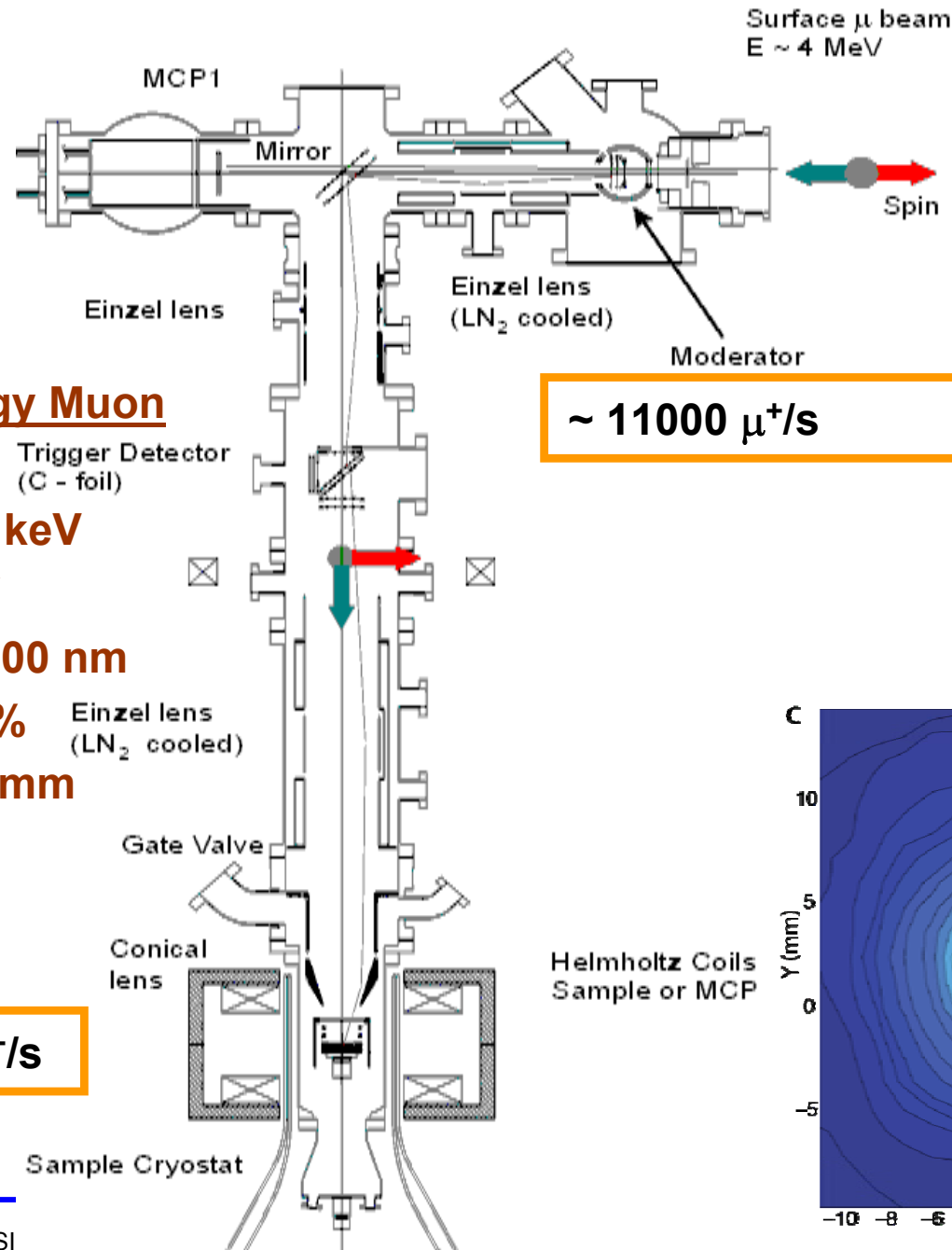
No polarization loss during moderation

(very fast slowing down time: ~ 10 ps, no depolarizing mechanism that fast)

$$\rightarrow P(0) \cong 1$$

*E. Morenzoni, F. Kottmann, D. Maden, B. Matthias,
M. Meyberg, Th. Prokscha, Th. Wutzke,
U. Zimmermann, Phys.Rev.Lett. 72, 2793 (1994).*

Low energy μ^+ beam and set-up for LE- μ SR



$\sim 1.9 \cdot 10^8 \mu^+/s$

from new μ E4 beam line

- UHV system ($\sim 10^{-10}$ mbar)

- Electrostatic transport, focussing and energy selection.

- All transport elements LN₂ cooled

$\sim 11000 \mu^+/s$

Polarized Low Energy Muon

Beam

Energy: 0.5-30 keV

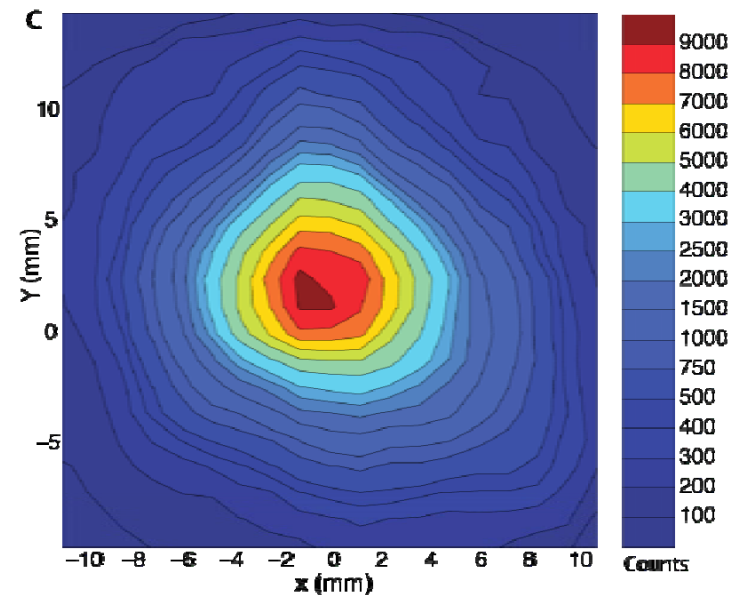
ΔE : 400 eV

Depth: $\sim 1 - 200$ nm

Polarization $\sim 100\%$

Beam Spot: 10-20 mm

$\sim 4500 \mu^+/s$



Thin film in the Meissner State

- $B_{\text{ext}} (< B_{c1}) \parallel$ surface, $T < T_c \rightarrow B=0$ in the bulk, but not at the surface
- If $\lambda \gg \xi$ electrodynamic response described by London equations:

$$1) \quad \frac{d\vec{j}}{dt} = \frac{1}{\mu_0 \lambda_L^2} \vec{E} \quad 2) \quad \text{rot} \vec{j} = -\frac{1}{\mu_0 \lambda_L^2} \vec{B} \quad (\vec{j} = -\frac{1}{\mu_0 \lambda_L^2} \vec{A})$$

From 2), $\text{rot} \vec{B} = \mu_0 \vec{j}$ and $\text{rot}(\text{rot} \vec{B}) = \text{grad div} \vec{B} - \Delta \vec{B}$ it follows

$$\Delta \vec{B} = \frac{1}{\lambda_L^2} \vec{B}$$

which in the thin film geometry $\vec{B}_{\text{ext}} \parallel \hat{x}$ gives

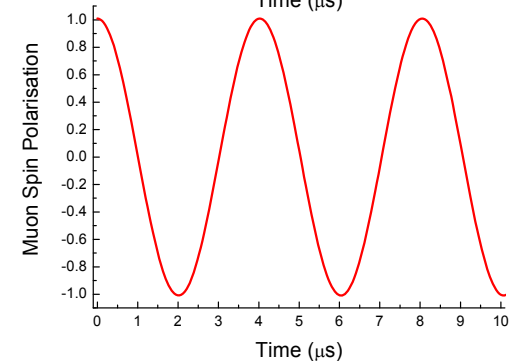
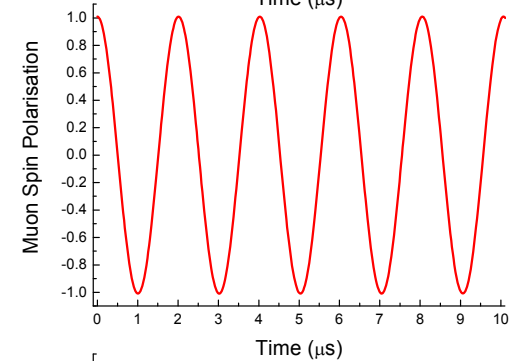
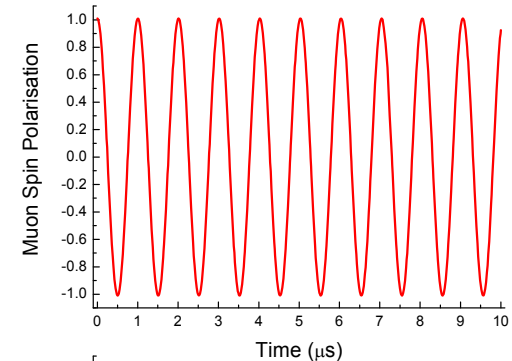
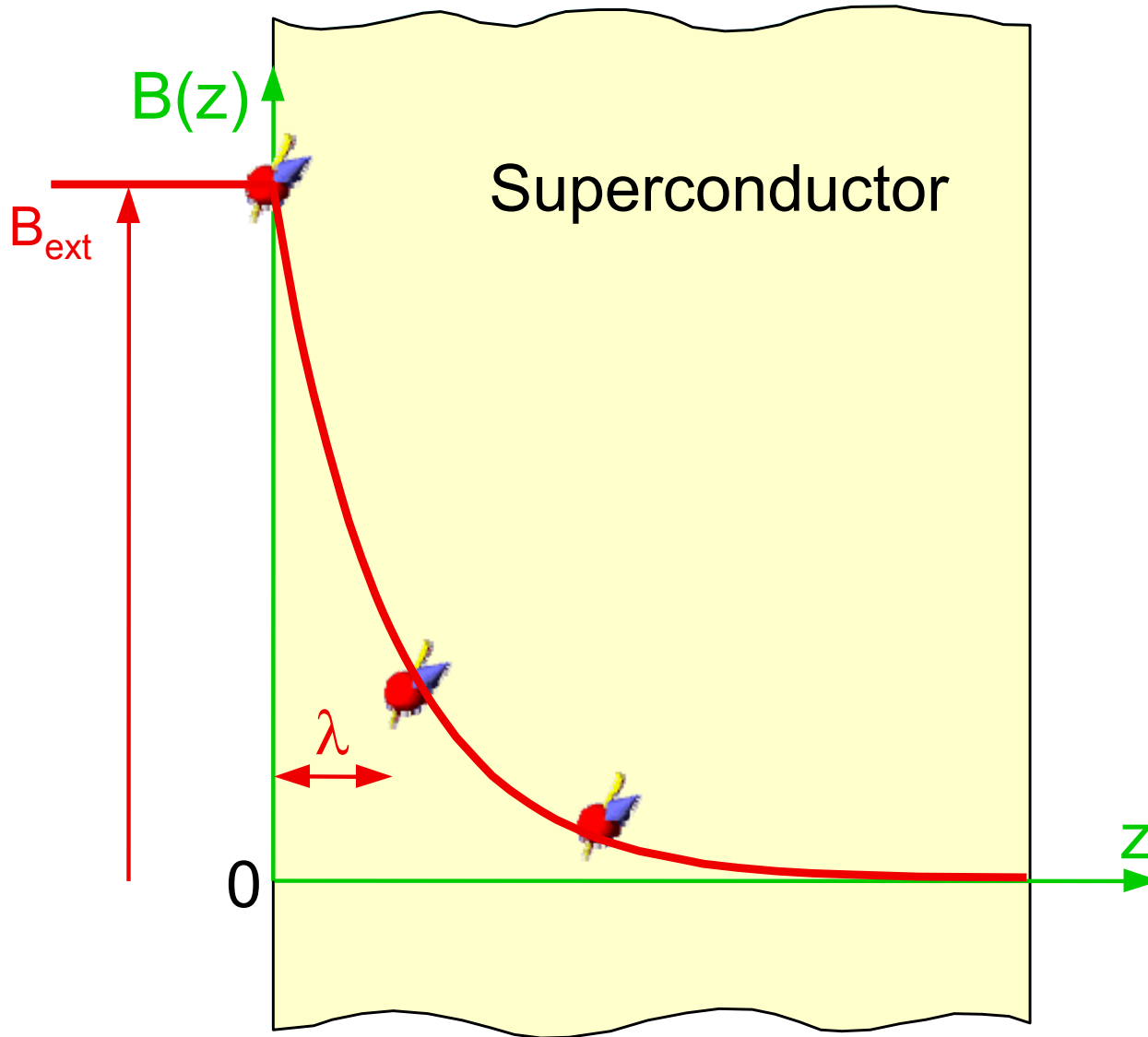
$$B(z) = B_{\text{ext}} e^{-\frac{z}{\lambda_L}} \quad \rightarrow \quad \lambda(T) = \sqrt{\frac{m^*}{\mu_0 e^2 n_s(T)}} \quad (\text{clean limit } \ell \gg \xi_0)$$

λ_L magnetic penetration depth (London)

m^* , n_s effective mass and density of superconducting carriers

*F. and H. London,
Proc. Roy. Soc. A149, 71 (1935)*

Depth dependent μ SR measurements

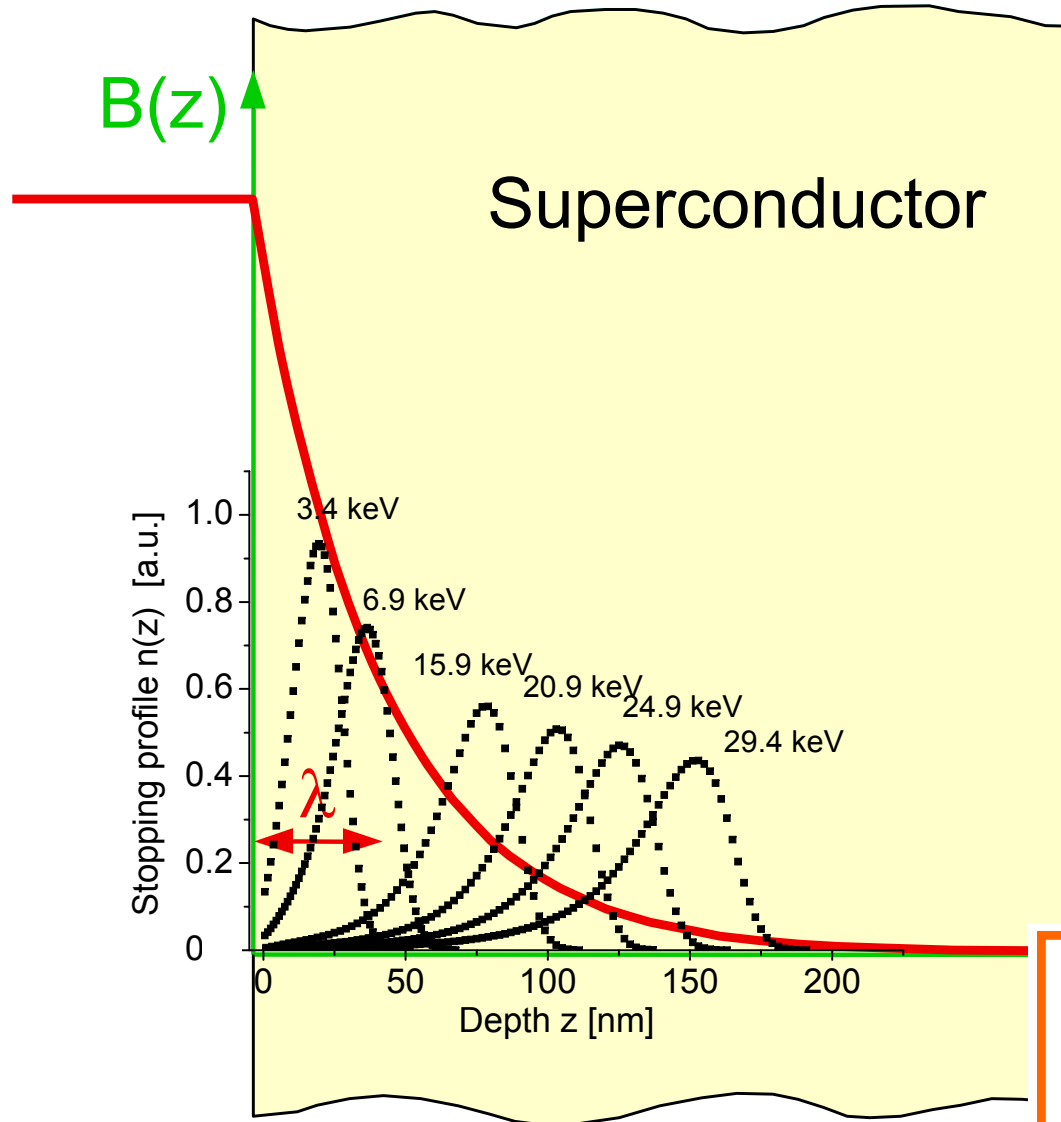


$$\omega_{\mu}(z) = \gamma_{\mu} B_{\text{loc}}(z)$$

$\langle B \rangle$ vs $\langle z \rangle \Rightarrow B(z)$

- Magnetic field profile $B(z)$ over nm scale
- Characteristic lengths of the sc λ, ξ

More precise: use known implantation profiles



$n(z, E)$: muon implantation profile for a particular muon energy E

μ SR experiment \Rightarrow magnetic field probability distribution $p(B, E)$ sensed by the muons



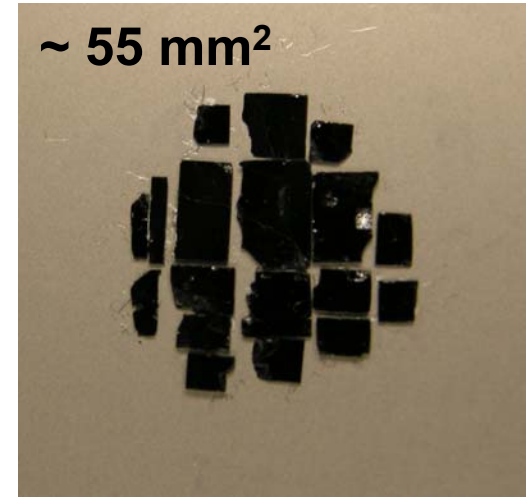
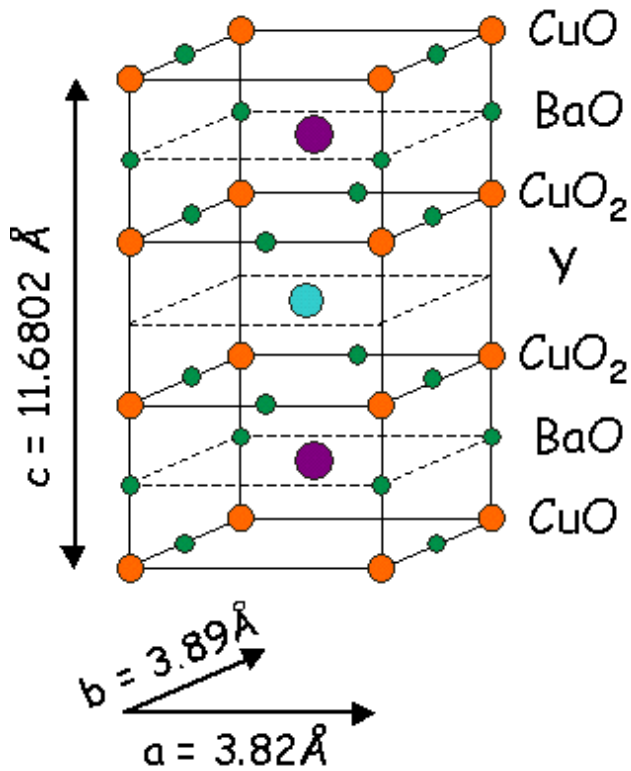
$$n(z, E) dz = p(B, E) dB$$

$$\int_0^z n(\zeta, E) d\zeta = \int_{B(z)}^{\infty} p(\beta, E) d\beta$$

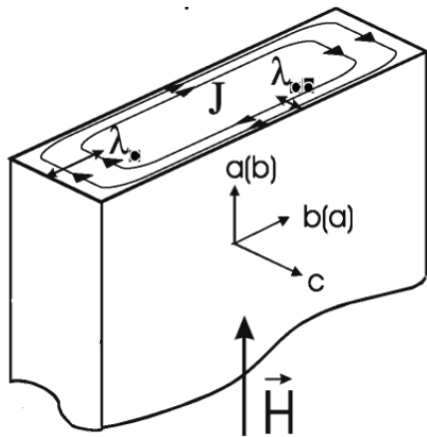
- \rightarrow Magnetic field profile $B(z)$ over nm scale
- \rightarrow Characteristic lengths of the sc λ, ξ

$\Rightarrow B(z)$

In plane anisotropy λ_a, λ_b in detwinned $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$



Detwinned (>95%) $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ crystals optimally doped
 ($T_c = 94.1 \text{ K}$, $\Delta T_c \leq 0.1 \text{ K}$)

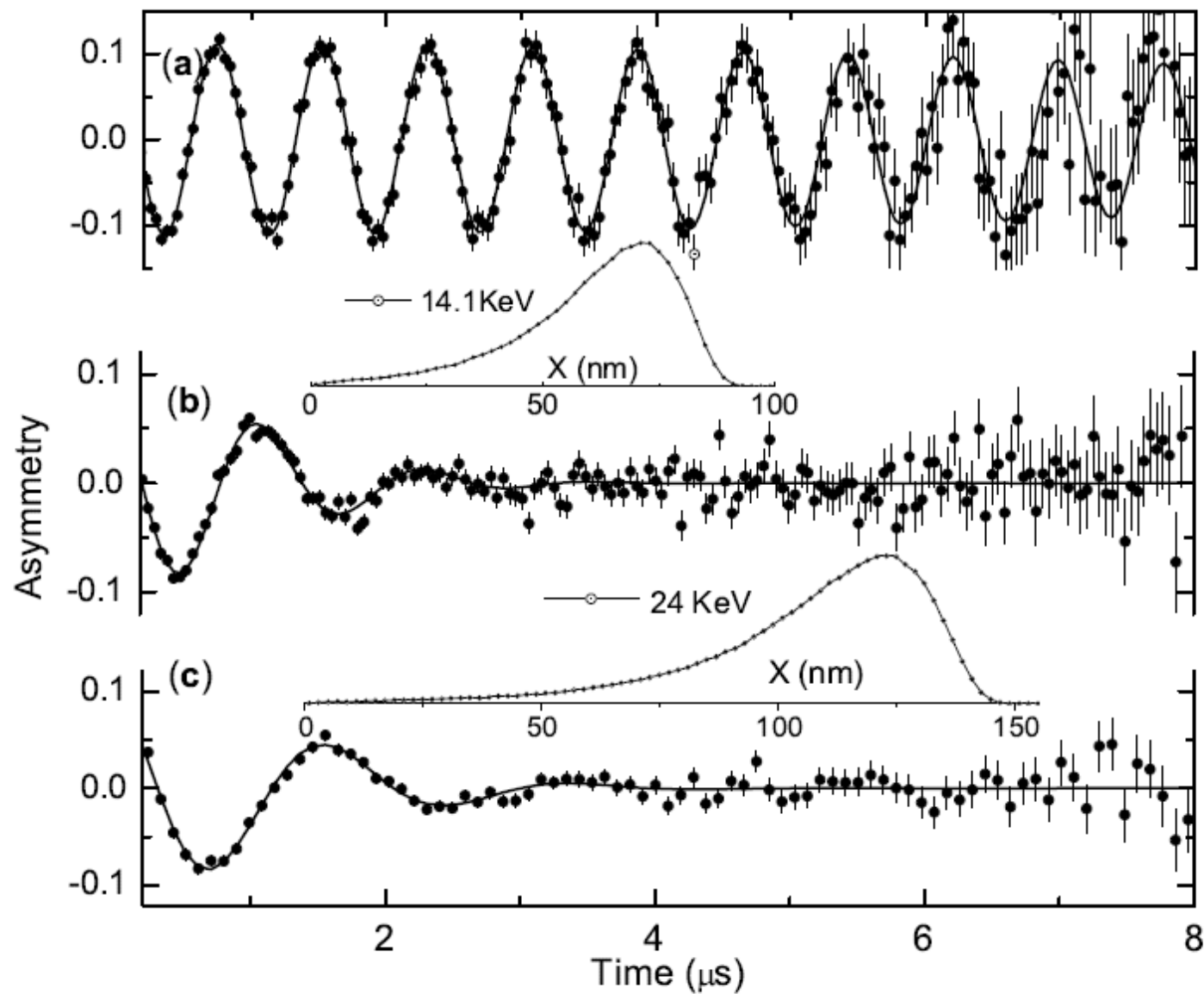


$$\vec{H}_{\text{ext}} \parallel \hat{a}\text{-axis} \rightarrow \lambda_b$$

$$\vec{H}_{\text{ext}} \parallel \hat{b}\text{-axis} \rightarrow \lambda_a$$

samples produced by R. Liang, W. Hardy, D. Bonn,
 Univ. of British Columbia

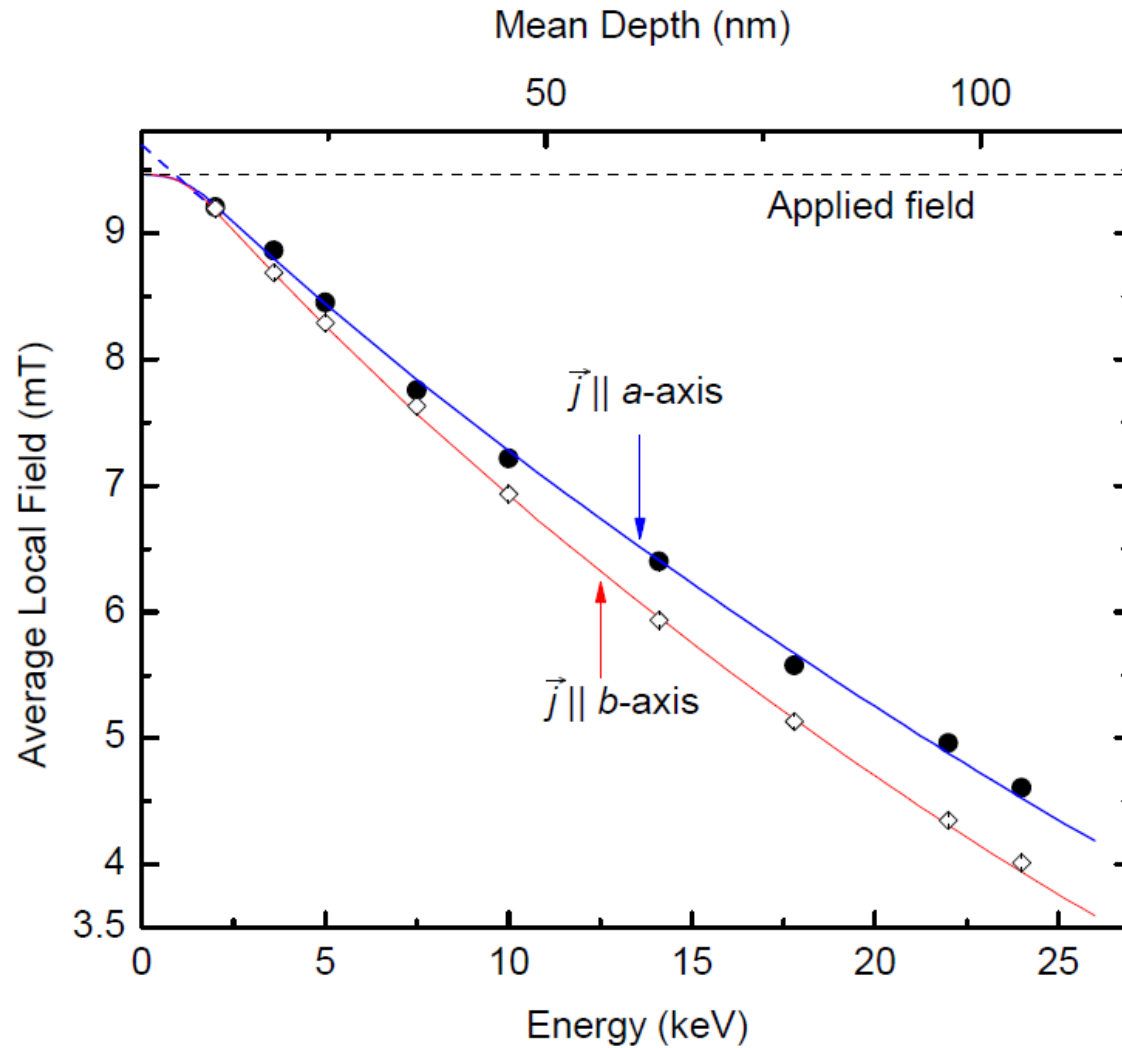
$$\vec{B}_{\text{ext}} = 9.47\text{mT} \quad \parallel \hat{a}\text{-axis}$$



T = 110 K

T = 8 K

$$\vec{B}_{\text{ext}} = 9.47 \text{ mT}, \quad T = 8 \text{ K}$$



$$B(z) = \begin{cases} B_0 \exp[-(z-d)/\lambda_{a,b}] & , z \geq d \\ B_0 & , z < d, \end{cases}$$

$$d = 10.3(4) \text{ nm.}$$

$$\mathcal{A}(t) = A \exp[-\sigma^2 t^2 / 2] \int \rho(z) \cos[\gamma_\mu B(z)t + \phi] dz$$

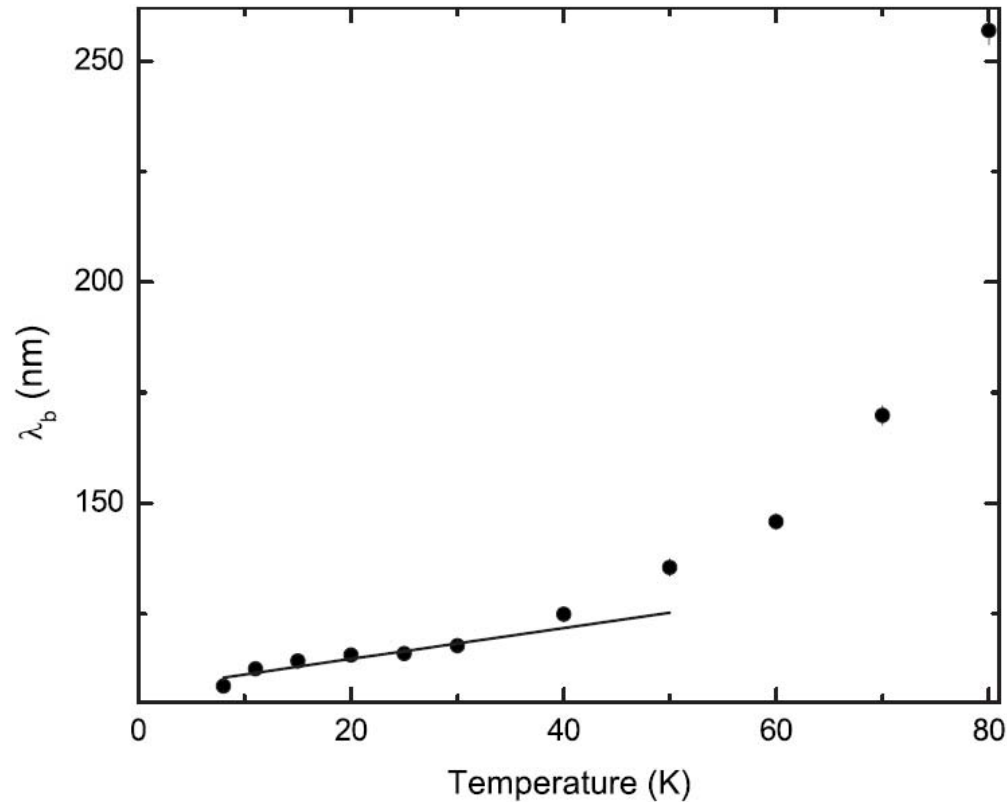
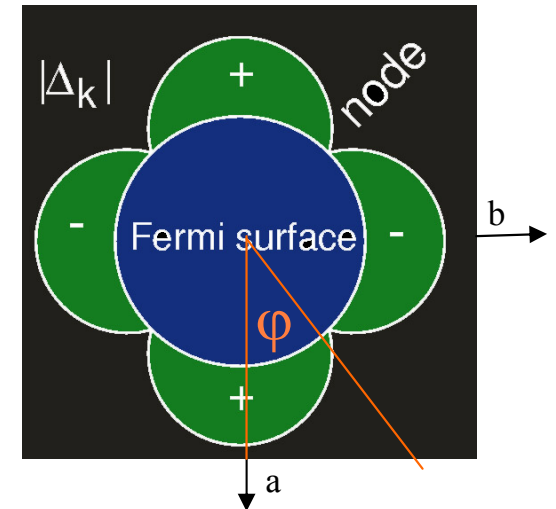


FIG. 3. Temperature dependence of the London penetration depth in an external magnetic field of 9.46 mT applied parallel to the a axis so that the shielding currents are in the b direction and parallel to the CuO chains.

d-wave superconductor



for $\Delta = \Delta(0)\cos(2\varphi)$

for low T:

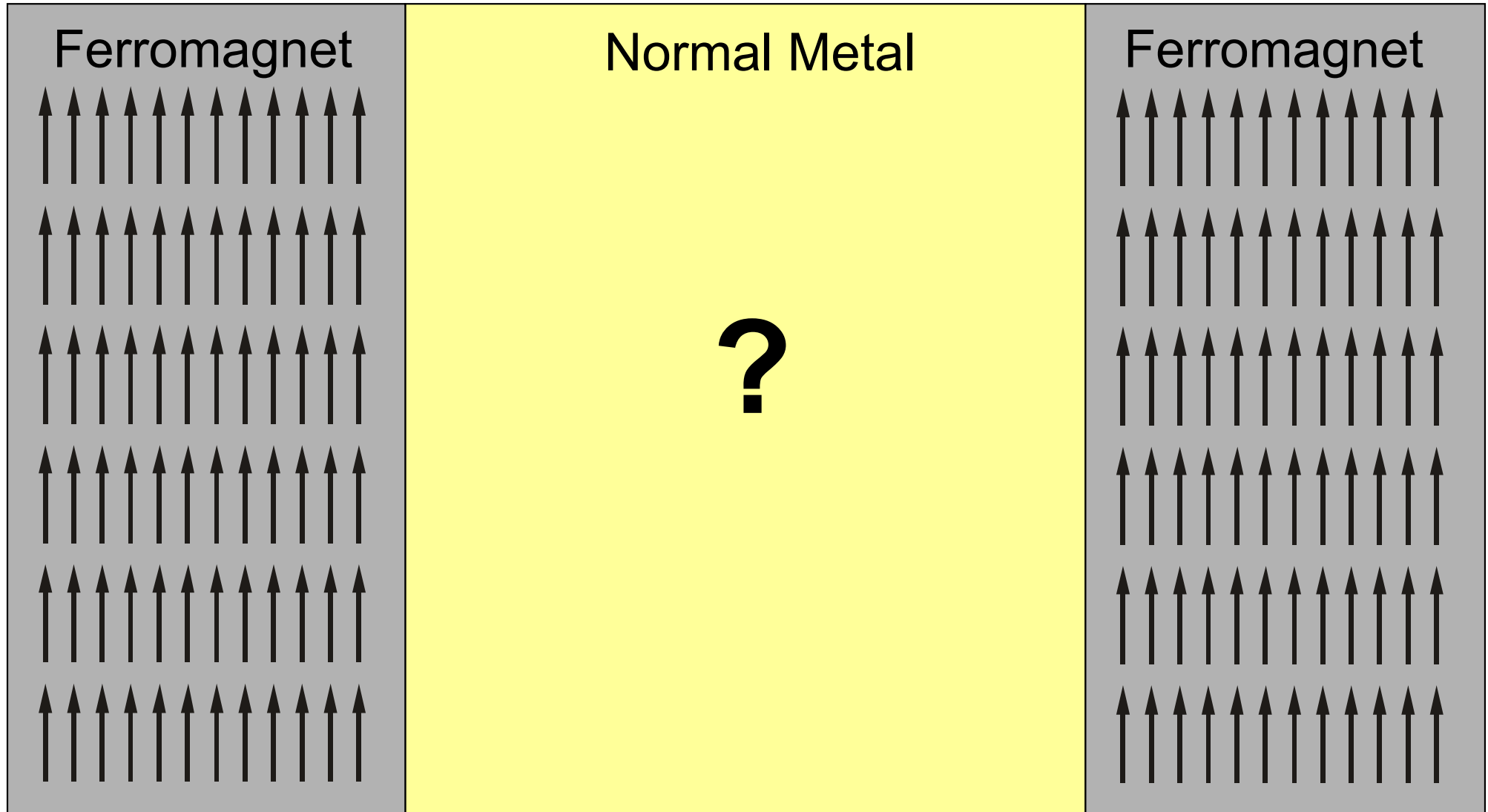
$$\lambda(T) \cong \lambda(0) \left(1 + \frac{\ln 2k_B T}{\Delta(0)} \right)$$

$$\frac{\Delta\lambda}{T} = 0.35(7) \frac{\text{nm}}{\text{K}}$$

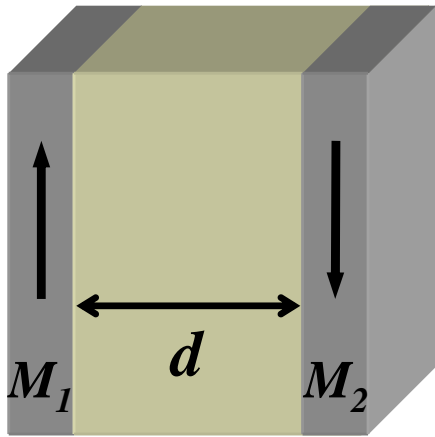
$$\lambda_a = 126 \pm 1.2 \text{ nm}, \lambda_b = 105.5 \pm 1.0 \text{ nm}, \lambda_{ab} = 115.3 \pm 0.8 \text{ nm}, \lambda_a/\lambda_b = 1.19 \pm 0.01$$

R.F. Kiefl et al. , Phys. Rev. B 81, 180502R (2010)

Magnetic multilayers (ML)

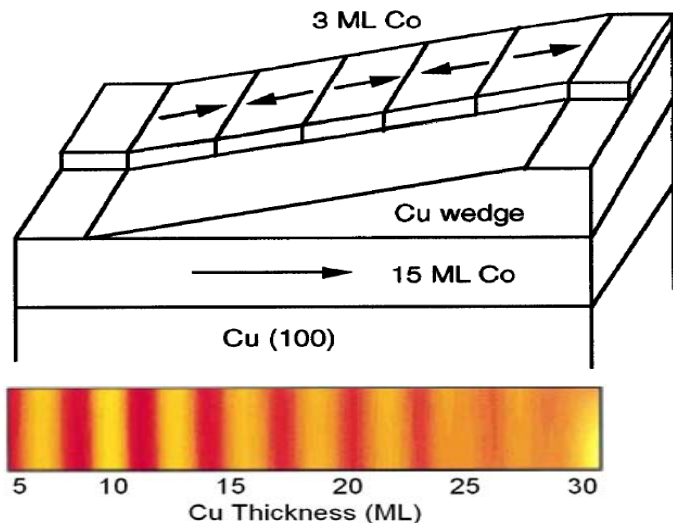


Interlayer exchange coupling in magnetic ML



$$\mathcal{H}_{\text{IEC}} = -J(d) \mathbf{M}_1 \cdot \mathbf{M}_2$$

IEC in trilayers with non-magnetic spacer:
Example: Co/Cu/Co



- IEC oscillates with spacer thickness (Model: RKKY)

$$J(d) \propto \frac{1}{d^2} \cos(qd + \phi)$$

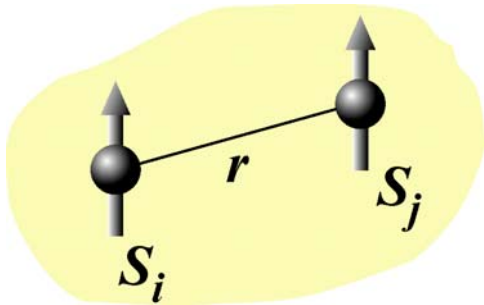
- Different techniques to probe the FM layer (polarization of secondary electrons, MOKE, ...)

→ oscillation period, coupling strength

- Muons can probe the spatially varying polarization of the nonmagnetic spacer (Spin Density Wave) mediating the coupling between the FM layers.

RKKY Model

Interaction between two moments via conduction electrons

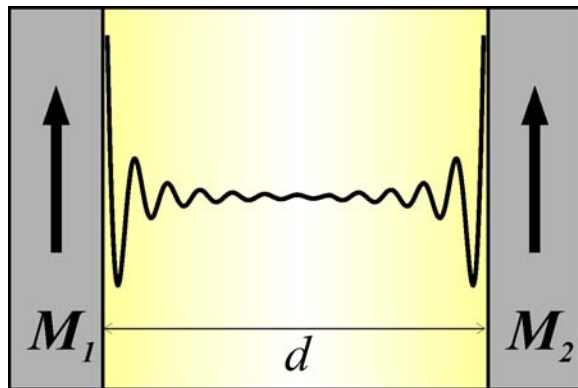


$$\mathcal{H}_{\text{RKKY}} = -J(r) \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J(r) \propto \frac{1}{r^3} \cos(2k_F r + \phi)$$

(leading term for spherical FS)

Interaction between two layers: Integrate over interfaces



$$J(d) \propto \frac{1}{d^2} \cos(qd + \phi)$$

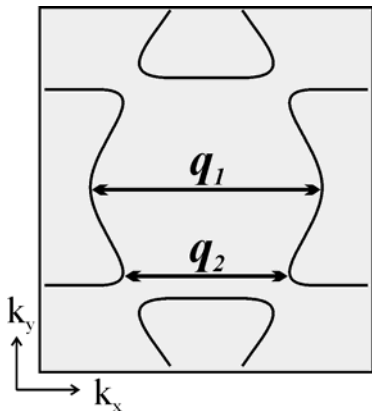
$$E = -J(d) \mathbf{M}_1 \cdot \mathbf{M}_2$$

In non-spherical Fermi surfaces, oscillations of IEC determined by critical spanning vectors

P. Bruno, C. Chappert, Phys. Rev. Lett. 67, 1602 (1991)

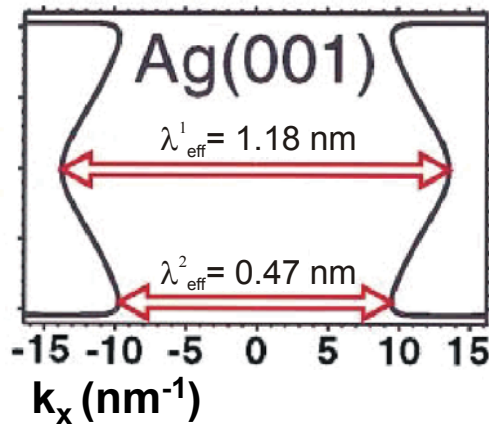
$$\rightarrow J(d) \propto \sum \frac{1}{d^{p_i}} \cos(q_i d + \phi) \quad p_i \leq 2$$

critical spanning vectors Ag: $\lambda_1^{eff} = 11.8 \text{ \AA}$, $\lambda_2^{eff} = 4.7 \text{ \AA}$

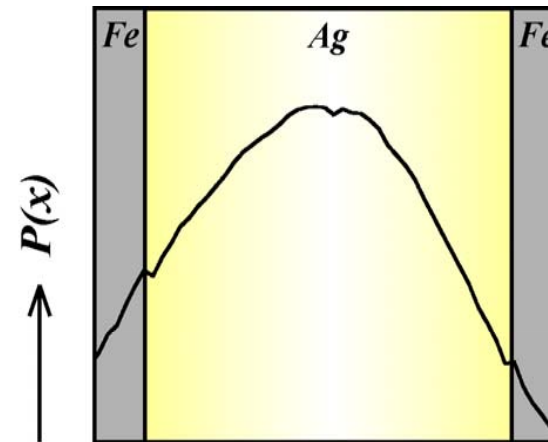


Oscillating polarization of conduction electrons

Critical spanning vectors in Ag:

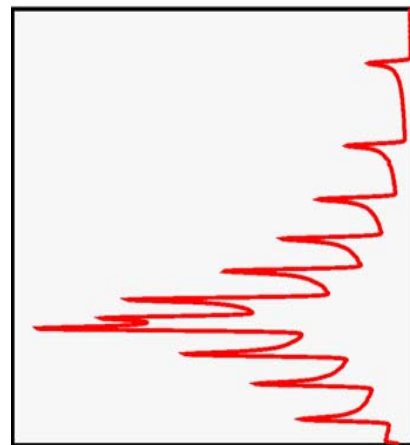


4nm 20nm 4nm

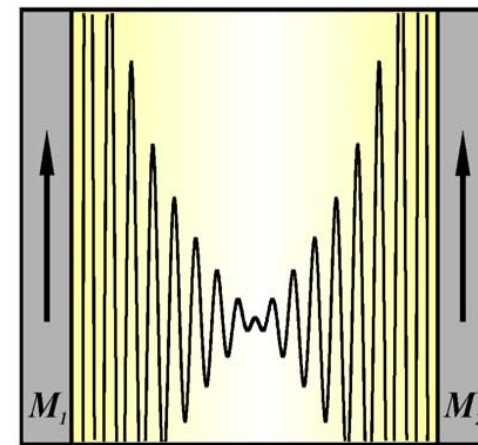


Fe/Ag/Fe
Implantation profile
of 3 keV muons.

$p(B)$ ←



→ x



$$P(x) \propto B(x)$$

We expect an oscillating spin polarization of conduction electrons $P(x)$ in Ag
→ The depth resolution of LE- μ SR cannot resolve the oscillations ($WL \sim 1$ nm or less), but the oscillating behavior if present is reflected in the field distribution $p(B)$ sensed by the muons. Turning points of oscillations should produce side bands to the B_{ext} .

H. Luetkens, J. Korecki, E. Morenzoni, T. Prokscha, M. Birke, H. Glückler, R. Khasanov, H.-H. Klauss, T. Slezak, A. Suter, E. M. Forgan, Ch. Niedermayer, and F. J. Litterst Phys Rev. Lett. **91**, 017204 (2003).

Oscillating polarization of conduction electrons

This is what is observed in the field distribution obtained by Maximum Entropy Fourier analysis.

Results:

- $P(x)$ and IEC oscillate with the same period

Attenuation of electron spin polarization:

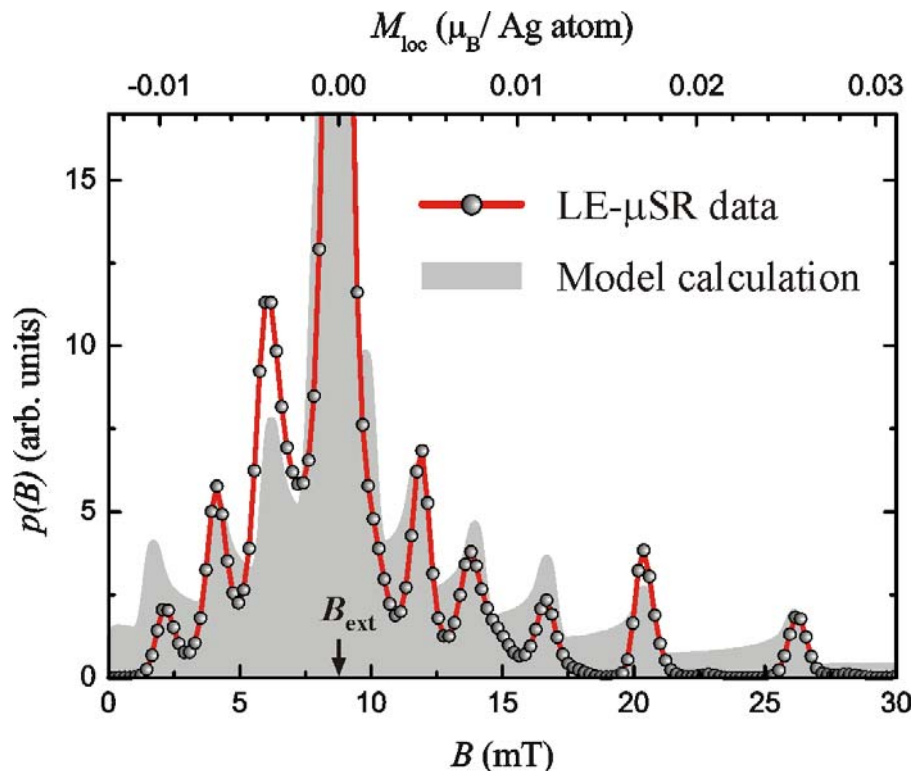
$$P(x) \propto \frac{1}{x^{0.8}}$$

significantly smaller than the one of IEC strength:

$$J(d) \propto \frac{1}{d^2}$$

(beyond RKKY: quantum well model)

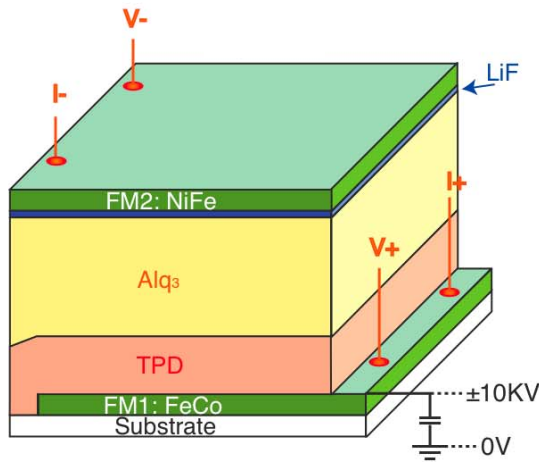
Magnetic field distribution in Ag



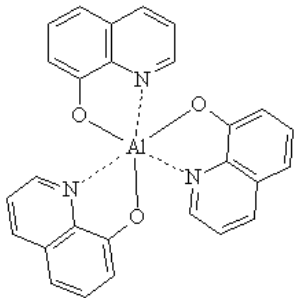
$$J(d) \propto \sum_{i=1}^2 A_i \frac{1}{d^2} \sin(q_{\perp}^i d + \theta_i)$$

$$P(x) = \sum_{i=1}^2 C_i \frac{1}{x^{n_i}} \sin(q_{\perp}^i x + \theta_i)$$

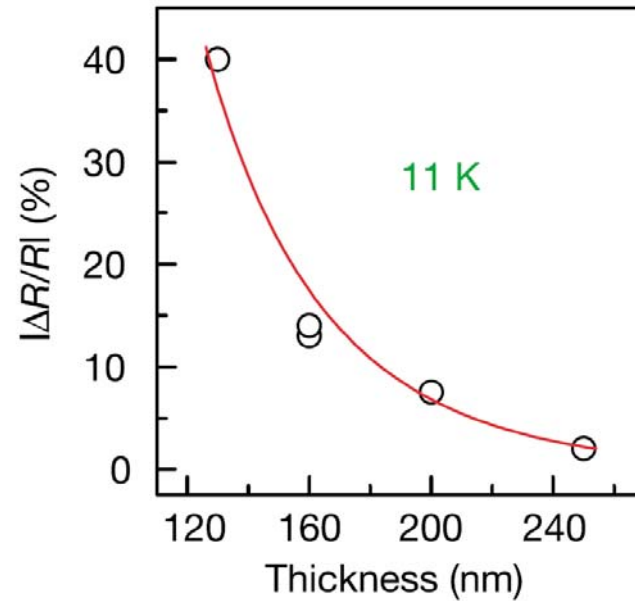
Spin Coherent Transport in Organic Spin Valves



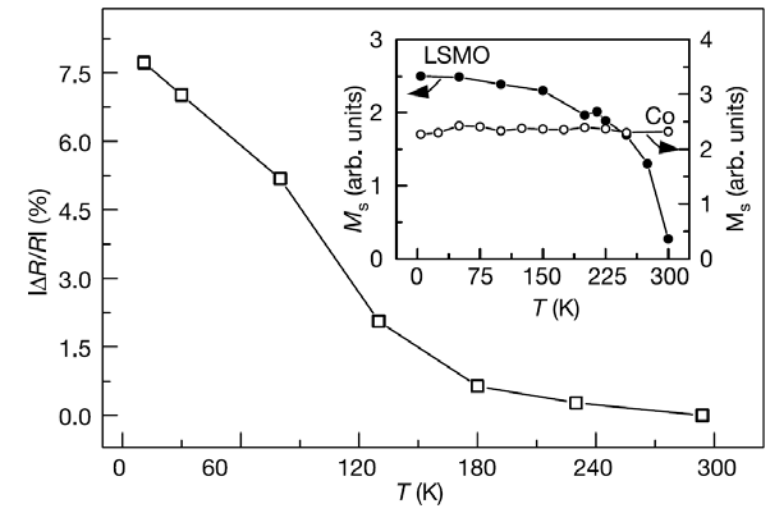
Spacer:
organic semiconductor
Alq₃: C₂₇ H₁₈ N₃ O₃Al



MR vs spacer thickness

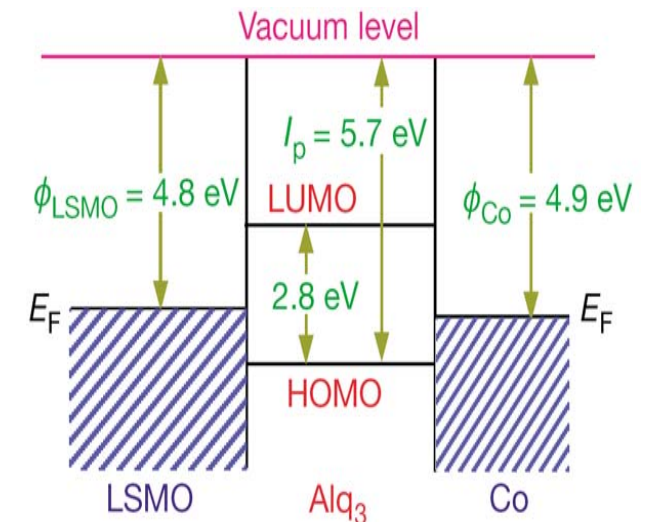


Magnetoresistance vs T

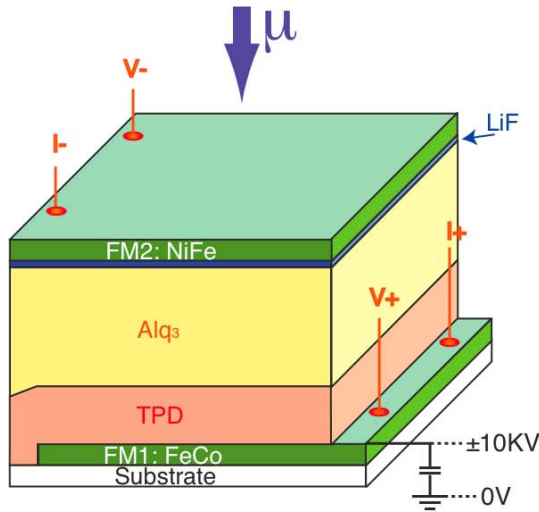


Z.H. Xiong et al., Nature **427**, 821 (2004)

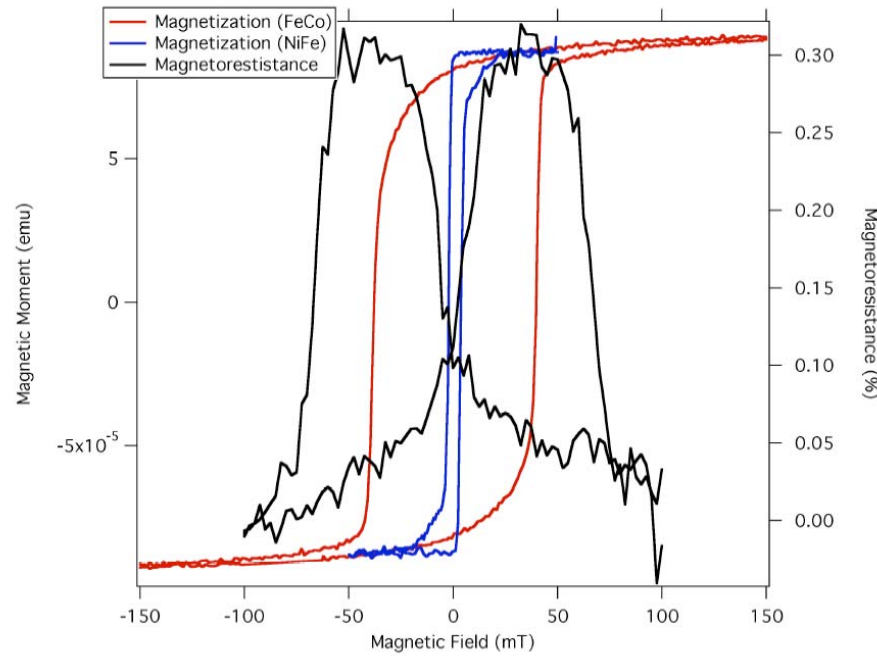
$$MR = \frac{\Delta R}{R} = \frac{R_{AP} - R_P}{R_{AP}}$$



Spin Coherent Transport in Organic Spin Valves

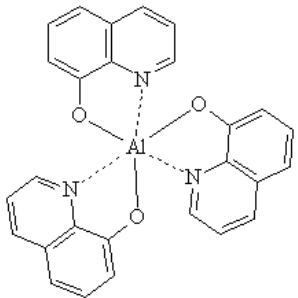


Magnetoresistance vs B

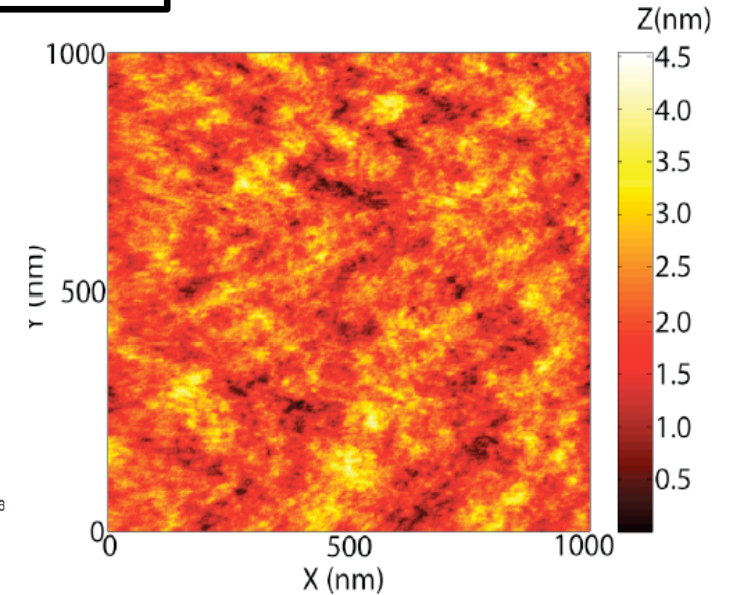
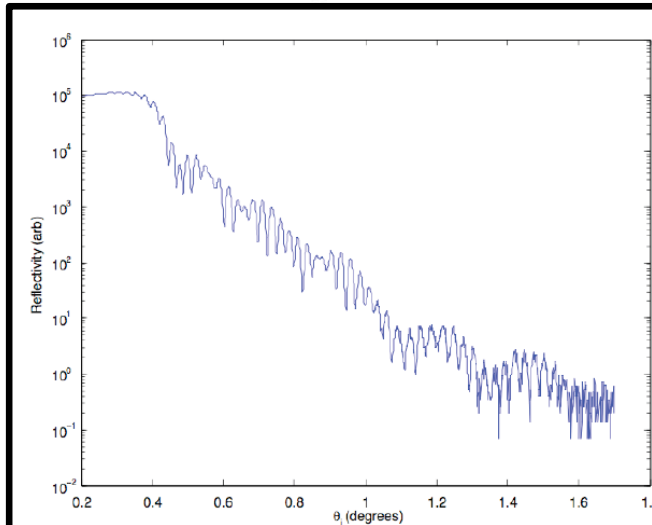


X-rays, n-reflectivity, AFM
 → very good structural quality, sharp layers and interfaces (rms < 0.5nm)

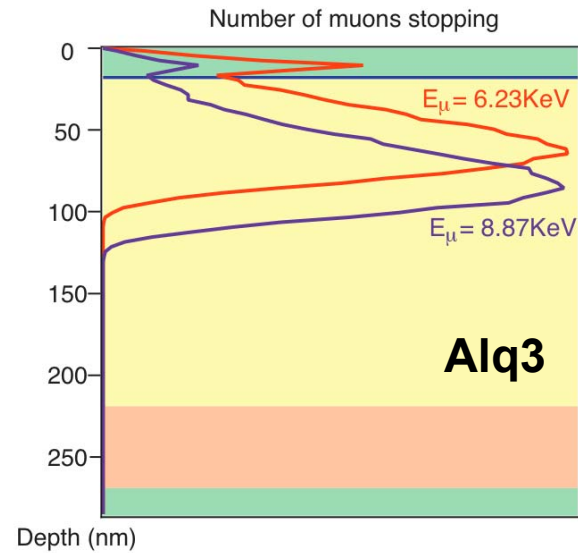
Spacer:
organic semiconductor
Alq3: $C_{27}H_{18}N_3O_3Al$



A. J. Drew, J. Hoppler, L. Schulz, F. L. Pratt, P. Desai, P. Shakya, T. Kreouzis, W. P. Gillin, A. Suter, N. A. Morley, V. K. Malik, H. Bouyanfif, K. Kim, A. Dubroka, F. Bourqui, C. Bernhard, R. Scheuermann, T. Prokscha, G. Nieuwenhuys, E. Morenzoni, Nature Materials **8**, 109-114 (2009)

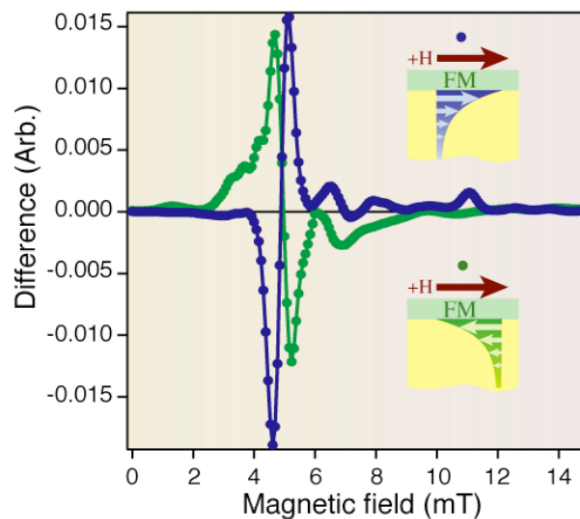


Spin diffusion length in organic spin valve

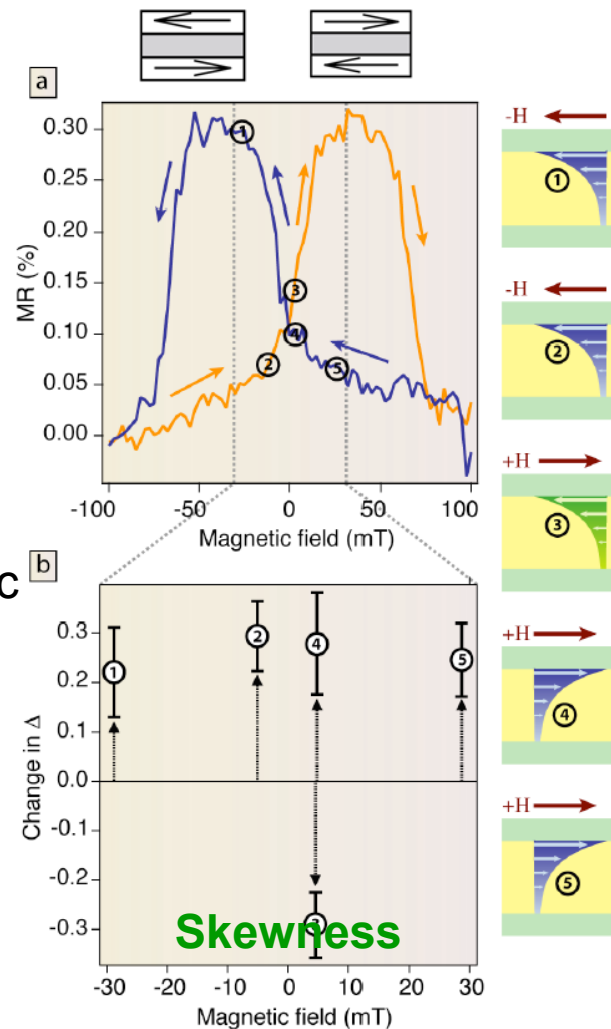


Long coherence time of injected spins $\sim 10^{-5}$ s \rightarrow measurable static field.

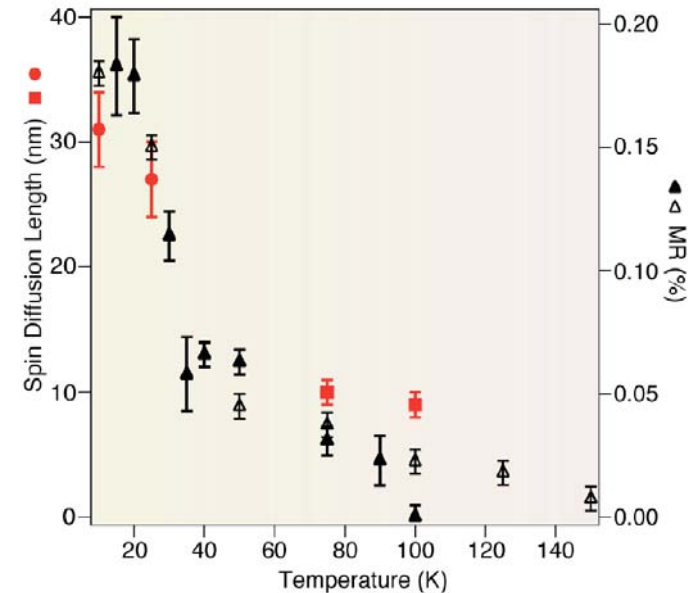
Field distribution: $I_{on} - I_{off}$



Magnetoresistance vs B



Spin diffusion length vs T correlates with Magnetoresistance



First direct measurement of spin diffusion length in a working spin valve.

