Tau Decays in Pythia 8

Philip Ilten

School of Physics, University College Dublin

Abstract

As of version 8.150 of Pythia, the isotropic decay model of τ-leptons has been replaced with sophisticated τ-lepton decay machinery. The decays and spin correlations for τ-leptons in Pythia 8 are described, including the spin correlation algorithm, the available τ-lepton production processes, the τ-lepton decay models, the user interface, and the implementation.

Keywords: Monte Carlo, tau decays, polarization, hadronic currents

1. Introduction

The role of τ-leptons in Higgs boson measurements [1] and beyond the Standard Model searches [2] is becoming increasingly important, due to the enhanced coupling of the τ-lepton in many of these physics models. Consequently, it is necessary for current Monte Carlo generators to ensure accurate modeling of τ-lepton decays.

Prior to version 8.150 of Pythia [3], τ-lepton decays in Pythia were performed using a leptonic or generic hadronic current matrix element without including spin correlations, and more sophisticated τ-lepton modeling was only possible through external packages such as Tauola [4]. Now, in Pythia version 8.150 and above, fully modeled hadronic currents with spin correlations are available, based on prior τ-lepton modeling work in Tauola and Herwig++ [5]. Currently, all known τ-lepton decays with a branching fraction greater than 0.04% are modeled.

In this review, the spin correlation algorithm used in Pythia for τ-leptons is described, as well as the available τ-lepton production mechanisms, summarized in Table 1, and τ-lepton decays, given in Table 2. An introduction to the user interface, as well as the technical implementation is given.

2. Correlations

The spin correlation algorithm used in Pythia for τ-lepton decays is based on the algorithm proposed by Collins [6] and Knowles [7], and expanded by Richardson [8]. The algorithm separates spin correlations from the hard process, parton shower, and hadronization phases of the Monte Carlo generator, while maintaining full correlations, and can be broken into the following steps.

1. The $2 \rightarrow n$ hard process is generated according to its matrix element $\mathcal{M}$.

2. One of the outgoing particles is selected and its helicity density matrix is calculated,

$$
\rho^j_{\lambda_j \lambda'_j} = \rho^1_{\kappa_1 \kappa'_1} \rho^2_{\kappa_2 \kappa'_2} \mathcal{M}_{\kappa_1 \kappa_2; \lambda_1 \ldots \lambda_n} \mathcal{M}^*_{\kappa'_1 \kappa'_2; \lambda'_1 \ldots \lambda'_n} \times \prod_{k \neq j} \mathcal{D}^k_{\lambda_k \lambda'_k}
\tag{1}
$$

and the trace is normalized. Here, $\rho^{1,2}$ are the helicity density matrices of the incoming particles with helicity $\kappa_{1,2}$, $\mathcal{M}$ is the matrix element with outgoing particle helicities λ_k, and $\mathcal{D}^k$ are the decay matrices of the outgoing particles, initialized to the identity. If the particle is from a decay, the helicity density matrix is the same as above but without ρ^2 and κ_2.

3. The selected particle is decayed using the weight,

$$
\mathcal{W} = \rho_{\lambda_0 \lambda'_0} \mathcal{M}_{\lambda_0; \lambda_1 \ldots \lambda_n} \mathcal{M}^*_{\lambda'_0; \lambda'_1 \ldots \lambda'_n} \prod_{k=1,n} \mathcal{D}^k_{\lambda_k \lambda'_k}
\tag{2}
$$

where ρ is the helicity density matrix of the decaying particle with helicity λ_0 and the decay matrix element $\mathcal{M}$.

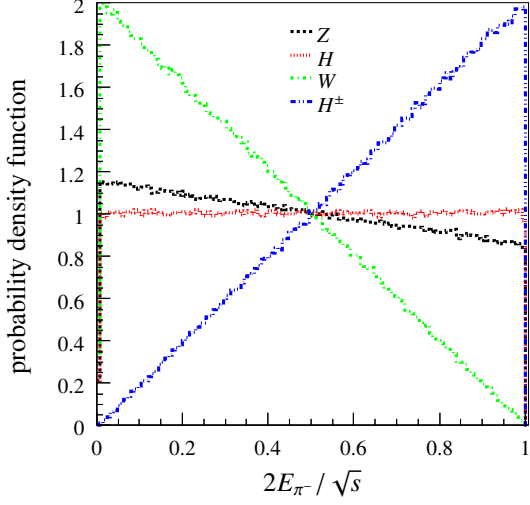4. Steps 2 through 3 are performed until a decay is reached with no unstable particles.

Figure 1: Distribution of the fractional energy of the pion from a $\tau^- \to \nu_\tau \pi^-$ decay in the rest frame of the producing boson. The τ-leptons are produced from Z (dashed black), H (dotted red), W (dash-dotted green), and $H^\pm$ (dash-dot-dotted blue) bosons.

5. The decay matrix of the last decayed particle is calculated,

$$\mathcal{D}_{\lambda_0 \lambda_0'} = \mathcal{M}_{\lambda_0; \lambda_1 ... \lambda_n} \mathcal{M}^*_{\lambda_0'; \lambda_1' ... \lambda_n'} \prod_{k=1,n} \mathcal{D}^k_{\lambda_k \lambda_k'} \qquad (3)$$

and the trace is normalized.

6. An undecayed particle from the decay above is randomly selected and steps 2 through 5 are repeated.

7. Step 6 is repeated until all unstable particles are decayed.

3. Production

Spin correlated τ-lepton decays can be generated in PYTHIA from a variety of hard processes, summarized in Table 1. The spin of the τ-lepton can also be provided from an externally generated hard process, or explicitly set for all τ-leptons or τ-leptons from a specified parent.

For electroweak processes, spin correlations are fully handled if the incoming fermions producing the γ, Z, or W are known. If the W is not produced from fermions or the fermions are unavailable, the τ-lepton is given a polarization of $\mathcal{P} = -1$, e.g. $H \to W \to \tau \nu_\tau$. For γ or Z hard processes not produced from fermions or with unavailable fermion information, the γ or Z is assumed to be unpolarized, e.g. $H \to ZZ \to 4\tau$.

Because the Higgs is spin zero, the production information is not necessary for calculating τ-lepton spin correlations. However, the type of Higgs must be known. Currently, spin correlations of τ-leptons produced from the $C\mathcal{P}$-even (H, h^0, H^0), $C\mathcal{P}$-odd (A^0), or charged ($H^\pm$) Higgses are calculated.

The τ-lepton can also be produced from a variety of B and D meson decays. For these decays, the constituent quarks of the meson are approximated, and the spin correlations are calculated using an $f\bar{f}' \to W \to f\bar{f}'$ matrix element. For any τ-lepton from an unknown process, the τ-lepton is assumed to be unpolarized.

The slope of the energy distribution for pions from $\tau^- \to \nu_\tau \pi^-$ decays in the rest frame of the parent bosons is proportional to the average τ-lepton polarization. In Figure 1 this distribution, from PYTHIA simulation, is given for τ-leptons produced from electroweak and Higgs processes. Note the the opposite polarization of τ-leptons produced from a W and $H^\pm$, the unpolarized τ-leptons produced from neutral Higgses, and the slight polarization of τ-leptons produced from Z bosons at a pp collider.

Type	Processes
electroweak	$f\bar{f} \to \gamma \to f\bar{f}, f\bar{f} \to Z \to f\bar{f},$ $f\bar{f} \to \gamma^*/Z \to f\bar{f}, f\bar{f}' \to W \to f\bar{f}',$ $Z \to f\bar{f}, W \to f\bar{f}'$
Higgs	$H \to f\bar{f}, h^0 \to f\bar{f}, H^0 \to f\bar{f}$ $A^0 \to f\bar{f}, H^\pm \to f\bar{f}'$
other	$B/D \to f\bar{f}' + X$

Table 1: Production mechanisms in PYTHIA for which full spin correlations of τ-lepton decays are automatically calculated.

4. Decays

The matrix element for the decay of the τ-lepton can be written as,

$$\mathcal{M} = \frac{g_W^2}{8m_W^2} L_\mu J^\mu \qquad (4)$$

where g_W is the $SU(2)$ coupling, m_W the W mass, L_μ the leptonic current of the τ-lepton, and J_μ a leptonic or hadronic current dependent upon the decay. The τ-lepton current L_μ is $\bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma^5) u_\tau$ where u_τ is the τ-lepton spinor, dependent upon momentum and helicity, and $\bar{u}_{\nu_\tau}$ is the τ-lepton neutrino spinor. A full list of the available J_μ currents modeled in PYTHIA is given in Table 2.

2

For the two-body decays of the τ-lepton, J_μ is fq, where f is a constant and q is the momentum of the hadron. For the three-body leptonic decays, $\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e$ and $\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu$, J_μ is of the same form as L_μ. Two hadronic three-body decay models are available, a decay via a vector resonance and a decay via a vector and scalar resonance.

Four-body τ-lepton decays in PYTHIA are implemented in four different models. The primary four-body decays, $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^-$ and $\tau^- \rightarrow \nu_\tau \pi^- \pi^- \pi^+$, are modeled using the CLEO fit. These decays can also be performed using a generic hadronic four-body model from Decker, et al. The four-body decays with kaons in the final state are calculated using a model from Finkemeier and Mirkes.

The five-body decays of the τ-lepton to pions are produced with the Novosibirsk model, a phenomenological fit of four pion production from electron-positron annihilation. The six-body decays of the τ-lepton are handled through a model proposed by Kühn and Wąs.

A comparison of the hadronic invariant mass distributions, generated with PYTHIA, between the dominant three, four, five, and six-body decays of the τ-lepton is given in Figure 2. The $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^-$ distribution was generated using the default CLEO model. The expected upwards shift for higher multiplicity decays can be seen, as well the distinct difference in shapes due to the differing propagators of the models.

5. Interface

Documentation for τ-lepton decays in PYTHIA can be found under the *Tau Decays* subsection of *Particle Decays* in the PYTHIA HTML manual. The τ-lepton decay mechanism is configured by setting the mode `sophisticatedTau` of `ParticleDecays`. This mode default is 1, where the decays are performed using the sophisticated τ-lepton decay machinery. When sophisticated decays are not necessary, or a comparison with the isotropic PYTHIA model is needed, the mode can be set to 0.

If the mode is set to 1, and a LHEF file is read, the spin of the τ-lepton will be set to the SPINUP information. However, if no SPINUP information is provided, and the τ-lepton is from a known production process, PYTHIA will automatically calculate the polarization. If the τ-lepton is from an unknown process the sophisticated decay will be performed assuming the τ-lepton is unpolarized.

It is possible to force the polarization of the τ-leptons and override either the PYTHIA calculated polarization or
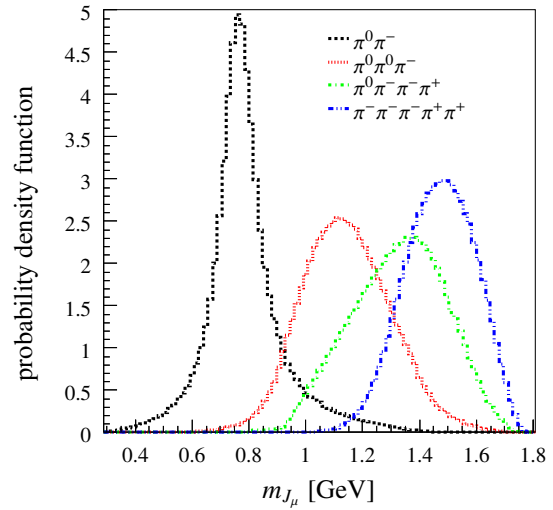

Figure 2: Distribution of the invariant mass of the hadronic current for the dominant three-body $\pi^0 \pi^-$ (dashed black), four-body $\pi^0 \pi^0 \pi^-$ (dotted red), five-body $\pi^0 \pi^- \pi^- \pi^+$ (dash-dotted green), and six-body $\pi^- \pi^- \pi^- \pi^+ \pi^+$ (dash-dot-dotted blue) decays of the τ-lepton.

the polarization from a LHEF file. If the mode is set as 2, all τ-leptons produced from a parent with an identification code specified by `tauMother` will be given a polarization set by the parameter `tauPolarization`. If mode 3 is selected, all τ-leptons will be produced with the polarization set by `tauPolarization`.

The models used to decay the τ-leptons for each channel can be changed by switching the matrix element mode of the channel. The syntax takes the form `15:channel:meMode = mode` where 15 is the τ-lepton particle identification code, `channel` specifies the decay channel number as listed under *Particle Data*, and `mode` is the new matrix element mode. For example, the default CLEO model used for the $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^-$ decay can switched to the Decker, et al. model using `15:9:meMode = 1543`. However, caution is advised in switching models, as it is possible to set a channel to use an incompatible matrix element, causing unpredicted behavior.

6. Implementation

The implementation of the τ-lepton decay machinery in PYTHIA is intended to be complete and fast, yet easily extensible for new decay models or production processes. A helicity framework is available in the PYTHIA source files `HelicityBasics` where methods for cal-

Multiplicity	Model		$\mathcal{M}$	Decay Products
2-body	single hadron		1521	π^-, K^-
3-body	leptonic		1531	$e^-\bar{\nu}_e$, $\mu^-\bar{\nu}_\mu$
	Kühn and Santamaria	[9]	1532	$\pi^0\pi^-$, K^0K^-, ηK^-
	Finkemeier and Mirkes	[10]	1533	$\pi^-\bar{K}^0$, π^0K^-
4-body	CLEO	[11]	1541	$\pi^0\pi^0\pi^-$, $\pi^-\pi^-\pi^+$
	Finkemeier and Mirkes	[12]	1542	$K^-\pi^-K^+$, $K^0\pi^-\bar{K}^0$, $K_S^0\pi^-K_S^0$, $K_L^0\pi^-K_L^0$, $K_S^0\pi^-K_L^0$, $K^-\pi^0K^0$, $\pi^0\pi^0K^-$, $K^-\pi^-\pi^+$, $\pi^-\bar{K}^0\pi^0$
	Decker, et al.	[13]	1543	$\pi^0\pi^0\pi^+$, $\pi^-\pi^-\pi^+$, $K^-\pi^-K^+$, $K^0\pi^-\bar{K}^0$, $K^-\pi^0K^0$, $\pi^0\pi^0K^-$, $K^-\pi^-\pi^+$, $\pi^-\bar{K}^0\pi^0$, $\pi^-\pi^0\eta$
	Jadach, et al.	[14]	1544	$\gamma\pi^0\pi^-$
5-body	Novosibirsk	[15]	1551	$\pi^0\pi^-\pi^-\pi^+$, $\pi^0\pi^0\pi^0\pi^-$
6-body	Kühn and Wąs	[16]	1561	$\pi^0\pi^0\pi^-\pi^-\pi^+$, $\pi^0\pi^0\pi^0\pi^0\pi^-$, $\pi^-\pi^-\pi^-\pi^+\pi^+$

Table 2: Summary of available τ-lepton decay models in Pythia 8 sorted by multiplicity. For each model the internal Pythia matrix element mode identifier ($\mathcal{M}$) is given, as well as the decays available through the model and the reference of the model. The implicit ν_τ is omitted for brevity.

culating the helicity dependent wavefunctions for particles, both fermions and bosons, is provided, as well as the corresponding γ-matrices.

Using this framework, the helicity matrix element for the decay of a τ-lepton into a pion,

$$\mathcal{M} = \bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma^5)u_\tau f_\pi q_\pi^\mu \qquad (5)$$

can be written in pseudo-code as,

$$\mathcal{M} = \sum_\mu \nu_\tau.\texttt{waveBar}(\lambda_{\nu_\tau}) * \texttt{GammaMatrix}(\mu)$$
$$*(1 - \texttt{GammaMatrix}(5))*\tau.\texttt{wave}(\lambda_\tau)*f_\pi \qquad (6)$$
$$*\texttt{GammaMatrix}(4)(\mu,\mu) * \texttt{Wave4}(q_\pi)(\mu)$$

where γ^4 provides the $+---$ metric.

Both the production matrix elements and τ-lepton decay matrix elements are written using this framework and are provided in the `HelicityMatrixElements` files of the Pythia source. For the τ-lepton decay matrix elements, it is only necessary to implement the J_μ current. The spin correlation and decays of the τ-leptons are implemented in the `TauDecays` files of the Pythia source.

7. Conclusion

Fully modeled τ-lepton decays with spin correlations are now available in Pythia, using fully extensible code. Further development of τ-lepton production mechanisms and decay models is underway, and feedback on requested features is welcome.

8. Acknowledgments

References

[1] ATLAS Collab., Tech. Rep. ATLAS-CONF-2012-160.
[2] CMS Collab., Tech. Rep. CMS-PAS-HIG-12-050.
[3] T. Sjöstrand, et al., Comput.Phys.Commun. 178 (2008) 852–867. arXiv:0710.3820.
[4] P. Golonka, et al., Comput.Phys.Commun. 174 (2006) 818–835. arXiv:hep-ph/0312240.
[5] D. Grellscheid, P. Richardson. arXiv:0710.1951.
[6] J. C. Collins, Nucl.Phys. B304 (1988) 794.
[7] I. Knowles, Comput.Phys.Commun. 58 (1990) 271–284.
[8] P. Richardson, JHEP 0111 (2001) 029. arXiv:hep-ph/0110108.
[9] J. H. Kühn, A. Santamaria, Z.Phys. C48 (1990) 445–452.
[10] M. Finkemeier, E. Mirkes, Z.Phys. C72 (1996) 619–626. arXiv:hep-ph/9601275.
[11] CLEO Collab., Phys.Rev. D61 (2000) 012002. arXiv:hep-ex/9902022.
[12] M. Finkemeier, E. Mirkes, Z.Phys. C69 (1996) 243–252. arXiv:hep-ph/9503474.
[13] R. Decker, et al., Z.Phys. C58 (1993) 445–452.
[14] S. Jadach, et al., Comput.Phys.Commun. 76 (1993) 361–380.
[15] A. Bondar, et al, Comput.Phys.Commun. 146 (2002) 139–153. arXiv:hep-ph/0201149.
[16] J. H. Kühn, Z. Wąs, Acta Phys.Polon. B39 (2008) 147–158. arXiv:hep-ph/0602162.