Supplementary Material (ESI) for Lab on a Chip

Supplementary information

Numerical modeling of the flow field in the comparator

We determine the relative differential flow rate $\Delta Q/Q$ in our comparator by measuring the relative width $\Delta Y/W$ at the edge of the outgoing channel splitting. To relate $\Delta Q/Q$ to $\Delta Y/W$ we calculate numerically the flow field in the comparator region using an extension of the Hele Shaw approximation. Defining the aspect ratio of the comparator as

$$\varepsilon = (H/B)^2$$

The dimensionless Stokes equations for the problem can be written as:

$$\partial_\xi \pi = \partial_\zeta^2 u_\xi + \varepsilon\,(\partial_\xi^2 u_\xi + \partial_\eta^2 u_\xi)$$
$$\partial_\eta \pi = \partial_\zeta^2 u_\eta + \varepsilon\,(\partial_\xi^2 u_\eta + \partial_\eta^2 u_\eta)$$
$$\partial_\zeta \pi = 0$$

where we used the following scaling: $p = P\pi$, $x = B\xi$, $y = B\eta$, $z = H\zeta$, and $v_i = Vu_i$, and $H^2P = \mu BV$. The coordinate x has been measured in the main flow direction, y and z perpendicular to that, where z is the out of plane coordinate. In the limit $\varepsilon = 0$, one obtains the Hele Shaw solution:

$$u_\xi = U_\xi(\xi,\eta)\, f(\zeta)$$
$$u_\eta = U_\eta(\xi,\eta)\, f(\zeta)$$

where U_ξ and U_η are the height averaged velocities in x and y direction and

$$f(\zeta) = 6(1/4 - \zeta^2)$$

represents the parabolic flow profile between the lower and upper boundary at $\zeta = \pm 1/2$. To obtain a solution for finite values of ε we use the same factorization which results in:

$$\partial_\xi \pi = -12\, U_\xi + \varepsilon\,(\partial_\xi^2 U_\xi + \partial_\eta^2 U_\xi)$$
$$\partial_\eta \pi = -12\, U_\eta + \varepsilon\,(\partial_\xi^2 U_\eta + \partial_\eta^2 U_\eta)$$

Introducing the stream function $\psi(\xi,\eta)$ such that $\partial_\eta \psi = U_\xi$ and $\partial_\xi \psi = -U_\eta$ both equations reduce to:

$$(12 - \varepsilon \nabla^2)\, \nabla^2 \psi = 0$$

where $\nabla^2 \psi = \partial_\xi^2 \psi + \partial_\eta^2 \psi$. This equation has been rewritten as:

$$\nabla^2 \psi = \Omega$$

$$(12 - \varepsilon \nabla^2)\, \Omega = 0$$

The boundary conditions on the closed parts of the boundary are given by $\psi(\xi,\eta) = \psi_B(\xi,\eta)$ and $\partial_n \psi(\xi,\eta) = 0$, where $\partial_n \psi(\xi,\eta)$ is the derivative perpendicular to the boundary, as dictated by the no slip condition. Using these we calculate also the boundary condition for $\Omega(\xi,\eta) = \nabla^2 \psi_B(\xi,\eta)$. At the entrance and exit parts we use the second derivative $\partial_\xi^2 \psi(\xi,\eta) = 0$, in stead of $\psi(\xi,\eta) = \psi_B(\xi,\eta)$, see Fig. Si-1. The equations has been solved using a finite difference scheme. In Fig. S2 and S3 we plot the flow field for our comparator with $\varepsilon = 0$ (the Hele Shaw solution) and $\varepsilon = 9/25$ (the full solution) with $\Delta Q/Q = 0.1$. For the full solution we checked the calculated flow profiles at the outlet region with the known analytical solutions. The difference between the numerical and analytical calculated velocities was at all positions smaller than 1%. By plotting the stream line $\psi(\xi,\eta) = Q - \Delta Q$, see Fig. S3, we obtain the interface between the dyed fluid (red) entering the comparator at the top right corner and the non-dyed fluid (blue) entering at the bottom right corner. From this streamline we determine the value for $\Delta Y/W$ at the edge of the outgoing channel split. For the comparator at hand, $\Delta Q/Q$ for a given $\Delta Y/W$ is very well described by the empirical formula:

$$\Delta Q/Q = 1.73\,(\Delta Y/W)^2 - 0.73\,(\Delta Y/W)^{4.6}$$

This expression has been used in our analysis of the flow comparator results.

Supplementary Figures

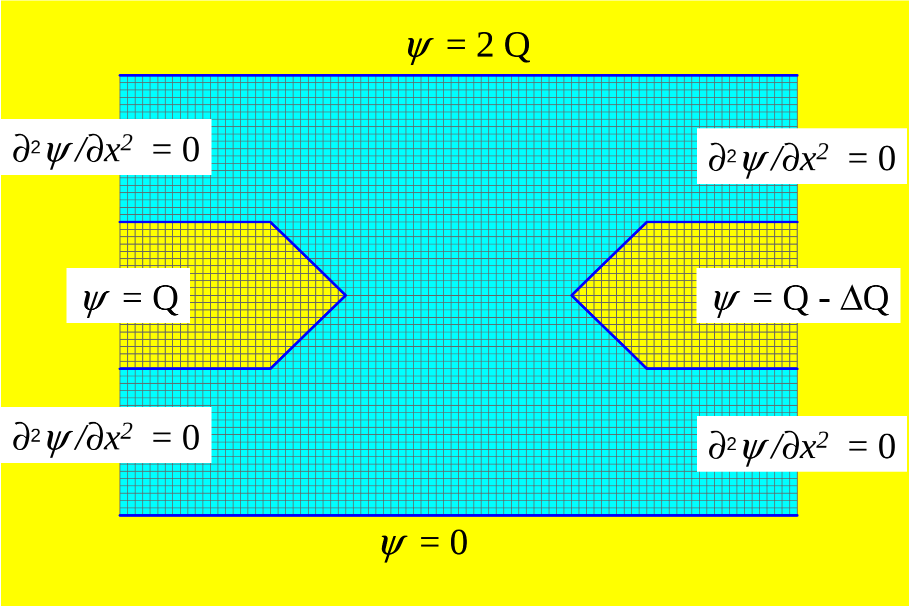

Figure S1. Boundary conditions for the stream function ψ. Besides these we also used the condition $\partial_n\psi = 0$, to calculate Ω on the boundary. The computational grid is shown in grey.

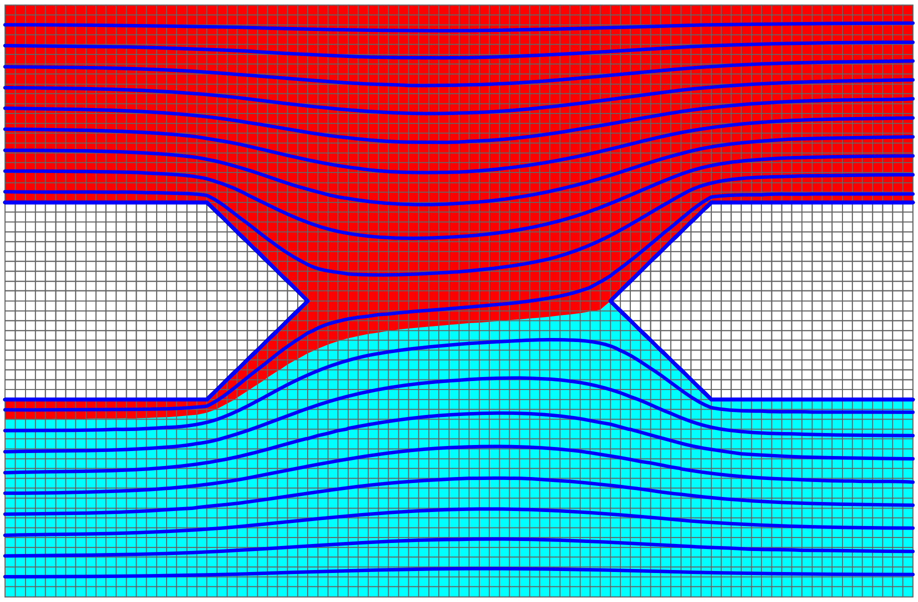

Figure S2. The Hele Shaw solution for $\varepsilon = 0$ and $\Delta Q/Q = 0.10$. Note that in this case the no slip condition on the fixed boundaries is not obeyed.

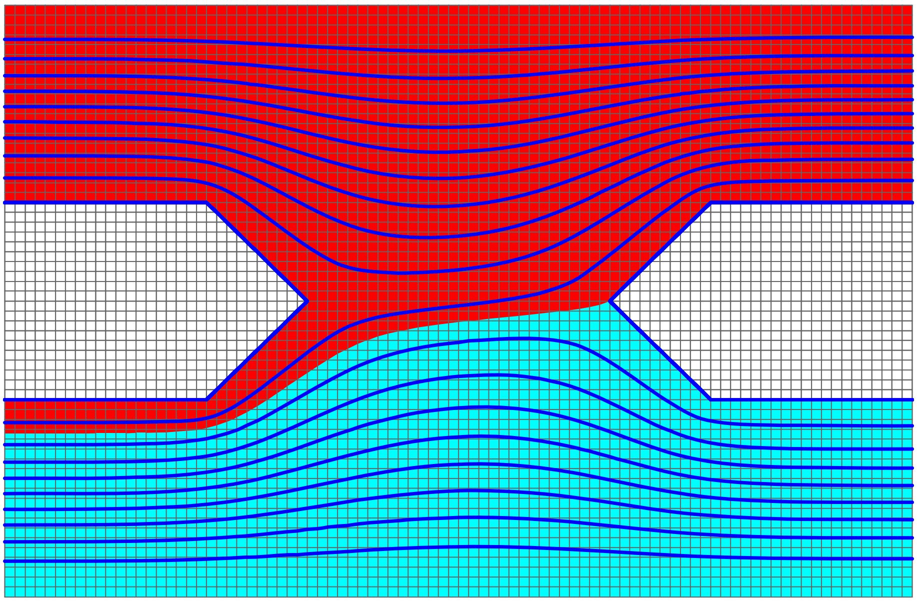

Figure S3. The full solution for $\varepsilon = 9/25$ and $\Delta Q/Q = 0.1$. Note that in this case the no slip condition on the fixed boundaries is obeyed. The interface between the red and blue flow indicates the separating stream line $\psi(\xi,\eta) = Q - \Delta Q$.