Fixed-Order Corrections in Sector Showers

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CoE Seminar
University of Jyväskylä
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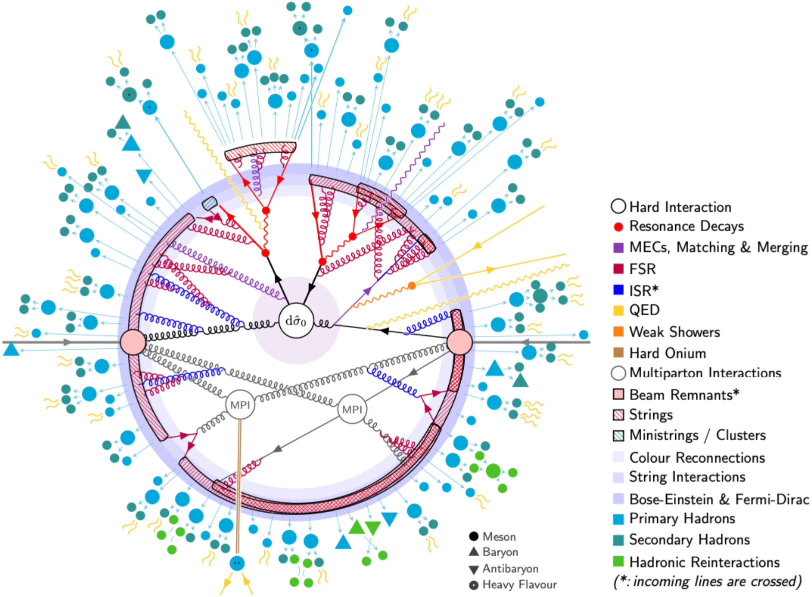
Event generation in PYTHIA



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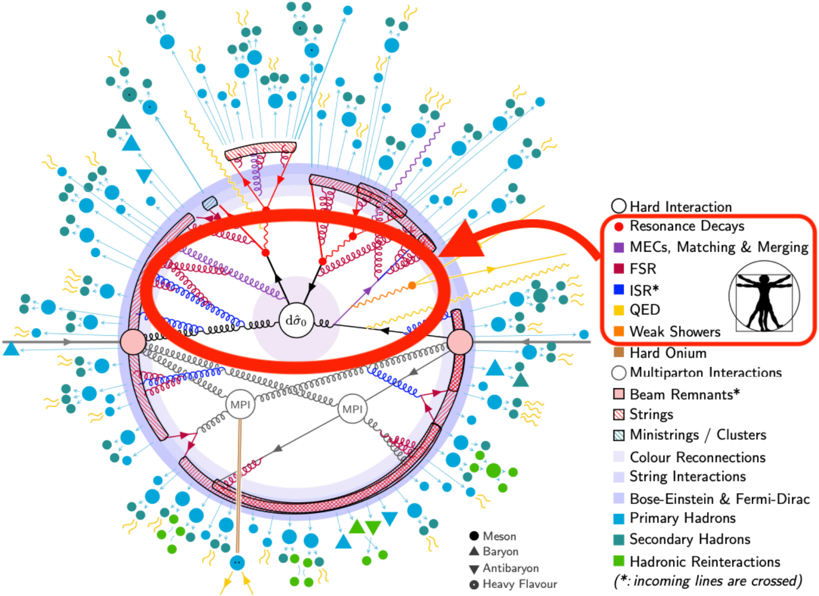


Vincia overview

C.T. Preuss, P. Skands, R. Verheyen

- **originally** developed as plug-in to Pythia 8.2
- **now** part of Pythia 8.3 (since October 2019) as one of three showers:
  - simple shower (`PartonShowers:model = 1`)
  - Vincia (`PartonShowers:model = 2`)
  - Dire (`PartonShowers:model = 3`)
- full-fledged **antenna** shower (FF,IF,II,RF)
- **exact** treatment of **mass corrections** (phase space and antenna functions)
- full **helicity dependence** in shower (and MECs)
- dedicated **default tuning** (similar to Pythia's Monash tune)

Recent and ongoing developments:

- dedicated **RF shower** for QCD radiation in **top decays** [Brooks, Skands 1907.08980]
- full-fledged **interleaved EW shower** [Brooks, Skands, Verheyen 2108.10786]
- fully-differential **NNLO QCD matrix element corrections** [Campbell, Höche, Li, CTP, Skands 2108.07133]
- based on **sector showers** (except EW) [H. Brooks, CTP, P. Skands 2003.00702]

Outline

1) Antenna showers on sectorised phase spaces [Brooks, CTP, Skands 2003.00702]

2) Efficient (CKKW-L-style) merging with sector showers [Brooks, CTP 2008.09468]
   - (POWHEG also possible but not shown here; see [Höche, Mrenna, Payne, CTP, Skands 2106.10987])

3) Towards NNLO+PS matching with sector showers [Campbell, Höche, Li, CTP, Skands 2108.07133]

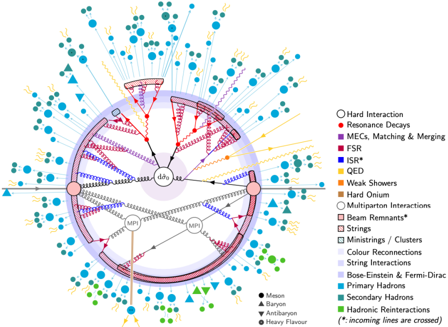
What is an event generator?

Particle-level event generators aim at **simulating** high-energy particle collisions in **full detail** by dividing events into three **energy regimes**:

Hard regime
(multiple) high-energy 2 → n processes with small n

Soft regime
forming and fragmentation of (visible) hadrons at low energies

Transition regime
QCD bremsstrahlung (+ QED/EW emissions)



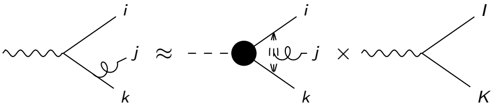
Sketch by Peter Skands

The "big players": PYTHIA, SHERPA, HERWIG

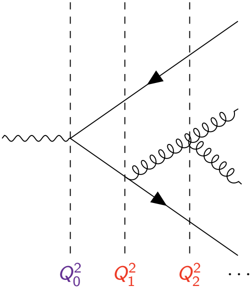
Parton showers

Parton showers dress a LO calculation with **additional radiation**, describing the evolution from **parton level** (quarks, gluons, . . .) to the **particle level** (hadrons).

- amplitudes **factorise** in limits where emissions are **soft** ($E_j \to 0$) or **collinear** ($\vartheta_{jk} \to 0$)



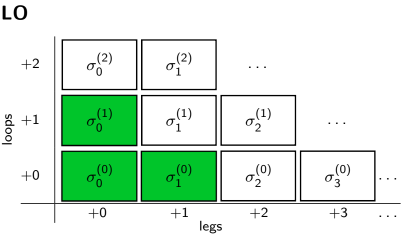
- starting from a **high scale** Q_0^2, further radiation is modelled under the assumption that it is **soft**/**collinear** and **ordered** ($Q_0^2 > Q_1^2 > Q_2^2 > \cdots > Q_{\mathrm{had}}^2$)



- evolves event from hard scale Q_0^2 to soft scale Q_{had}^2 but introduces **logarithms** of the form

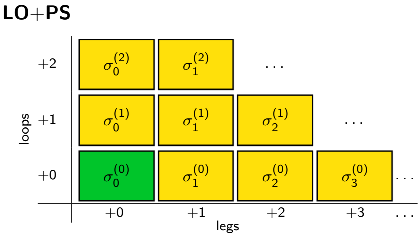
$$\alpha_S^n \to \alpha_S^n \log^m \left(\frac{Q_0^2}{Q^2} \right) , n \leq 2m , \quad \textbf{large} \text{ if } Q^2 \ll Q_0^2$$

Parton showers vs fixed-order calculations



NLO

loops \ legs	+0	+1	+2	+3	...
+2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	...		
+1	🟩 $\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	...	
+0	🟩 $\sigma_0^{(0)}$	🟩 $\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	...



LO+PS

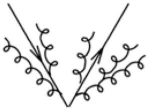
loops \ legs	+0	+1	+2	+3	...
+2	🟨 $\sigma_0^{(2)}$	🟨 $\sigma_1^{(2)}$	...		
+1	🟨 $\sigma_0^{(1)}$	🟨 $\sigma_1^{(1)}$	🟨 $\sigma_2^{(1)}$	...	
+0	🟩 $\sigma_0^{(0)}$	🟨 $\sigma_1^{(0)}$	🟨 $\sigma_2^{(0)}$	🟨 $\sigma_3^{(0)}$	...

Fixed-order calculations → hard jets

- reliable at **high scales** if **no large scale hierarchies** are present
- **accurate** predictions for **limited number** of legs (+ loops)
- determines **perturbative accuracy** (LO, NLO, NNLO, . . .)

Showers → jet substructure

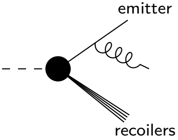
- reliable in **soft/collinear** regions if **large scale hierarchies** are present
- **approximate** predictions for **many** particles
- determines **logarithmic accuracy** (LL, NLL, NNLL, . . .)



⇒ largely **complementary**, so ideally **combine them**!

Many ways to skin a cat...

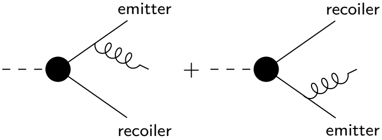
DGLAP



e.g. PYTHIA simple shower, HERWIG $\tilde{q}$

- recoil **independent** of colour partners
- coherent upon **angular ordering**

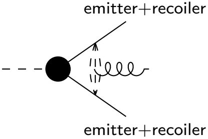
Dipoles



e.g. SHERPA CSS, HERWIG dipole, DIRE

- recoil taken by **opposite dipole end**
- **intrinsically** coherent

Antennae

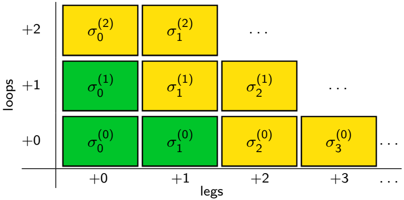


e.g. ARIADNE, VINCIA

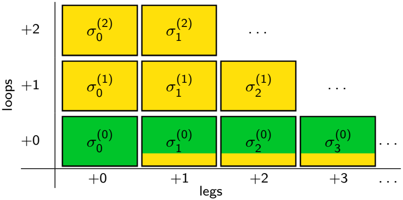
- **both parents** absorb transverse recoil
- **intrinsically** coherent

Combining showers and fixed-order calculations

NLO+PS Matching



LO$_m$+PS Merging



Some disambiguation:

Matching combine a fixed-order (typically NLO) calculation with a parton shower, **avoiding double-counting** in **overlap regions**

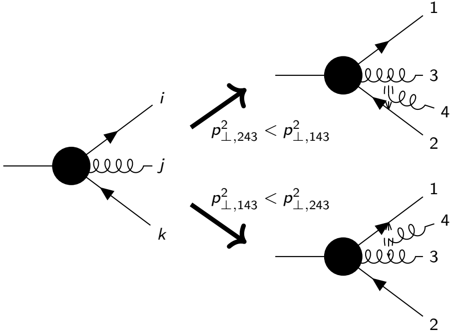
Merging combine **multiple** inclusive (N)LO event samples into a **single** inclusive one with additional **shower radiation**, **accounting** for **Sudakov suppression** and **avoiding double-counting** in **overlap regions** (typically via **phase-space slicing**)

Part I: Sector-Antenna Showers

Sector showers [Brooks, CTP, Skands 2003.00702; Lopez-Villarejo, Skands 1109.3608]

Idea: combine antenna shower with deterministic jet-clustering algorithm

- let shower only generate emissions that would be clustered by a $(3 \mapsto 2)$ jet algorithm ($\sim$ ARCLUS [Lönnblad Z.Phys.C 58 (1993)])



$$p^2_{\perp,ijk} = \frac{2(p_i \cdot p_j)(p_j \cdot p_k)}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}$$

⇒ **softest gluon** always regarded as the emitted one

⇒ only **one** (most singular) splitting kernel contributes per phase space point

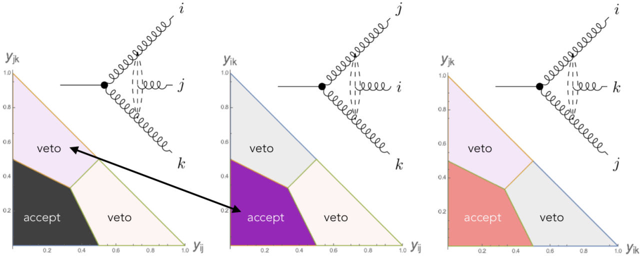
Since PYTHIA 8.304: **full-fledged*** implementation of sector showers in VINCIA

*including FSR, ISR, resonance-decay showers

Phase space sectors

Branching phase space gets divided into **non-overlapping sectors**.

- e.g. first emission in $H \to gg$:



- branchings in the shower are accepted **if and only if** they correspond to the **correct sector**
- sectors defined by minimal $p_{\perp}$ in event, but always contain:
  - soft endpoint
  - "full" collinear region for qg
  - "half" of the collinear region for gg with boundary at $z = \frac{1}{2}$

Note: in general, non-trivial sector boundaries away from the singular limits!

Sector antenna functions

Splitting kernels have to incorporate **full** single-unresolved limits for given PS point (KOSOWER subtraction terms [Kosower PRD 57 (1998) 5410, PRD 71 (2005) 045016])

- e.g. (FF) $qg \mapsto qgg$ ($s_{ij} = 2p_i \cdot p_j$):

$$A^{\mathrm{sct}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\,\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\,\frac{2(1-z(1-z))^2}{z(1-z)} & \text{if } j_g \parallel k_g \end{cases}$$

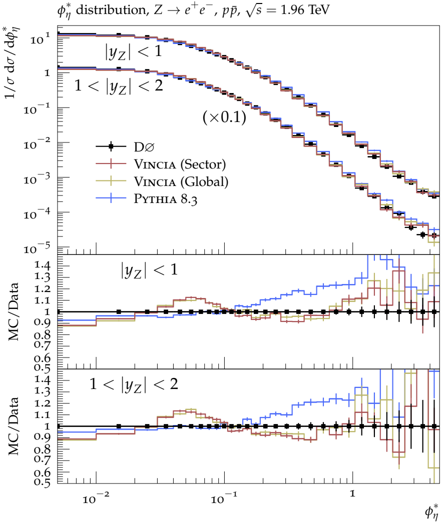
Compare to **global** antenna functions (aka *sub-antenna* functions):

- only "half" of the $j_g \parallel k_g$ limit contained in the splitting kernel:

$$A^{\mathrm{gl}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\,\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\,\frac{1+z^3}{1-z} & \text{if } j_g \parallel k_g \end{cases}$$

- "rest" of the jk-collinear limit reproduced by neighbouring antenna ($z \leftrightarrow 1 - z$)

Sector showers vs global showers



The sector approach is merely an **alternative way** to fraction singularities, so **formal accuracy** of the shower should be **retained**.

Note: same "global shower" tune in Vincia, no MECs here

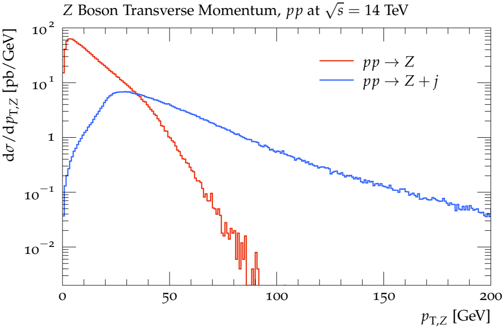
Part II: Efficient Merging with Sector Showers

Merging with traditional showers: illustration

Merging: introduce (arbitrary) **merging scale** Q_{MS} and let each calculation populate the phase space where it does best:

Parton shower generates **soft/collinear** radiation → reject hard branchings ☺

Fixed-order calculation generates **hard** jet(s) → reweight by shower Sudakovs ☹



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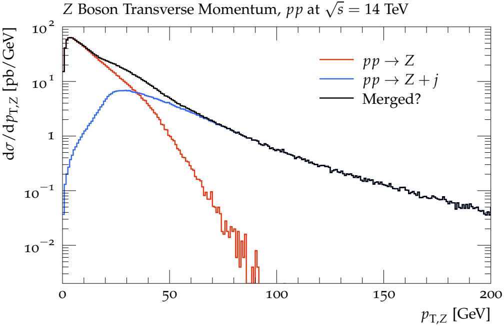

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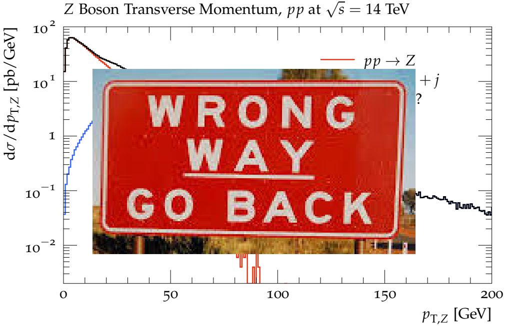

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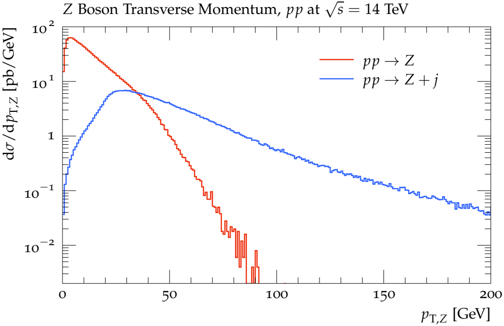

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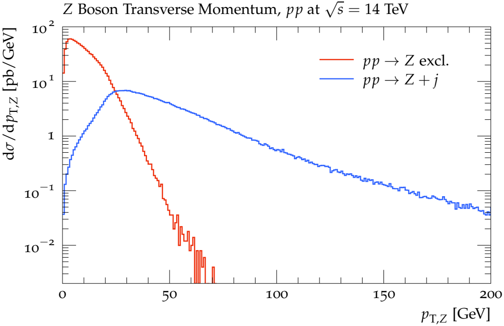

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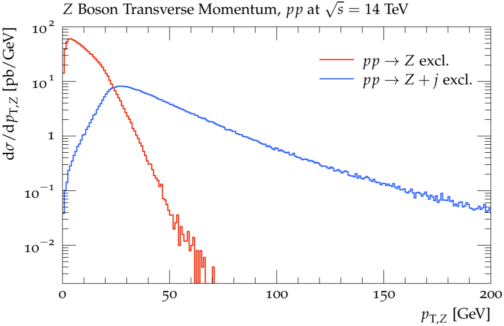

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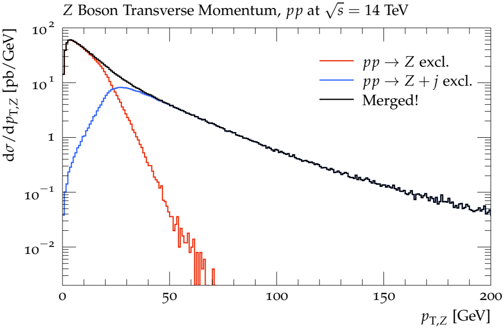
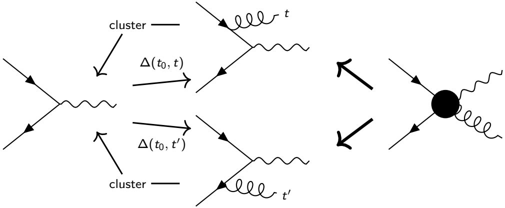


Merging with traditional showers: CKKW-L

Basic CKKW-L idea [Catani, Krauss, Kuhn, Webber hep-ph/0109231], [Lönnblad hep-ph/0112284]

- construct **all possible** shower histories, choose **most likely**
- let (truncated) **trial showers** generate Sudakov factors
- re-weight event by Sudakov factors



Number of histories **scales factorially** with number of legs

Number of Histories for n Branchings

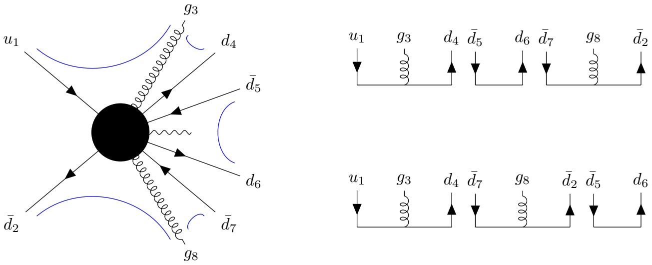
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040

⇒ quickly **increasing complexity** with multiplicity!

Merging with sector showers (MESS) [Brooks, CTP 2008.09468]

Tree-level merging with sector showers straight-forward:
start from CKKW-L and modify **history construction** (could be extended to NLO).

- sector showers have a **single** (!) history for **gluon emissions at LC**
- to account for **gluon splittings** $g \mapsto q\bar{q}$, find all viable quark permutations

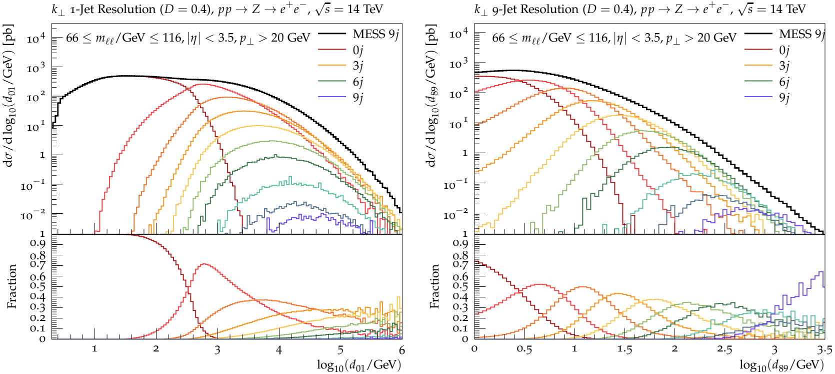


- for each colour-ordering, shower history again **uniquely** defined by sectors
- if multiple colour-orderings possible, choose one that **maximises** branching probability

Since PYTHIA 8.304: sector merging available with VINCIA

Merging with sector showers: validation

Parton-level results for merging in $pp \to Z$ with up to **9 jets**
(using HDF5 event samples from [Höche, Prestel, Schulz 1905.05120])

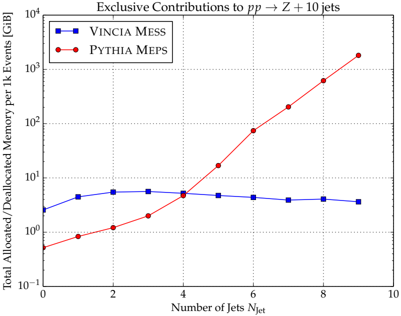


$\Rightarrow$ smooth transitions, no "sector effects" visible

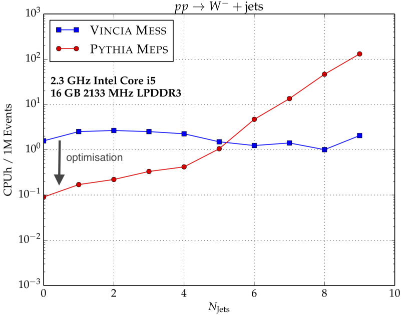
Merging with sector showers: efficiency

Gauge **efficiency gains** in $pp \to Z + 9j$ merging @ parton level
(using HDF5 event samples from [Höche, Prestel, Schulz 1905.05120]).

memory allocation/deallocation:



CPU time per event:



⇒ ∼ **constant** runtime and memory footprint in multi-jet merging

⇒ overall **optimisation** of the sector shower **possible**

Merging in DIS: a multi-scale problem [Helenius, Laulainen, CTP WIP]

Factorisation implies that showers start at **factorisation scale**

$$Q_0^2 \equiv \mu_F^2 > Q_1^2 > Q_2^2 > \dots$$

For **DIS**, small photon virtuality $\mu_F^2 = Q^2$ **severely restricts** shower phase space!

Matrix-element domain consists of two disparate regions:

- high-Q^2, intermediate-$E_{T,B}^2$ events
- low-Q^2, high-$E_{T,B}^2$ events

Cannot be covered with **fixed** merging scale.

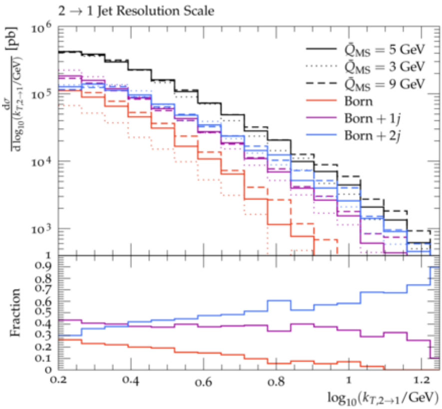


Figure adapted from Joni Laulainen

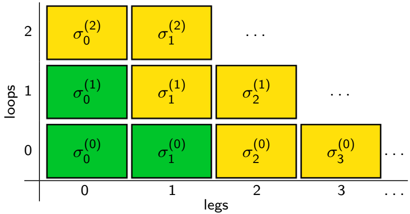
Solution: **dynamic merging scale** [Carli, Höche, Gehrmann 0912.3715]

$$Q_{MS}^2 = \bar{Q}_{MS}^2 \left(1 + \frac{\bar{Q}_{MS}^2}{S^2 \mu_F^2}\right)^{-\frac{1}{2}} \quad \text{with fixed } \bar{Q}_{MS}^2 \text{ and } 0.4 \leq S \leq 0.8$$

(other choices possible)

Part III: Towards NNLO+PS Matching with Sector Showers

NLO+PS matching



Strategy developed $\gtrsim$ 20 years ago [Norrbin, Sjöstrand hep-ph/0010012] nowadays known as Powheg matching [Nason hep-ph/0409146]

Alternative strategy: MC@NLO [Frixione, Webber hep-ph/0204244] (not discussed here)

Powheg master formula (for 2 Born partons):

$$\langle O\rangle^{\textsc{Powheg}}_{\text{NLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, \underset{\text{local }K\text{-factor}}{\boxed{k_{\text{NLO}}(\Phi_2)}}\, \underset{\text{shower operator}}{\boxed{\mathcal{S}_2(t_0,O)}}$$

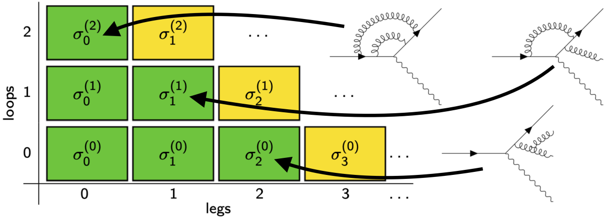
where $k_{\text{NLO}}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \int \mathrm{d}\Phi_{+1}\frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)}$.

Main trick: matrix-element correction (MEC) in first shower emission

$$\mathcal{S}_2(t_0,O) = \Delta_2(t_0,t_{\mathrm{c}})O(\Phi_2) + \int_{t_{\mathrm{c}}}^{t_0} \mathrm{d}\Phi_{+1}\,\frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)}\Delta_2(t,t_{\mathrm{c}})O(\Phi_3)$$

$$\Delta_2(t,t') = \exp\left(-\int_{t'}^{t}\mathrm{d}\Phi_{+1}\,A_{2\mapsto3}(\Phi_{+1})w^{\text{MEC}}_{2\mapsto3}(\Phi_2,\Phi_{+1})\right)\,,\quad w^{\text{MEC}}_{2\mapsto3} = \frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{A_{2\mapsto3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$$

Towards NNLO+PS [Campbell, Höche, Li, CTP, Skands 2108.07133]



Idea: "POWHEG at NNLO"

$$\langle O \rangle^{\textsc{Vincia}}_{\text{NNLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, \underbrace{\boxed{k_{\text{NNLO}}(\Phi_2)}}_{\text{local }K\text{-factor}}\, \underbrace{\boxed{\mathcal{S}_2(t_0, O)}}_{\text{shower operator}}$$

Need:

(1) Born-local NNLO *K*-factors (including R, V, RR, RV, VV corrections)
(2) shower filling ordered and unordered regions of 1- and 2-emission phase space
(3) tree-level MECs in ordered and unordered shower paths
(4) NLO MECs in the first emission

Valid for **all shower components** (FF, IF, II, RF), so far implemented only for FF.

NNLO+PS with sector showers

Key aspect

up to matched order, include process-specific **NLO corrections** into shower evolution:

(1) correct first branching to exclusive ($< t'$) NLO rate:

$$\Delta^{\mathrm{NLO}}_{2\mapsto 3}(t_0, t') = \exp\left\{ -\int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \mathrm{A}_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \right\}$$

(2) correct second branching to LO ME:

$$\Delta^{\mathrm{LO}}_{3\mapsto 4}(t', t) = \exp\left\{ -\int_{t}^{t'} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{3\mapsto 4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_3, \Phi'_{+1}) \right\}$$

(3) add direct $2 \mapsto 4$ branching and correct it to LO ME:

$$\Delta^{\mathrm{LO}}_{2\mapsto 4}(t_0, t) = \exp\left\{ -\int_{t}^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \mathrm{A}_{2\mapsto 4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_2, \Phi_{+2}) \right\}$$

⇒ entirely based on **MECs** and **sectorisation**

⇒ **by construction**, expansion of extended shower **matches** NNLO singularity structure

But shower kernels **do not** define **NNLO subtraction terms**[1] (!)

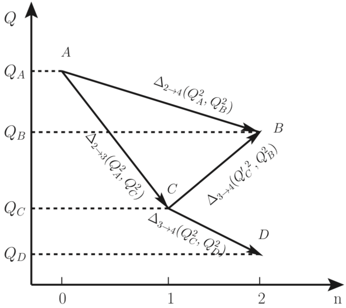
---

[1] This would be required in an "Mc@Nnlo" scheme, but difficult to realise in antenna showers.

Interleaved single and double branchings

A priori, direct 2 ↦ 4 and iterated 2 ↦ 3 branchings **overlap in ordered** region.

In **sector showers**, iterated 2 ↦ 3 branchings are **always strictly ordered**.



Divide double-emission phase space into **strongly-ordered** and **unordered** region: [Li, Skands 1611.00013]

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi_{+2}^{>}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi_{+2}^{<}}_{\text{s.o.}}$$

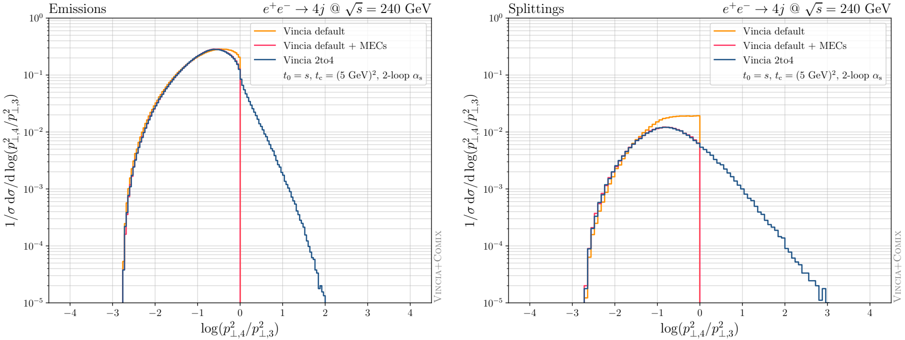
$\mathrm{d}\Phi_{+2}^{<}$: **single-unresolved** limits ⇒ iterated 2 ↦ 3
$\mathrm{d}\Phi_{+2}^{>}$: **double-unresolved** limits ⇒ direct 2 ↦ 4

Restriction on double-branching phase space enforced by additional veto:

$$\mathrm{d}\Phi_{+2}^{>} = \sum_j \theta\left(p_{\perp,+2}^2 - \hat{p}_{\perp,+1}^2\right) \Theta_{ijk}^{\mathrm{sct}}\, \mathrm{d}\Phi_{+2}$$

Real and double-real corrections

Direct $2 \mapsto 4$ shower component fills **unordered region** of phase space $p^2_{\perp,4} > p^2_{\perp,3}$.



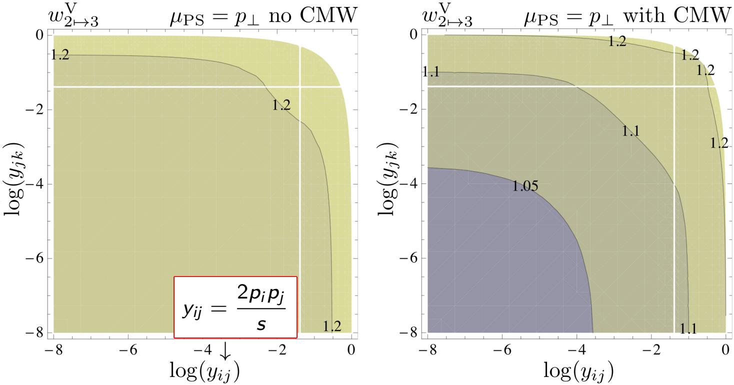
Sectorisation enforces **strict** cutoff at $p^2_{\perp,4} = p^2_{\perp,3}$ in iterated $2 \mapsto 3$ shower. **No recoil effects**!

Real-virtual corrections

Real-virtual correction factor ("local K-factor")

$$w^{\text{NLO}}_{2\mapsto3} = w^{\text{LO}}_{2\mapsto3}\left(1 + w^{\text{V}}_{2\mapsto3}\right)$$

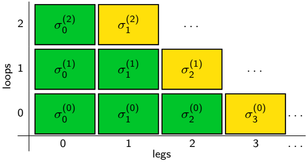
studied **analytically** in detail for $Z \to q\bar{q}$ in [Hartgring, Laenen, Skands 1303.4974]:



$$y_{ij} = \frac{2p_ip_j}{s}$$

Now: **generalisation** & **(semi-)automation** in VINCIA in form of NLO MECs

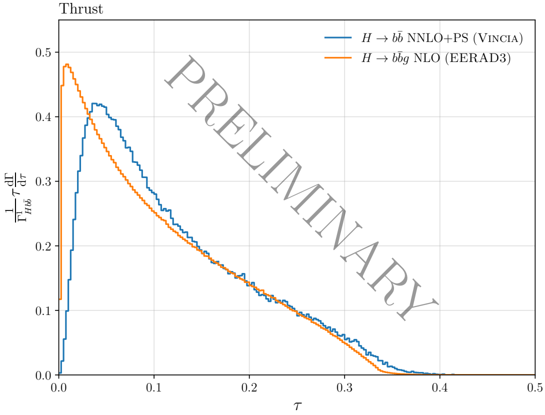
NNLO+PS matching in resonance decays



By construction, partial width is accurate to **NNLO**.

NNLO accuracy at Born level also implies
NLO correction in first emission and
LO correction in second emission.

E.g. $H \to b\bar{b}$ at parton level (vs NLO from [Coloretti, Gehrmann-De Ridder, CTP 2202.07333]):



Conclusions

Sector showers combine shower **evolution** with jet **clustering** to become **maximally bijective**

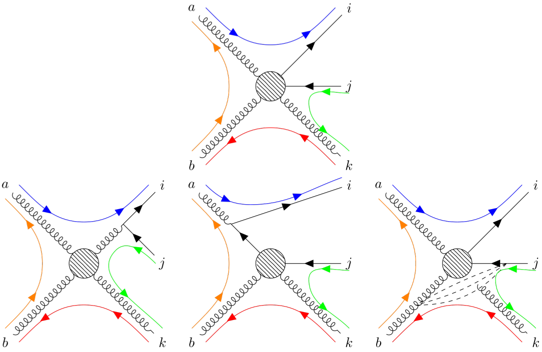
- "sectorised" VINCIA well validated against "global" VINCIA and PYTHIA
  (discontinuities? still searching...)
- sector merging has ∼**constant** overall **run time** and **memory usage**
- sector showers **default option** in VINCIA as of PYTHIA 8.304

This is just the beginning...

- sector **merging** easily extendable to **NLO**
  (lack of time that it hasn't been done yet...)
- sector decomposition facilitates inclusion of **NLO antenna functions** in shower evolution
  (including direct $2 \mapsto 4$ branchings covering double-unresolved limits)
- antenna-based **(N)NLO matching** and shower **evolution at NLO** ongoing developments
  (currently on a proof-of-concept level for $e^+e^- \to 2j$, but can be extended!)

Backup

Sector definitions



For massless particles, the sector resolution is defined by:

$$Q^2_{\text{res},j} = \begin{cases} \frac{s_{ij}s_{jk}}{s_{ijk}} & \text{if } j \text{ is a } g \\ s_{ij}\sqrt{\frac{s_{jk}}{s_{ijk}}} & \text{if } (i,j) \text{ is a } q\bar{q} \text{ pair} \end{cases}$$

Sectors defined by:

$$\Theta_{\text{sct},j} = \theta(\min\{Q^2_{\text{res},i}\} - Q^2_{\text{res},j})$$