The Moving K Diversified Nearest Neighbor Query

Yu Gu*, Guanli Liu, Jianzhong Qi, Hongfei Xu, Ge Yu, Rui Zhang

Abstract—As a major type of continuous spatial queries, the moving k nearest neighbor (kNN) query has been studied extensively. However, most existing studies have focused on only the query efficiency. In this paper, we consider further the usability of the query results, in particular the diversification of the returned data points. We thereby formulate a new type of queries named the *moving k diversified nearest neighbor query (MkDNN)*. This type of queries continuously report the k diversified nearest neighbors while the query object is moving. Here the degree of diversity of the kNN set is defined on the distance between the objects in the kNN set. Computing the k diversified nearest neighbors is an NP-hard problem. We propose an algorithm to maintain incrementally the k diversified nearest neighbors to reduce the query processing costs. We further propose two approximate algorithms to obtain even higher query efficiency with precision bounds. We verify the effectiveness and efficiency of the proposed algorithms both theoretically and empirically. The results confirm the superiority of the proposed algorithms over the baseline algorithm.

Index Terms—Moving nearest neighbor query, spatial diversity, safe region, approximate algorithm

1 Introduction

As a major type of moving queries, the *moving k nearest neighbor (MkNN)* query has been studied extensively [1], [2], [3], [4]. This query assumes a moving query object q and a set of static data objects O. When q is moving, the MkNN query reports its k nearest neighbors (kNN) continuously. Common applications of this query include finding nearest petrol stations continuously while one drives on highway, or nearest points of interest (POI) continuously while a tourist is walking around the city [4]. However, MkNN queries tend to return kNNs that cluster together, i.e., data objects near q tend to be near each other as well. These clustering kNNs may share common features, e.g., all in a less pleasant suburb, which make them all unsatisfactory query answers. To avoid clustering kNNs and to improve result usability, *query result diversification* is a popular technique. Originated from information retrieval [5], [6], [7], this technique tries to return results as different from each other as possible while all satisfying the query predicate.

Fig. 1 illustrates how query result diversification may improve the result of an MkNN query. The query user q is looking for a restaurant for dining as she is on a road trip. There are 9 restaurants $p_1, p_2, ..., p_9$ to be chosen from. An M3NN query will return $\{p_1, p_2, p_3\}$ as the answer since they are the nearest to q. However, they are all in a busy area. While it is convenient to shop or to get a cup of coffee there, the traffic is congested and parking is difficult. Alternatively, a query result diversification technique may take the distance between the restaurants themselves into consideration, and return $\{p_2, p_4, p_7\}$ as the answer. This

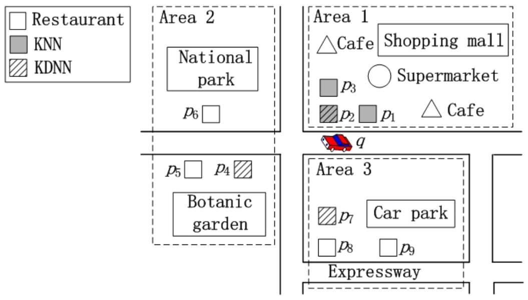

Fig. 1. An MkDNN query ($k = 3$)

answer is more preferable. The three restaurants are all relatively close to the query user. Meanwhile, they are far away from each other and are distributed in different areas. The query user can choose the one in her preferred area, e.g., p_4, which is in a nice garden area. The example can also be thought of as a scenario of tourist hotspot recommendation, where the small squares in the figure represent the tourist hotspots. The MkNN query may return tourist hotspots clustering together, which the query user may have all visited already. In this case, the user may prefer recommendations that are far away from each other. To find such query answers, we will need a new type of queries that consider the diversity of the query results.

Efforts [8], [9], [10] have been made to diversify the results for *static* kNN queries. However, there is still a lack of studies to diversify the results for moving queries. We aim to fill this gap by proposing a new type of queries called the *moving k diversified nearest neighbor (MkDNN)* query. As the query object q is moving, this query *continuously* returns k objects that are the *nearest* to q and are as *diversified* as possible. Here, "nearest" is defined on spatial proximity, while "diversified" is defined on the difference between the nearest neighbors, which can have different meanings if viewed from different perspectives. In this study, we define diversity as the distance between the nearest neighbors, al-

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though we can also use other types of attributes for diversification (e.g., type of cuisine in the restaurant example). The MkDNN query is then to find nearest neighbors that are as far away from each other as possible. This query definition will help avoid clustering kNNs and find diversified nearest neighbors as illustrated in Fig. 1.

The *safe region* technique [3], [4], [11], [12] is commonly used in processing moving queries. It first treats the query as a static one and computes the query answer together with a "safe region". As long as the query point is still in this safe region, it is guaranteed that the current query answer is still valid. The query processing then becomes continuously checking whether the query point is still in the safe region. Only when the query point moves out of the safe region that the static query is computed again and the query answer as well as the safe region are updated. We follow this procedure to process the MkDNN query. Every time the current query answer becomes invalid, we recompute a static *k diversified nearest neighbor (kDNN)* query.

As processing a static kDNN query has been shown to be NP-hard [13], our initial focus is to reduce the frequency of recomputing it as much as possible. Existing studies on moving queries have mostly used Voronoi diagram based safe regions to reduce the recomputation frequency. However, these safe regions are defined only on spatial proximity. They cannot handle diversity. We overcome this limitation by prefetching top-m kDNN sets instead of only the top-1 kDNN set when the kDNN query is recomputed, where m is a system parameter. Intuitively, nearby points should have similar kDNN sets. The prefetched kDNN sets will serve as a "cache". When the query point moves, we first try to find the new kDNN set from this "cache". Only when no prefetched set is valid that a query recomputation is needed. This reduces the recomputation frequency.

To further reduce the recomputation frequency, we propose an approximate algorithm that computes only the top-1 kDNN set, and keeps it as the query answer as long as it is a ρ-approximation of the true query answer, where ρ is user defined parameter. We derive safe regions based on spatial proximity as well as diversity to efficiently determine validity of the current kDNN set.

In addition, we adapt a greedy algorithm to compute approximate kDNN sets at recomputation, and propose a strategy to maintain the approximate query answer with a precision bound. This strategy also uses prefetching. We design safe regions that can be computed efficiently to guard the validity of the prefetched candidate query answers. The resultant algorithm can reduce the cost of recomputation and the recomputation frequency at the same time, and hence achieves even higher efficiency.

We make the following contributions in this paper:

- We propose a new query type, the moving k diversified nearest neighbor query (MkDNN), which ads diversity to the nearest neighbors.
- Since the static kDNN query is NP-hard, we approach the MkDNN query first by reducing the frequency of recomputing the kDNN queries when the query point moves. We propose a prefetching-based precise algorithm as well as a bounded approximate algorithm to achieve this goal.
- We further propose an algorithm that computes approximate kDNN query answers, and hence further reduces the overall processing costs.
- We analyze the costs of the proposed algorithms and evaluate the algorithm performance on real data sets. The result confirms the effectiveness and efficiency of the proposed algorithms.

The rest of the paper is organized as follows. We discuss related studies in Section 2. We present the problem setting in Section 3. We propose three MkDNN query algorithms in Sections 4 and 5. We study the performance of the proposed algorithms in Section 6 and conclude the paper in Section 7.

2 Related Work

2.1 Moving k nearest neighbor queries

MkNN queries have been studied extensively [2], [3], [14], [15], [16]. Early studies (e.g., [14]) have used sampling based approaches. They consider the query object's trajectory as formed by discrete points, and process a static kNN query at each point. Between the discrete points, the query answer is inaccurate. To obtain more accurate query answer, two adjacent discrete points have to be very close, and the kNN query has to be computed frequently, which is less efficient.

Later studies [3], [16] use *safe regions* to reduce the query recomputation frequency. The safe region is a region where the query object can move freely without causing the current kNN answer to change. This allows only recomputing the kNN answer when the query object leaves the current safe region. *Voronoi diagrams* are commonly used to construct safe regions. In an order-k Voronoi diagram [17], each cell is a safe region. As long as the query object is in an order-k Voronoi cell, the k data points forming this cell are the kNN answer. By definition, the order-k Voronoi cell is the largest possible safe region. However, this cell is expensive to compute. The *Retrieve-Influence-Set kNN (RIS-kNN)* algorithm [16] computes an order-k Voronoi cell by a few (six on average) time-parameterized kNN queries (which are still quite expensive) when the kNN answer becomes invalid. The *V*-Diagram* approach [3] sacrifices the size of the safe region for higher efficiency of safe region computation. It approximates an order-k Voronoi cell with the *integrated safe region*, which is more efficient to compute but smaller, and hence the query recomputation frequency is higher. The *influential neighbor set* approach [4] is the state-of-the-art MkNN algorithm. This approach uses *safe guarding objects* rather than safe regions, which are a small set of data points surrounding the current kNN set. Conceptually, the safe guarding objects still define a safe region, which has been shown to be equivalent to an order-k Voronoi cell. Meanwhile, the safe guarding objects are much more efficient to compute. Therefore, the influential neighbor set approach achieves high overall query efficiency.

Other MkNN query studies (e.g., [15]) assume predefined linear query trajectories. The safe regions are then reduced to line segments on the trajectories. These studies have focused on the efficiency aspect of query processing. They cannot compute k diversified nearest neighbors and hence are not applicable to our MkDNN queries.

2.2 Query result diversification

Originated from information retrieval, studies [5], [6], [7], [18] on query result diversification try to return results as different from each other as possible while all satisfying the query predicate, to improve query result usability.

Diversification has been considered *static* spatial queries such as the kNN query. For example, Lee et al. [8] find the nearest objects surrounding a static query point. Kreveld et al. [19] consider diversification in ranking spatial objects. They rank all objects in the database for every query, which has high costs. Zhang et al. [9] study the problem of diversified spatial keyword search on road networks. Their query result diversity is also defined on the distance between the objects. A signature-based inverted index is proposed to solve their problem, which utilizes both keyword-based and diversity-based pruning techniques to reduce the search space. Abbar et al. [20] find the k-diverse near neighbors within a given circle around the query object. They use the distance between the objects to define diversity as well, but their solution is based on Hamming space which is not applicable under our problem settings. Haritsa [21] finds the k nearest objects that satisfy a diversity constraint defined based on the *Gower coefficient* [22]. Kucuktunc and Ferhatosmanoglu [10] investigate the diversified kNN problem based on the *angular similarity*. They propose a *Gabriel graph*-based method to solve the problem, which scales well with dimensionality. Ference et al. [23] also study query result diversification based on angular similarity. They propose a dynamic programming algorithm to find the optimal kDNNs and two heuristic algorithms, Distance-based Browsing (DistBrow) and Diversity-based Browsing (DivBrow), to explore the search space prioritized on spatial proximity and spatial diversity, respectively.

All these studies targeted diversifying a single (static) query, while we aim to diversify the kNN query continuously. Thus, the existing approaches do not apply.

3 Preliminaries

We assume two dimensional point data. A query point q moves in an Euclidean space where there is a set of n static data points P. Given a query parameter k, the goal is to report a size-k subset S of P, such that among all the size-k subsets of P, S contains the data points that are the nearest to q and are the most diversified. Note that the set S may change over time as q moves. We write the optimization goal as a function $f_q(S)$:

$$f_q(S) = \lambda \times DIS(S,q) + (1-\lambda) \times DIV(S).$$

Here, $DIS(S,q)$ is a function that returns the distance between q and the data points in S; $DIV(S)$ is a function that computes the *degree of diversity* among the data points in S; $\lambda(\lambda \in [0,1])$ is a user-defined parameter that represents the preference on spatial proximity over diversity:

$$DIS(S,q) = \frac{1}{k}\sum_{p_i \in S}(1 - \frac{dis(p_i,q)}{dis_m})$$

$$DIV(S) = \frac{2}{k(k-1)}\sum_{p_i,p_j \in S}\frac{div(p_i,p_j)}{div_m}$$

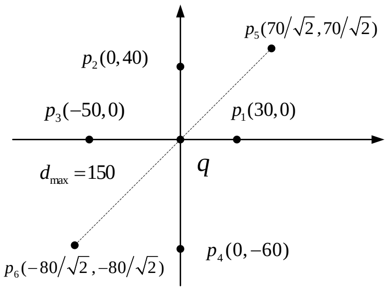


Fig. 2. A 3DNN query ($\lambda = 0.5$)

In these two functions, $dis(p_i,q)$ and $div(p_i,p_j)$ compute the distance and the degree of diversity between two points. We use dis_m and div_m to denote the maximum distance and the maximum degree of diversity, respectively. These two maximum values help normalize the two function values into the range $[0,1]$. We compute the average ($\frac{1}{k}$, $\frac{2}{k(k-1)}$) of these function values to convert them into a similar scale. The full function $f_q(S)$ can be written as follows.

$$f_q(S) = \frac{\lambda}{k}\sum_{p_i \in S}(1-\frac{dis(p_i,q)}{dis_m}) + \frac{2(1-\lambda)}{k(k-1)}\sum_{p_i,p_j \in S}\frac{div(p_i,p_j)}{div_m} \tag{1}$$

In this paper we focus on spatial diversity. We define the *degree of diversity* between two data points p_i and p_j as the spatial distance between p_i and p_j, i.e.,

$$div(p_i,p_j) = dis(p_i,p_j).$$

The function $dis(p_i,p_j)$ returns the Euclidean distance between p_i and p_j. As a result, we have $div_m = dis_m$. Equation 1 can then be rewritten as follows.

$$f_q(S) = \frac{\sum_{p_i,p_j \in S}(\lambda(2 - \frac{dis(q,p_i)+dis(q,p_j)}{dis_m}) + \frac{2(1-\lambda)}{dis_m}dis(p_i,p_j))}{k(k-1)} \tag{2}$$

Now we can define the *k diversified nearest neighbor query* and the *moving k diversified nearest neighbor query*.

***Definition 1** (K **diversified nearest neighbor** (k**DNN**) **query**).* Given a set of static data objects P, a query point q, a query parameter k, and a user preference parameter λ, the k diversified nearest neighbor query returns a size-k set $S \subseteq P$, for any size-k set $S' \subseteq P$, $f_q(S) \geq f_q(S')$.

Fig. 2 is an example, where $dis(q,p_1) = 30$, $dis(q,p_2) = 40$, $dis(q,p_3) = 50$, $dis(q,p_4) = 60$, $dis(q,p_5) = 70$, $dis(q,p_6) = 80$. Assuming $k = 3$ and $\lambda = 0.5$, then the kDNN set of q should be $\{p_3,p_4,p_5\}$ instead of any other subsets of P. For example, $f_q(\{p_3,p_4,p_5\}) = 0.6438 > f_q(\{p_1,p_2,p_3\}) = 0.5822$. Intuitively, even though $\{p_1,p_2,p_3\}$ is slightly nearer to q, $\{p_3,p_4,p_5\}$ is more diversified (the points are farther away from each other). Overall, $\{p_3,p_4,p_5\}$ has a higher optimization function value.

When the query point q is moving, i.e., q may be at different locations at different timestamps, the query becomes a moving k diversified nearest neighbor query.

***Definition 2** (**Moving** k **diversified nearest neighbor** (MkDNN) **query**).* Given a set of static data objects P, a moving query point q, a query parameter k, and a user preference parameter λ, the moving k diversified nearest neighbor query returns the kDNN set of q for every timestamp.

Following existing studies on moving kNN queries (e.g., [3], [4], [16]), we consider the general scenario where there is no constraint on the moving pattern of the query point q, i.e., q can move towards any direction with any speed at any time. When an MkDNN query is issued, we first compute a static kDNN query at q, and then continuously validate the kDNN set as q is moving. We recompute the kDNN set when it becomes invalid. We present both precise and approximate algorithms for kDNN computation and validation in the following sections.

4 MkDNN Algorithms based on Precise KDNN Computation

Computing the kDNN query is essentially to solve the Maximum Dispersion problem, which is NP-hard [13]. In this section we assume that there exists an algorithm[1] to compute the kDNN query, and focus on optimizing the kDNN validation phrase. The goal is to reduce the frequency of recomputing the kDNN to reduce the overall query costs.

4.1 A Precise Algorithm for kDNN Maintenance

We first reduce the kDNN computation frequency by prefetching the top-m kDNN sets, where m is a system parameter and will be chosen empirically. Here, the intuition is that two nearby locations l_1 and l_2 should share similar kDNN sets. It is likely that the top-1 kDNN set of l_1 is also among the top m kDNN sets of l_2. As such, the prefetched the top-m kDNN sets would serve as a "cache" for the MkDNN query. Only when none of the prefetched kDNN sets is still valid that the kDNN sets need to be recomputed. Note that if the enumeration based algorithm is used to compute the kDNN set, then the prefetching requires maintaining a size-m priority queue, although this would have very little overhead when m is small.

We use a priority queue denoted by Q to store the m kDNN sets prioritized by their $f_q()$ function values. When the query point q moves, we first try to find the new kDNN set from Q. If found, then we return it, and update Q according to the new $f_q()$ function values of the kDNN sets. Otherwise, we compute the new top-m kDNN sets, store them in Q, and return the top one as the query answer.

Next we detail how to maintain Q and compute the new kDNN set from it.

4.1.1 Incremental kDNN Maintenance

Safe radius. We introduce the *safe radius* to help maintain Q. This concept defines a region to guarantee that a size-k set $S_i \subseteq P$ is "nearer" to q than another size-k set $S_j \subseteq P$.

Definition 3 (Safe radius). Let S_i, S_j be two size-k subsets of P satisfying $f_q(S_i) > f_q(S_j)$. The safe radius for S_i and S_j, denoted by $r_{i,j}$, is defined to guarantee $f_{q'}(S_i) > f_{q'}(S_j)$ for the query point to move from q to any new location q' as long as $dis(q,q') < r_{i,j}$.

1. A straightforward enumeration based algorithm can enumerate all C_n^k combinations of the n data points to find the optimal kDNN set with $O(n^k)$ time.

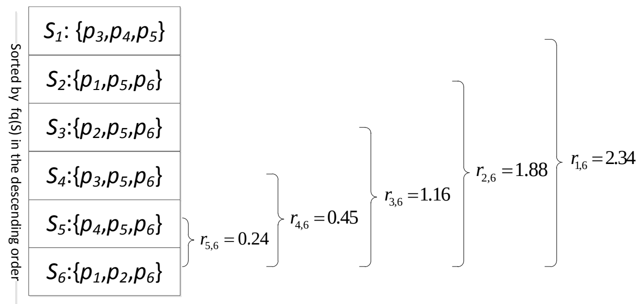


Fig. 3. Top-m kDNN sets and safe radius

We derive a mathematical definition for $r_{i,j}$. The goal is to find $r_{i,j}$ such that $f_{q'}(S_i) > f_{q'}(S_j)$, where

$$f_{q'}(S_i) = \frac{\sum_{p_x,p_y\in S_i}(\lambda(2-\frac{dis(q',p_x)+dis(q',p_y)}{dis_m})+\frac{2(1-\lambda)}{dis_m}dis(p_x,p_y))}{k(k-1)}$$

$$f_{q'}(S_j) = \frac{\sum_{p_x,p_y\in S_j}(\lambda(2-\frac{dis(q',p_x)+dis(q',p_y)}{dis_m})+\frac{2(1-\lambda)}{dis_m}dis(p_x,p_y))}{k(k-1)}$$

According to triangle inequality, $\forall p \in P$:

$$dis(q,p) + dis(q,q') \geq dis(q',p)$$
$$dis(q,p) - dis(q,q') \leq dis(q',p).$$

We replace $dis(q',p_x)$ and $dis(q',p_y)$ in $f_{q'}(S_i)$ by $dis(q,p_x)+dis(q,q')$ and $dis(q,p_y)+dis(q,q')$, respectively:

$$f_{q'}(S_i) \geq \frac{\sum_{p_x,p_y\in S_i}\lambda(2-\frac{dis(q,p_x)+dis(q,q')+dis(q,p_y)+dis(q,q')}{dis_m})}{k(k-1)} + \frac{\sum_{p_x,p_y\in S_i}\frac{2(1-\lambda)}{dis_m}dis(p_x,p_y)}{k(k-1)} = f_q(S_i) - \frac{\lambda dis(q,q')}{dis_m}.$$

Similarly, we replace $dis(q',p_x)$ and $dis(q',p_y)$ in $f_{q'}(S_j)$ by $dis(q,p_x) - dis(q,q')$ and $dis(q,p_y) - dis(q,q')$:

$$f_{q'}(S_j) \leq \frac{\sum_{p_x,p_y\in S_j}\lambda(2-\frac{dis(q,p_x)-dis(q,q')+dis(q,p_y)-dis(q,q')}{dis_m})}{k(k-1)} + \frac{\sum_{p_x,p_y\in S_j}\frac{2(1-\lambda)}{dis_m}dis(p_x,p_y)}{k(k-1)} = f_q(S_j) + \frac{\lambda dis(q,q')}{dis_m}.$$

To let $f_{q'}(S_i) > f_{q'}(S_j)$, we need:

$$f_q(S_i) - \frac{\lambda dis(q,q')}{dis_m} > f_q(S_j) + \frac{\lambda dis(q,q')}{dis_m} \Rightarrow$$

$$f_q(S_i) - f_q(S_j) > \frac{2\lambda dis(q,q')}{dis_m} \Rightarrow$$

$$dis(q,q') < \frac{(f_q(S_i) - f_q(S_j))dis_m}{2\lambda} \Rightarrow$$

$$r_{i,j} = \frac{(f_q(S_i) - f_q(S_j))dis_m}{2\lambda}$$

Fig. 3 shows an example of the priority queue Q and the safe radius, where $m = 6$. We denote the kDNN sets in Q by $S_1, S_2, ..., S_m$, according to their ranks in Q. The kDNN set $S_1 = \{p_3, p_4, p_5\}$ has the largest $f_q()$ function value,

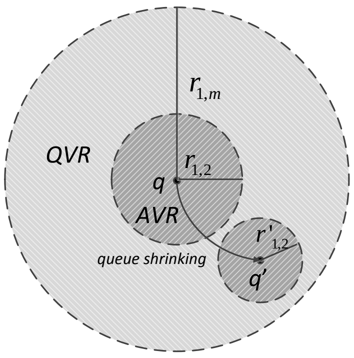


Fig. 4. Queue valid region

and hence it the top item in Q. We compute the safe radius $r_{1,m}, r_{2,m}, ..., r_{m-1,m}$ between the last item in the queue, S_m, and each of the other sets $S_1, S_2, ..., S_{m-1}$, respectively.

Safe regions. The safe radius helps define two circular safe regions centered at q. The first safe region has a radius of $r_{1,2}$ (cf. the smaller circle centered at q in Fig. 4), which guarantees that the kDNN set stays unchanged as long as the query point stays in this region. We call this region the *answer valid region (AVR)*.

The second safe region guarantees that the new kDNN set is still among the m kDNN sets in Q, which means that Q is still valid. We call this region the *queue valid region (QVR)*. As shown by the larger circle centered at q in Fig. 4, the radius of QVR is $r_{1,m}$. Assume that the query point has moved to q'. Then as long as $dis(q, q') < r_{1,m}$, by the definition of the safe radius, $f_{q'}(S_1) > f_{q'}(S_i)$ for any $S_i \notin Q$. This means that any $S_i \notin Q$ cannot be the top kDNN set. Thus, the top kDNN set must still be in Q.

Query updates. Next we derive two properties of the safe radius as specified by the following two theorems. They support our query update algorithm.

Theorem 1. Assume that all size-k subsets of P are sorted in the descending order of their $f_q()$ function values, denoted by S_1, S_2, ... S_m, ..., $S_{C_n^k}$. Then, given three integers i, j, and pt:

1) If $1 \le i < j < pt \le C_n^k$, then $r_{i,pt} > r_{j,pt}$.
2) If $1 \le pt < i < j \le C_n^k$, then $r_{pt,i} < r_{pt,j}$.

Proof 1. See Appendix A.

Suppose that we have computed the kDNN sets and obtained the queue Q at q. When the query point moves to q', we can compare $dis(q', q)$ with the safe radius of all the kDNN sets in Q. For a kDNN set S_j, if its safe radius value $r_{j,m} (1 \le j \le m)$ satisfies $r_{j,m} > dis(q', q)$, then $f_{q'}(S_j) > f'_q(S_m)$ still holds and S_j should still be a top-m kDNN set. We put all the kDNN sets satisfying this condition into a new queue Q' prioritized by their $f_{q'}()$ function values. We denote the last entry in Q' by $S'_{|Q'|}$. For each remaining kDNN set $S_x \in Q$, we compute $f_{q'}(S_x)$, and compare it with $f_{q'}(S'_{|Q'|})$. If $f_{q'}(S_x) > f_{q'}(S'_{|Q'|})$, we also insert S_x into Q'. Otherwise we simply omit S_x since we can no longer guarantee it to be a top kDNN set.

The queue Q' will then become the new Q in the query maintenance, with the top entry S'_1 being the new top kDNN set at q', as guaranteed by the following theorem.

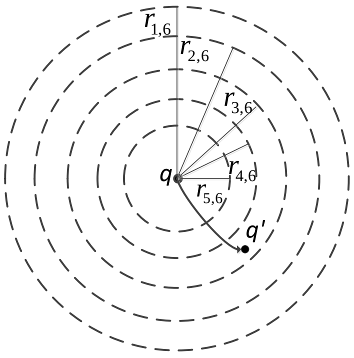


Fig. 5. Query update processing

Theorem 2. Let $S'_{|Q'|}$ be the last entry in Q', i.e., $f_{q'}(S'_{|Q'|})$ is the smallest among the entries in Q'. Then $\forall S \notin Q'$ that is a size-k subset of P: $f_{q'}(S'_{|Q'|}) > f_{q'}(S)$.

Proof 2. See Appendix A.

According to this theorem, when the query point moves, we can keep updating Q to Q' and use Q' as the new Q. Note that every time Q is updated, we might remove some entries from it, and hence Q will shrink. When Q becomes empty, we need to recompute the top-m kDNN sets.

Fig. 5 shows an example. When the query point moves to q', $r_{1,6}$, $r_{2,6}$, and $r_{3,6}$ are greater than $dis(q', q)$. We insert S_1, S_2, and S_3 into a new queue Q', prioritized by $f_{q'}(S_i), i = 1, 2, 3$. We then compare $f_{q'}(S_x), x = 4, 5, 6$ with $f_{q'}(S'_{|Q'|})$, and only keep the ones with $f_{q'}(S_x) > f_{q'}(S'_{|Q'|})$.

4.1.2 The PCPM Algorithm

Algorithm 1 summarizes the query processing procedure. This algorithm provides precise query answers, and we call it the ***Precise Computation and Precise Maintenance (PCPM)*** algorithm. When a query comes, we first perform a kDNN computation, obtain Q and the valid regions, and return the first entry of Q as the answer (lines 1 to 3). Then we start query maintenance (lines 4 to 24). When the query point moves to a new location q', we first test whether q' is still in the AVR. If yes then the query answer has not changed and no further processing is needed (lines 6 to 8). Otherwise we further test whether q' is in the QVR. If not then Q has become invalid and we need to recompute the top-m kDNN sets (lines 9 to 13). If q' is still in the QVR, then Q is still valid, i.e., the new kDNN set is still in Q. We update Q with an auxiliary queue Q' to locate the new kDNN set. We dequeue entries from Q. If an entry S_j's safe radius $r_{j,m}$ is greater than $dis(q', q)$, it is still valid and we re-insert it back to Q' prioritized by the new function value $f_{q'}(S_j)$ (lines 14 to 18). Otherwise we only re-inserted it into Q' if $f_{q'}(S_j) > f_{q'}(S'_{|Q'|})$ (lines 19 to 21). When this is done we replace Q and q by Q' and q', respectively, update the valid regions, and return S_1 in the new Q (lines 22 to 24).

Complexity. The main cost in the above algorithm lies in $kdnnSearch()$, which is a function that computes the top-m kDNN sets and it takes $O(n^k \log m)$ time. Next we derive how frequent this function is computed.

Assume that the query point moves at a constant speed v. In the worst cast, the query point keeps moving away from the last point q where $kdnnSearch()$ is called. When the query point reaches a new location q' where

Algorithm 1: PCPM

```
Input  : $q$: query location
Output: $S_1$: $k$DNN set
1  $Q \leftarrow kdnnSearch()$
2  $AVR \leftarrow circle(q, r_{1,2}), QVR \leftarrow circle(q, r_{1,|Q|})$
3  $reportResult(S_1)$
4  while query continues do
5      $q' \leftarrow$ query point's new location
6      if $q' \in AVR$ then
7          $reportResult(S_1)$
8          continue
9      if $q' \notin QVR$ then
10         $q \leftarrow q'$, $Q \leftarrow kdnnSearch()$
11         $AVR \leftarrow circle(q, r_{1,2}), QVR \leftarrow circle(q, r_{1,|Q|})$
12         $reportResult(S_1)$
13         continue
14     $Q'.clear()$
15     while not $Q.empty()$ do
16         $S_j \leftarrow Q.dequeue()$
17         if $r_{j,|Q|} > dis(q', q)$ then
18             $Q'.enqueue(S_j)$
19         else
20             if $f_{q'}(S_j) > f_{q'}(S'_{|Q'|})$ then
21                 $Q'.enqueue(S_j)$
22     $Q \leftarrow Q'$, $q \leftarrow q'$
23     $AVR \leftarrow circle(q, r_{1,2}), QVR \leftarrow circle(q, r_{1,|Q|})$
24     $reportResult(S_1)$
```

$dis(q', q) > r_{1,m}$, the queue Q becomes invalid. This takes at least $t_1 = \frac{r_{1,m}}{v} = \frac{(f_q(S_1) - f_q(S_m))dis_m}{2v\lambda}$ time. Before the query point reaches q', Q is still valid. Updating Q at every timestamp takes a traversal on the queue ($O(m)$), recomputing the optimization function value for each entry ($O(k^2)$), and (possible) re-insertion of the entry into a new queue ($O(\log m)$). This takes $O(mk^2 \log m)$ time.

Therefore, in a period of t_1 timestamps, the worst case computation cost is $O((t_1 - 1)mk^2 \log m + n^k \log m)$. The amortized computation cost per timestamp is $O(\frac{(t_1-1)mk^2 \log m + n^k \log m}{t_1})$, which is:

$$\frac{((f_q(S_1) - f_q(S_m))dis_m - 2v\lambda)mk^2 \log m + 2v\lambda n^k \log m}{(f_q(S_1) - f_q(S_m))dis_m}.$$

Note that compared with $n^k \log m$, $(t_1 - 1)mk^2 \log m$ is negligible when $k > 2$. Therefor, the amortized computation cost can be simplified to be $\frac{2v\lambda n^k \log m}{(f_q(S_1) - f_q(S_m))dis_m}$.

The I/O cost of computing the top-m kDNN sets by enumeration depends on the size of the main memory and the data set. If the whole data set can fit in the memory, then the I/O cost is just to load the whole data set into memory, the cost of which is $O(\frac{n}{B})$, where n denotes the size of the data set and B denotes the number of data points per disk page. The amortized I/O cost per timestamp is $\frac{2v\lambda n}{(f_q(S_1) - f_q(S_m))Bdis_m}$. If the memory can only hold part of the data set, we need a disk-based algorithm for the enumeration, which is beyond the scope of this paper and will not be discussed further.

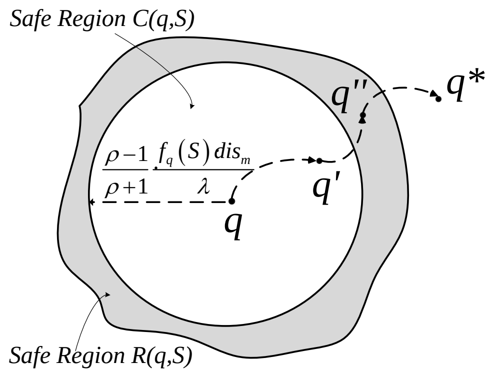


Fig. 6. Safe regions for ρ-approximation

The algorithm requires the whole data set to be stored in the memory to compute the kDNN sets, plus two circles (safe regions) and a queue that "caches" mk data points. Thus, the space complexity of the algorithm is $O(n + mk)$.

Assuming a client-server based system, we analyze the communication cost. In t_1 timestamps, there will be only one time that m kDNN sets are sent from the query server to the client. Thus, the communication cost is $O(mk)$, and the amortized communication cost is $\frac{2v\lambda mk}{(f_q(S_1) - f_q(S_m))dis_m}$.

4.2 A ρ-Approximate Algorithm for kDNN Maintenance

In this subsection we propose an approximate algorithm to further reduce the frequency of kDNN recomputation. This algorithm does not prefetch the top-m kDNN sets. Instead, it just computes the top-1 kDNN set. When the query point moves, this algorithm keeps checking whether the previously computed kDNN set is still a "*good*" approximation of the true kDNN set. If yes then the previously computed kDNN set will be returned as the query answer. Otherwise the kDNN set is recomputed. Here, we use ρ ($\rho > 1$) to denote the approximation ratio bound that defines a "good" approximation. We allow query users to set this bound.

4.2.1 Safe Regions for ρ-Approximation

Let the kDNN set computed at q be S, and the kDNN set when the query point has moved to q' be S_{new}. Then as long as $f_{q'}(S_{new}) \leq \rho \cdot f_{q'}(S)$, S is a ρ-approximation of S_{new}. According to Section 4.1.1,

$$f_{q'}(S_{new}) \leq f_q(S_{new}) + \frac{\lambda dis(q, q')}{dis_m} \leq f_q(S) + \frac{\lambda dis(q, q')}{dis_m},$$

$$f_{q'}(S) \geq f_q(S) - \frac{\lambda dis(q, q')}{dis_m}.$$

Thus, if $f_q(S) + \frac{\lambda dis(q,q')}{dis_m} \leq \rho(f_q(S) - \frac{\lambda dis(q,q')}{dis_m})$, then $f_{q'}(S_{new}) \leq \rho \cdot f_{q'}(S)$ will hold. This means:

$$dis(q', q) \leq \frac{\rho - 1}{\rho + 1}\frac{f_q(S)dis_m}{\lambda} \quad (3)$$

This inequality defines a circular safe region centered at q with a radius of $\frac{\rho-1}{\rho+1}\frac{f_q(S)dis_m}{\lambda}$. We denote this safe region by $C(q, S)$, as shown in Fig. 6. When the query point is in this region, S is a ρ-approximate query answer.

When $C(q, S)$ is derived, we have relaxed both sides of the inequality $f_{q'}(S_{new}) \leq \rho \cdot f_{q'}(S)$. We can derive a stricter safe region by only relaxing $f_{q'}(S_{new})$ to $f_q(S) + \frac{\lambda dis(q,q')}{dis_m}$, which means $f_q(S) + \frac{\lambda dis(q,q')}{dis_m} \leq \rho \cdot f_{q'}(S)$:

$$dis(q', q) \leq \frac{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}{\lambda} \quad (4)$$

This inequality defines a stricter safe region denoted by $R(q, S)$. As Fig. 6 shows, $R(q, S)$ is a region with an irregular shape. This safe region is more expensive to compute compared with $C(q, S)$ because it requires $f_{q'}(S)$, which is changing as the query point moves.

Fig. 6 shows how the two safe regions are used in query processing. When the query point moves, we first test whether it is still in $C(q, S)$ or $R(q, S)$. If it is in either safe region (e.g., at q' or q''), then there is no further computation. Only when the query point has moved out of both safe regions (e.g., at q^*) that we need to recompute the top kDNN set and the corresponding safe regions.

4.2.2 The PCρAM Algorithm

Algorithm 2 summarizes the above procedure, where $topKdnnSearch()$ is a function that computes the top-1 kDNN set (not top-m). Since it maintains answers with an approximation ratio of ρ, we call it the ***Precise Computation and ρ-Approximate Maintenance (PCρAM)*** algorithm.

Algorithm 2: PCρAM

```
Input  : q: query location, ρ: approximation ratio
Output: S: kDNN set
1 $S \leftarrow topKdnnSearch()$
2 while query continues do
3     $q' \leftarrow$ query point's new location
4     if $q' \in C(q, S)$ or $q' \in R(q, S)$ then
5         $reportResult(S)$
6     else
7         $q \leftarrow q'$
8         $S \leftarrow topKdnnSearch()$
9         $reportResult(S)$
```

Complexity. Similar to the cost of the PCPM algorithm, the main cost in the above algorithm is in $topKdnnSearch()$, which takes $O(n^k)$ time.

Assume that the query point moves at a constant speed v. In the worst cast, the query point keeps moving away from the last point q where $topKdnnSearch()$ is computed. When the query point reaches a new location q' where $dis(q', q) > \frac{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}{\lambda}$, the safe regions become invalid. This takes $t_\rho = \frac{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}{v\lambda}$ time. Before the query point reaches q', testing whether it is still in the safe regions requires computing $f_{q'}(S)$ with $O(k^2)$ time.

Therefore, in a period of t_ρ timestamps, the worst case computation cost is $O((t_\rho - 1)k^2 + n^k)$. The amortized computation cost per timestamp is $O(\frac{(t_\rho - 1)k^2 + n^k}{t_\rho})$, i.e.,

$$\frac{((\rho \cdot f_{q'}(S) - f_q(S))dis_m - v\lambda)k^2 + v\lambda n^k}{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}.$$

Compared with n^k, $(t_\rho - 1)k^2$ is negligible when $k > 2$. Therefor, the amortized computation cost can be simplified to be $\frac{n^k v\lambda}{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}$.

The I/O cost of computing the top-1 kDNN sets by enumeration, assuming that the whole data set can fit in the main memory, is $O(\frac{n}{B})$, where n denotes the size of the data set and B denotes the number of data points per disk page. The amortized I/O cost per timestamp is $\frac{v\lambda n}{(\rho \cdot f_{q'}(S) - f_q(S))Bdis_m}$.

The algorithm requires the whole data set to be stored in the memory to compute the kDNN sets, plus the current top kDNN set and two circles (safe regions). The space complexity is $O(n + k)$.

If a client-server based system is used, in t_ρ timestamps, the kDNN set only needs to be sent to the client for once. The communication cost is $O(k)$ and the amortized communication cost is $\frac{kv\lambda}{(\rho \cdot f_{q'}(S) - f_q(S))dis_m}$.

5 MkDNN Algorithm based on Approximate KDNN Computation

5.1 The MaxSumDispersion Algorithm

As the cost analysis above shows, even with approximate maintenance the overall cost is still quite high due to the enumeration cost O(n^k) in kDNN recomputation. In this section we further propose an algorithm to avoid this high recomputation cost by approximation.

We first revisit the kDNN query. Similar to [9], [23], we define $\theta_q(p_i, p_j)$ to be a function of two points p_i and p_j:

$$\theta_q(p_i, p_j) = \lambda(2 - \frac{dis(q, p_i) + dis(q, p_j)}{dis_m}) + \frac{2(1-\lambda)}{dis_m}dis(p_i, p_j).$$

We rewrite Equation 2 with $\theta_q(p_i, p_j)$ which is the optimization function of the kDNN query. The equation becomes:

$$f_q(S) = \frac{1}{k(k-1)} \sum_{p_i, p_j \in S} \theta_q(p_i, p_j) \quad (5)$$

This rewritten equation is very similar to the optimization function of the *Maximum Dispersion* problem [13]. The Maximum Dispersion problem finds a size-k subset S of P to maximize the following optimization function $f(S)$:

$$f(S) = \frac{1}{k(k-1)} \sum_{p_i, p_j \in S} \theta(p_i, p_j) \quad (6)$$

Equations 5 and 6 only differ in that $\theta(p_i, p_j)$ in Equation 6 involves two points p_i and p_j, while $\theta_q(p_i, p_j)$ in Equation 5 involves a third point q.

The Maximum Dispersion problem has been shown to be NP-hard [13], and a 2-approximate algorithm named *MaxSumDispersion* [24] has been proposed to solve the problem.

As summarized in Algorithm 3, the MaxSumDispersion algorithm simply generates all pairs of data points in P, and insert the pairs (the two data points) into S according to their $\theta()$ function values in the descending order (lines 2 to 4). Whenever a pair (p_x, p_y) is chosen for the insertion, any remaining pairs formed by either p_x or p_y will be discarded from further consideration (line 5). This procedure ends when $\lfloor \frac{k}{2} \rfloor$ pairs have been chosen, and S will then contain $2\lfloor \frac{k}{2} \rfloor$ elements. If k is odd, $2\lfloor \frac{k}{2} \rfloor = k - 1$. In this case, a random data point from P is added to S (lines 6 and 7).

Algorithm 3: MaxSumDispersion

```
Input  : $P$: data set, $k$: query parameter
Output: $S$: a size-$k$ subset of $P$ that maximizes $f(S)$
1 $S \leftarrow \emptyset$
2 for $i = 1$ to $\lfloor \frac{k}{2} \rfloor$ do
3     find $(p_x, p_y) = argmax_{p_i, p_j \in P} \theta(p_i, p_j)$
4     $S \leftarrow S \bigcup \{p_x, p_y\}$
5     $P \leftarrow P \setminus \{p_x, p_y\}$
6 if k is an odd number then
7     $S \leftarrow S \bigcup$ {a random point in $P$}
8 return $S$
```

To process a kDNN query, we just need to replace $\theta(p_i, p_j)$ in the algorithm with $\theta_q(p_i, p_j)$. When k is odd, instead of adding a random data point, we add the one from P that is the nearest to the query point q. Since these modifications are straightforward, we omit the pseudo-code due to the space limit. For simplicity, in the following discussion, we still call this modified algorithm the MaxSumDispersion algorithm as long as the context is clear.

Next we prove that this algorithm is a 2-approximation algorithm for the kDNN query, following a similar procedure that the original MaxSumDispersion algorithm was proven to be 2-approximate [13].

Theorem 3. The MaxSumDispersion algorithm is a 2-approximation algorithm for the kDNN query.

Proof 3. See Appendix A.

Next we discuss how to process the MkDNN query with the MaxSumDispersion algorithm.

5.2 A 2-Approximate Algorithm for kDNN Maintenance

We use prefetching to maintain the kDNN set computed by the MaxSumDispersion algorithm. In particular, when the MaxSumDispersion algorithm is called, we use a priority queue Q to store not only the $\lfloor \frac{k}{2} \rfloor$ pairs found by the algorithm but also a few extra point pairs as a "cache" for query updates. Let p_i be a data point in the $\lfloor \frac{k}{2} \rfloor$ pairs, and $(p_{\lfloor \frac{k}{2} \rfloor_1}, p_{\lfloor \frac{k}{2} \rfloor_2})$ be the $\lfloor \frac{k}{2} \rfloor^{th}$ pair. Then Q also stores a point pair (p_x, p_y) if the pair contains p_i and $\theta_q(p_x, p_y) \geq \theta_q(p_{\lfloor \frac{k}{2} \rfloor_1}, p_{\lfloor \frac{k}{2} \rfloor_2})$. Further, we store m more pairs that have the largest $\theta_q()$ function values smaller than $\theta_q(p_{\lfloor \frac{k}{2} \rfloor_1}, p_{\lfloor \frac{k}{2} \rfloor_2})$. These extra point pairs are stored in Q together with the top $\lfloor \frac{k}{2} \rfloor$ point pairs in the descending order of their $\theta_q()$ function values. We will study the impact of the value of m in the experimental section.

We modify the MaxSumDispersion algorithm to obtain Q as shown in Algorithm 4. The modified algorithm only contains a few addition lines (lines 7 to 10) to collect the point pairs and store them into Q. It does not change the data points to be added to the kDNN set S, and hence is still a 2-approximate algorithm.

After this algorithm is called we can return the kDNN set S. When the query point moves, we keep finding the new kDNN set from the point pairs in Q until they cannot guarantee a 2-approximation, i.e., Q becomes invalid.

Safe region. Next we define a safe region to guarantee the validity of Q with the *safe radius for the data point pairs*.

Algorithm 4: MaxSumDispersion-MkDNN

```
Input  : $P$: data set, $q$: query location, $k$: query parameter
Output: $S$: a size-$k$ subset of $P$ that maximizes $f(S)$, $Q$: a
        priority queue to store the point pairs for query
        maintenance
1  $S \leftarrow \emptyset$, $Q \leftarrow \emptyset$, $i \leftarrow 1$
2  while $i \leq \lfloor \frac{k}{2} \rfloor$ do
3      find $(p_x, p_y) = argmax_{p_i, p_j \in P, (p_i, p_j) \notin Q} \theta_q(p_i, p_j)$
4      if $p_x, p_y \notin S$ then
5          $S \leftarrow S \bigcup \{p_x, p_y\}$
6          i++
7      $Q \leftarrow Q \bigcup \{(p_x, p_y)\}$
8  for $i = 1$ to $m$ do
9      find $(p_x, p_y) = argmax_{p_i, p_j \in P, (p_i, p_j) \notin Q} \theta_q(p_i, p_j)$
10     $Q \leftarrow Q \bigcup \{(p_x, p_y)\}$
11 if k is an odd number then
12     $S \leftarrow S \bigcup$ {the point nearest to $q$ in $P \setminus S$}
13 return $S, Q$
```

Definition 4 (Safe radius for two data point pairs). Let the query point be at q. Two pairs of data points (p_{i_1}, p_{i_2}) and (p_{j_1}, p_{j_2}) satisfy $\theta_q(p_{i_1}, p_{i_2}) > \theta_q(p_{j_1}, p_{j_2})$. The safe radius for this two pairs, denoted by $rp_{i,j}$, is defined to guarantee $\theta_{q'}(p_{i_1}, p_{i_2}) > \theta_{q'}(p_{j_1}, p_{j_2})$ for the query point to move to a new location q' as long as $dis(q, q') < rp_{i,j}$.

Following the same procedure in Section 4.1.1 but replacing $f_q(S_i), f_q S_j, f_{q'}(S_i)$, and $f_{q'}(S_j)$ with $\theta_q(p_{i_1}, p_{i_2}), \theta_q(p_{j_1}, p_{j_2}), \theta_{q'}(p_{i_1}, p_{i_2})$, and $\theta_{q'}(p_{j_1}, p_{j_2})$, we can derive that if $\theta_q(p_{i_1}, p_{i_2}) > \theta_q(p_{j_1}, p_{j_2})$, then $\theta_{q'}(p_{i_1}, p_{i_2}) > \theta_{q'}(p_{j_1}, p_{j_2})$ as long as $dis(q, q') < \frac{(\theta_q(p_{i_1}, p_{i_2}) - \theta_q(p_{j_1}, p_{j_2}))dis_m}{4\lambda}$, i.e.,

$$rp_{i,j} = \frac{(\theta_q(p_{i_1}, p_{i_2}) - \theta_q(p_{j_1}, p_{j_2}))dis_m}{4\lambda}.$$

We omit the detailed derivation as it is straightforward.

Similar to Theorem 1, we can derive the following property of $rp_{i,j}$.

Theorem 4. Assume that all point pairs of P are sorted in the descending order of their $\theta_q()$ function values, denoted by (p_{1_1}, p_{1_2}), (p_{2_1}, p_{2_2}), ... , $(p_{C^2_n 1}, p_{C^2_n 2})$. Then, given three integers i, j, and pt:

1) If $1 \leq i < j < pt \leq C^2_n$, then $rp_{i,pt} > rp_{j,pt}$.
2) If $1 \leq pt < i < j \leq C^2_n$, then $rp_{pt,i} < rp_{pt,j}$.

Proof 4. See Appendix A.

Let (p_{i_1}, p_{i_2}) be a point pair in Q where both p_{i_1} and p_{i_2} are in the current kDNN set S. Then $rp_{i,i+1} < rp_{i,j}, \forall j > i+1$. When the query point moves from q to q',

$$dis(q, q') < rp_{i,i+1} \Rightarrow dis(q, q') < rp_{i,j}, \forall j > i+1.$$

This means that the point pair (p_{i_1}, p_{i_2}) is still safe, and that no point pairs after (p_{i_1}, p_{i_2}) will be chosen over (p_{i_1}, p_{i_2}) by the MaxSumDispersion algorithm.

Further, if this holds for every point pair in Q where both points are in the current kDNN set S, then S (and hence Q) is still valid, i.e.,

$$dis(q, q') < \min \{rp_{i,i+1} | p_{i_1}, p_{i_2} \in S\}.$$

We denote $\min\{rp_{i,i+1}|p_{i_1}, p_{i_2} \in S\}$ by rp_{min} and further refine rp_{min} as follows.

- If $p_{i_1}, p_{i_2}, p_{(i+1)_1}, p_{(i+1)_2}$ are all in S, then $rp_{i,i+1}$ can be excluded from rp_{min} since swapping (p_{i_1}, p_{i_2}) and $(p_{(i+1)_1}, p_{(i+1)_2})$ will not change S.
- If either $p_{(i+1)_1}$ or $p_{(i+1)_2}$ is in S, then $rp_{i,i+1}$ can be excluded from rp_{min} as well. This is because according to the MaxSumDispersion algorithm, the pair $(p_{(i+1)_1}, p_{(i+1)_2})$ will not be added to S.

The refined rp_{min} defines the *safe region* to guard S and Q.

Computing the new kDNN set from Q. When the query point has moved out of the safe region defined by rp_{min}, S becomes invalid. We first attempt to compute the new set of S from the data pairs in Q. We divide Q into three parts:

- The last point pair of Q, i.e., $(p_{|Q|_1}, p_{|Q|_2})$.
- Sub-queue Q_1 that contains every point pair (p_{i_1}, p_{i_2}) satisfying $dist(q, q') < rp_{i,|Q|}$, i.e., $Q_1 = \{(p_{i_1}, p_{i_2})|dist(q, q') < rp_{i,|Q|}\}$.
- Sub-queue Q_2 that contains the rest of the point pairs of Q, i.e., $Q_2 = \{(p_{i_1}, p_{i_2})|dist(q, q') \geq rp_{i,|Q|}\}$.

The point pairs in Q_1 still have larger $\theta_q()$ function values than that of any pair *not* in Q, as formalized by the following theorem.

Theorem 5. $\forall(p_{i_1}, p_{i_2}) \in Q_1, (p_{j_1}, p_{j_2}) \notin Q$, when the query point has moved from q to a new location q', we have $\theta_{q'}(p_{i_1}, p_{i_2}) > \theta_{q'}(p_{j_1}, p_{j_2})$.

Proof 5. See Appendix A.

We compute $\theta_{q'}()$ for the point pairs in Q_1, and insert them into a new priority queue Q' in the descending order of the new function values. The last point pair $(p_{|Q'|_1}, p_{|Q'|_2})$ in Q' has the smallest $\theta_{q'}()$ value, which is still larger than that of any point pair not in Q, according to Theorem 5.

We add a point pair from Q_2 into Q' as well if its $\theta_{q'}()$ function value is larger than or equal to $\theta_{q'}(p_{|Q'|_1}, p_{|Q'|_2})$.

Then we run the modified MaxSumDispersion algorithm on Q' and try to find $\lfloor \frac{k}{2} \rfloor$ point pairs to form the new S and Q. If successful then the new sets will be returned. Based on Theorem 5, the new set S is still a 2-approximate answer set. If less than $\lfloor \frac{k}{2} \rfloor$ valid point pairs can be found from Q', then a full query recomputation is needed. The modified MaxSumDispersion algorithm will run on the full data set to compute the new sets of S and Q again.

5.2.1 The 2AC2AM Algorithm

Algorithm 5 summarizes the above procedure. Since this algorithm computes and maintains 2-approximate query answers, we call it the ***2-Approximate Computation and 2-Approximate Maintenance (2AC2AM)*** *algorithm*. At start, this algorithm computes an approximate kDNN set S and the priority queue Q using the MaxSumDispersion-MkDNN algorithm (line 1). Then query maintenance begins. When the query point moves from q to a new location q', if $dist(q, q') < rp_{min}$, the query result stays unchanged and no further processing is needed (lines 4 to 8). Otherwise, the algorithm first updates the ranking of the point pairs in Q (line 9). Then it tries to find $\lfloor \frac{k}{2} \rfloor$ pairs from the updated queue Q' that can contribute to the new kDNN set S (line 11). If successful then Q' becomes the new Q and the new set S is returned as the query result (lines 12 to 14). If not then the MaxSumDispersion-MkDNN algorithm is called again to obtain the new S and Q (line 16). This process repeats and updated query answers will be produced continuously.

Algorithm 5: 2AC2AM

```
Input  : $P$: data set, $q$: query location, $k$: query parameter
Output: $S$: $k$DNN set
1  $S, Q \leftarrow$ MaxSumDispersion-M$k$DNN($P, q, k$)
2  $reportResult(S)$
3  while query continues do
4      $q' \leftarrow$ query point's new location
5      $rp_{min} \leftarrow \min\{rp_{i,i+1}|p_{i_1}, p_{i_2} \in S\}$
6      if $dist(q', q) < rp_{min}$ then
7          $reportResult(S)$
8          continue
9      $Q' \leftarrow$ update $Q$
10     $q \leftarrow q'$
11     compute $S$ from $Q'$
12     if $|S| = k$ then
13         $Q \leftarrow Q'$
14         $reportResult(S)$
15     else
16         $S, Q \leftarrow$ MaxSumDispersion-MkDNN($P, q, k$)
17         $reportResult(S)$
```

Complexity. Storing the whole data set in the memory takes $O(n)$ space. In addition, the algorithm will cache every point pair (p_x, p_y) if $p_x \in S$ and $p_y \notin S$, and that $\theta_q(p_x, p_y) > \theta_q(p_i, p_j)$ where both $p_i, p_j \in S$. In the worst case, this will result in $(k-2)(n-1)$ pairs being cached. This occurs when there are two points (p_1^*, p_2^*) in S that form a pair with an extremely small $\theta_q()$ function value, which leads to every point $p_x \in S, p_x \neq p_1^*, p_2^*$ (a total of $k-2$ points) to bring a point pair into the cache with every point in the data set (except for p_x itself, a total of $n-1$ pairs). The algorithm further caches another m pairs (p_a, p_b) where $p_a, p_b \notin S$. Also, it will need to store the current top kDNN set. Therefore, the overall worst case space complexity is $O(n+2(k-2)(n-1)+2m+k) = O(kn-k+m)$. However, this extreme case rarely occurs in reality. For all the data sets tested in our experiments, the cache size is reasonably small that incurs little extra time to maintain.

MaxSumDispersion-MkDNN takes $O((kn-k+m)n^2\log(kn-k+m))$ time to generate the cached pairs. Assume that the query point moves at a constant speed v. In the worst cast, the query point keeps moving away from the last point q where the approximate kDNN set S is computed. When the query point reaches a new location q' where $dis(q', q) \geq rp_{min} = \min\{rp_{i,i+1}|p_{i_1}, p_{i_2} \in S\}$, the set S becomes invalid. This takes at least $t_2 = \dfrac{rp_{min}}{v}$ time. Before the query point reaches q', computing rp_{min} for testing whether $dis(q', q) < rp_{min}$ takes $O(k)$ time.

Therefore, in a period of t_2 timestamps, the worst case computation cost is $O((t_2-1)k + (k+m)n^2\log n)$. The amortized computation cost is $O(\frac{(t_2-1)k+(k+m)n^2\log n}{t_2})$, i.e.,

$$\frac{(rp_{min}-v)k + v(k+m)n^2\log n}{rp_{min}}.$$

Compared with $v(k+m)n^2 \log n$, $(rp_{min} - v)k$ is negligible. Therefor, the amortized computation cost can be simplified to be $\frac{v(k+m)n^2 \log n}{rp_{min}}$.

Assuming that the whole data set can fit in the main memory, the I/O cost of the MaxSumDispersion-MkDNN algorithm is $O(\frac{n}{B})$, where n denotes the size of the data set and B denotes the number of data points per disk page. The amortized I/O cost per timestamp is $\frac{vn}{rp_{min}B}$.

If a client-server based system is used, in t_2 timestamps, the kDNN set only needs to be sent for once. The communication cost is $O(k)$ and is amortized to be $O(\frac{kv}{rp_{min}})$.

6 Experiments

TABLE 1
Experiment parameters

Parameter	Default	Values
m for PCPM	40	5, 10, 20, 30, 40, 50, 60, 70
m for 2AC2AM	30	5, 10, 20, 30, 40, 50, 60, 70
k	6	3, 6, 12, 24, 48
λ	0.5	0.1, 0.3, 0.5, 0.7, 0.9
trajectory interval	900	100, 300, 900, 2700, 8100
ρ	1.5	1.1, 1.2, ..., 2.0
data set	LA	LA, NY
data set size	500	500, 1000, 10000, 50000, 500000
query trajectory	directional	directional, random

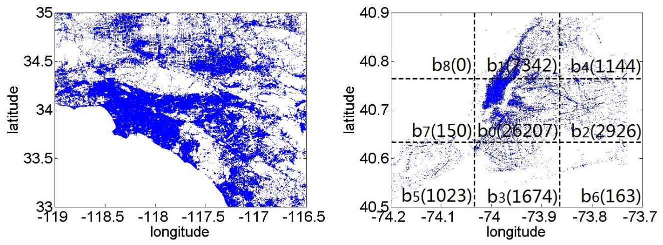
6.1 Settings

We empirically compare the three proposed algorithms, PCPM (Section 4.1), PCρAM (Section 4.2), and 2AC2AM (Section 5) with a sampling-based algorithm denoted by $BASE$, which computes a kDNN query at every timestamp.

The algorithms are implemented in Java, and ran on a desktop computer with a 3.40GHz Intel Core i7-2600 CPU, 8GB memory, and 64-bit Windows operating system.

We use two real data sets. The first data set contains 501,841 Twitter check-in locations in Los Angeles and nearby regions extracted based on coordinates from the data set used in [25]. The second data set contains 40,629 Foursquare check-in venues in New York City and nearby suburbs extracted based on coordinates from the data set used in [26]. We denote the two data sets by "**LA**" and "**NY**", respectively. Fig. 7 visualizes the two sets. We see that NY contains fewer data points overall and is more skewed. To test the effect of data density and distribution, we generate more data sets by randomly sampling data points from these two data sets (i.e., varying density), and by taking data points from different partitions of the data space (i.e., varying distribution). By default the LA data set is used.

We generate two types of trajectories for the query point, "random" and "directional". In the random trajectories, the query point starts at a random point in the data space, and moves towards a randomly chosen new direction at every timestamp. In the directional trajectories, the query point also starts at a random point and chooses a random direction, but it then keeps moving towards this direction until reaching the boundary of the space, where a new direction towards within the space is chosen. By default, between two timestamps (i.e., two query computations) the query point moves for a distance interval that is randomly



(a) LA (b) NY

Fig. 7. The data sets

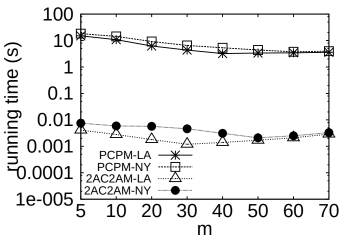
generated between 1 and 900 meters. We did not use a smaller distance interval for the following reason. The data points are distributed in a large space. A smaller distance interval will result in highly stable MkDNN query answers and large safe regions, which benefits the proposed algorithms but not the BASE algorithm.

We generate 20 trajectories (100 timestamps each) for each set of experiments. We report the average CPU time per timestamp, the average total I/O cost (number of page accesses assuming a page size of 1KB and 50 data points per page) for processing each query, and the average total number of times the kDNN set is recomputed for each query. Here, this number of recomputation would represent the communication cost if a client-server based system were used. Further, to verify the effectiveness of the proposed approximation algorithms we report the precision of the algorithms computed by $\frac{f_q(S)}{f_q(S_*)}$, where S denotes the kDNN set returned by the approximation algorithms and S_* denotes the true kDNN set.

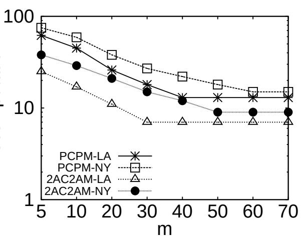
We vary the query parameter k, queue size m, optimization weight parameter λ, approximation ratio ρ, query computation distance interval, and data set cardinality in the experiments. The value ranges and default values of these parameters are summarized in Table 1. Due to the inherently high computation complexity of the precise kDNN sets in PCPM and PCρAM, we use a data set of 500 data points and $k = 6$ by default so that we can test PCPM and PCρAM together with 2AC2AM under different settings of other parameters. We argue that this is not an unreasonable setting. In real applications such as restaurant or tourist hotspot finding, while the total number of restaurants or tourist hotspots may be large, we can request for additional keywords or categories (e.g., "pizza" or "museum") to reduce the number of possible candidates to a few hundred. In addition, as the query user is moving it is difficult to browse through a large number of answers, especially when using a mobile device. Therefore, a small k value such as 6 is not unreasonable. In such scenarios, both PCPM and PCρAM may be used. When the data set or k is larger 2AC2AM may be used to provide approximate results.

6.2 Results

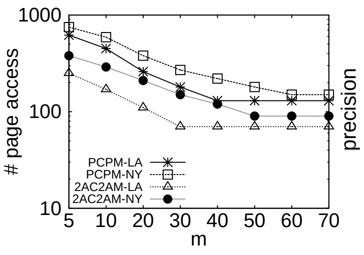
Effect of prefetching. PCPM and 2AC2AM both use prefetching to reduce kDNN recomputation. In particular, PCPM prefetches top-m kDNN sets; 2AC2AM prefetches m extra data pairs. We test the effect of m and aim to find its optimal value to be used in the rest of the experiments.



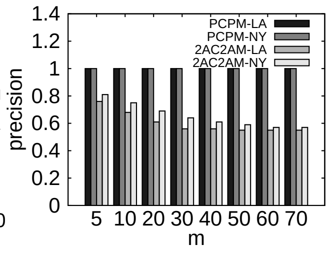
(a) CPU time



(b) kDNN recomputation

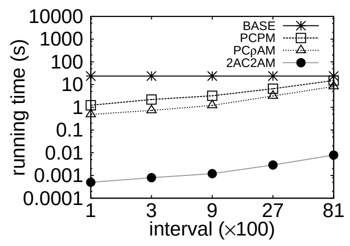


(c) I/O

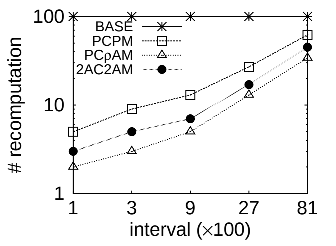


(d) Precision

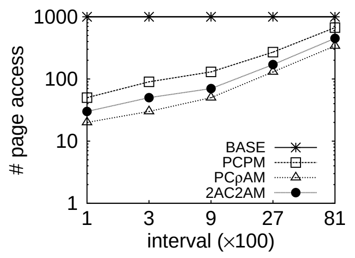
Fig. 8. Effect of prefetched data size, m



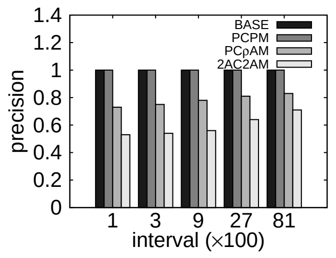
(a) CPU time



(b) kDNN recomputation



(c) I/O



(d) Precision

Fig. 9. Effect of query computation distance interval (directional)

As Fig. 8 shows, the CPU time, the number of kDNN recomputation, and the I/O cost of both algorithms drop initially as m increases, which is expected because a larger "cache" reduces the frequency of kDNN recomputation. However, when m continues to increase, the benefit of "caching" increases little and higher queue maintenance cost is required. As such, the algorithm costs rise back. We observe that: PCPM shows the best performance at $m = 40$ on the LA data set and $m = 60$ on the NY data set; 2AC2AM shows the best performance at $m = 30$ on the LA data set and $m = 50$ on the NY data set. The larger optimal values of m on a more skewed data set (i.e., the NY data set) can be explained by that, on a more skewed data set, the query answer becomes invalid more frequently in the denser area. It requires a larger prefetched queue to reduce the recomputation frequency. In the rest of the experiments, we use these optimal values as the default values.

Fig. 8 (d) shows the precision of the two algorithms. As we can see, PCPM provides accurate query answers all the time (precision = 1), while 2AC2AM provides 2-approximation results (precision ≥ 0.5). Note that as m increases, the accuracy of 2AC2AM decreases. This is because a large "cache" discourages kDNN recomputation as long as the current answer is still a 2-approximation, which means that the precision tends to stay at 0.5.

Effect of query computation distance interval. In Fig. 9, we compare the three proposed algorithms with the BASE algorithm on the LA data set using directional query trajectories, where the query computation distance interval is varied from 100 to 8100 meters. Note that, for every interval value used (e.g., 900), the query point speed is *not* fixed at the interval value per timestamp (e.g., 900 meters per timestamp). Instead, the query point speed is randomly chosen between 1 and the interval value at every timestamp. When the distance interval increases, the CPU cost, the number of kDNN recomputation, and the I/O cost increase for the three proposed algorithms while those of BASE stay stable. This is because BASE simply recomputes kDNN at every timestamp, which is not affected by the computation interval; the proposed algorithms rely on safe regions to reduce the recomputation frequency, which become invalid more frequently as the query distance interval increases. However, the three proposed algorithms outperform BASE consistently, which validates the effectiveness of our proposed techniques to reduce the recomputation frequency. In particular, 2AC2AM has the lowest CPU cost (Fig. 9 (a)), because it uses an approximate kDNN algorithm with a low complexity. Meanwhile, PCρAM has the lowest recomputation cost (Fig. 9 (b)). This is because of the strict bounds used by PCρAM that define large safe regions. Correspondingly, PCρAM also has the lowest I/O cost. In Fig. 9 (d), both BASE and PCPM show constant precision values of 1 since they are both precise algorithms. The two approximate algorithm 2AC2AM and PCρAM (ρ=1.5) show precision values that are no less than 0.5 and 0.66, respectively. These confirm the effectiveness of the proposed algorithms.

We have also tested the algorithms on random query trajectories. The comparative performance of the algorithms is similar to that shown in Fig. 9. To keep the paper concise we have put the figure in Appendix B. In the following experiments we omit the figures on random query trajectories. A further observation is that the costs for the proposed algorithms are generally lower on random query trajectories. This is because when the query point moves randomly instead of directionally, its probability of staying in the current safe region is higher and hence the kDNN recomputation frequency is lower.

On the NY data set, the comparative performance of the algorithms also show a similar pattern, we omit the figures due to space limit (same in the following experiments).

Effect of data set size. Next we vary the data set size. In Fig. 10 we randomly sample data points from the LA data set to obtain data set of different sizes. As Fig. 10 (a) shows, computing a single kDNN query takes more than 20 seconds for the BASE algorithm on 500 data points. This is because the MkDNN query is inherently complex. The

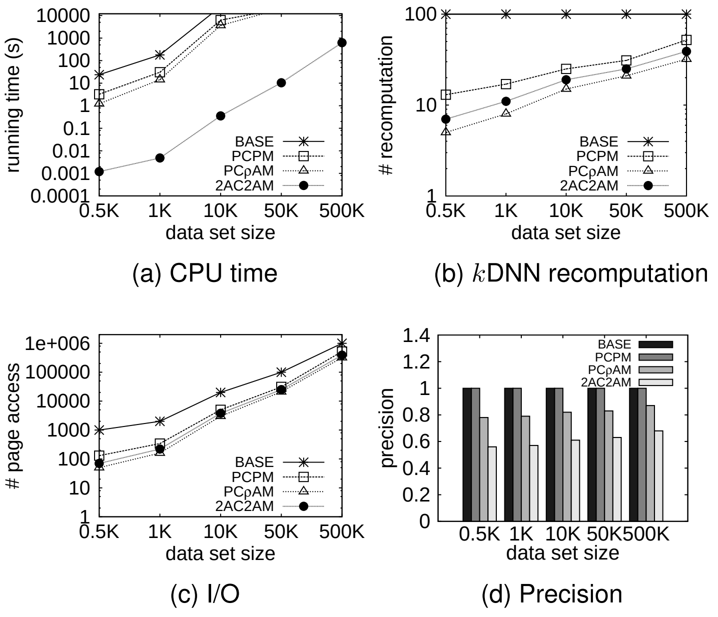


Fig. 10. Effect of data set size (by random sampling)

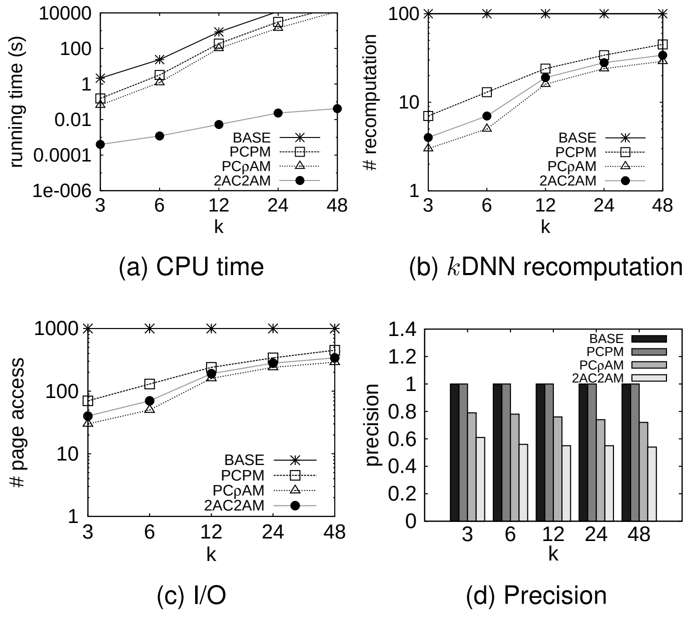


Fig. 12. Effect of k

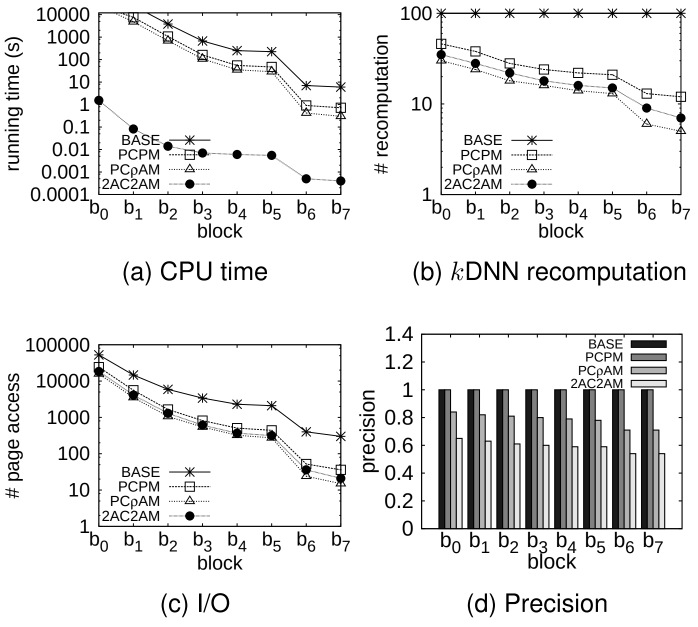


Fig. 11. Effect of data set size (by space partitioning)

two proposed algorithms PCPM and PCρAM have lower *average* running time (around 1 second) per timestamp, but at timestamps where kDNN recomputation is needed, they would also need to run for more than 20 seconds. Only 2AC2AM can process the query in time for the larger data sets, which demonstrates the scalability of this algorithm.

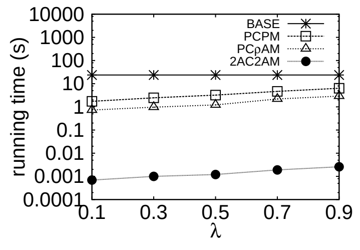
The costs are even higher when larger data sets are used, since the safe regions will become smaller and kDNN set recomputation will become more frequent. Note that in Fig. 10 (a) we have omitted the points beyond 10000 seconds, as they are too high to be practical.

Fig. 11 shows the algorithm performance on data sets obtained by partitioning the NY data set. In particular, we partition the space by a 3×3 regular grid. As a result we obtain 9 blocks denoted by $b_0, b_1, ..., b_8$, each with 26207, 7342, 2926, 1674, 1144, 1023, 163, 150, 0 data points, respectively. Note that b_8 is the top left block which does not contain any data points and hence no experiment is performed on it. As the figure shows, in b_0, only 2AC2AM can produce the query answers. All the other algorithms rely on precise kDNN computation, and cannot produce query answers in time on such dense and skewed data set. In the other blocks, the algorithms' costs drop as the data points become fewer and the distribution becomes more even. The proposed algorithms again show better performance consistently.

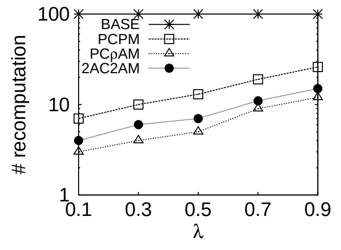
Effect of k. Fig. 12 shows the results when we vary k from 3 to 48. Note that BASE, PCPM, and PCρAM all involve computing precise kDNN answers, which is NP-hard. As k increases their computational costs increase dramatically. In Fig. 12 (a) we have also omitted the points beyond 10000 seconds. Meanwhile, 2AC2AM uses an approximation algorithm for kDNN recomputation with much lower complexity. It scales much better with larger k values.

Effect of λ. In the optimization function we use a parameter λ to allow query users to set the preference on spatial proximity over diversity. Fig. 13 shows the impact of λ. Again the proposed algorithms outperform BASE. An observation is that the costs of the proposed algorithms increase as λ increases. This can be explained by that a larger λ value will lead to smaller safe regions, as shown by the safe region definition equations. This will lead to higher kDNN recomputation frequency and hence higher costs.

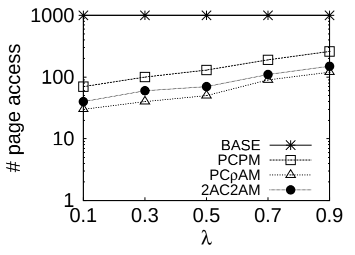
Effect of ρ. In PCρAM, we use a parameter ρ to allow a query user to set the required approximation ratio. Fig. 14 shows the impact of ρ. We see that a larger value of ρ can reduce the costs, because the required precision drops and less frequent kDNN recomputation is needed. Note that the precision value is still bounded by $\frac{1}{\rho}$ in all cases shown in Fig. 14 (d). This confirms the effectiveness of PCρAM. When $\rho = 1$, PCρAM produces precise query answers at every timestamp, which are the same as those produced by PCPM. However, its performance is worse because it does not "cache" any kDNN candidates, while PCPM has a queue Q to serve as a "cache". When $\rho = 2$,



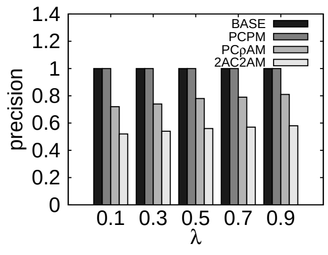
(a) CPU time



(b) kDNN recomputation

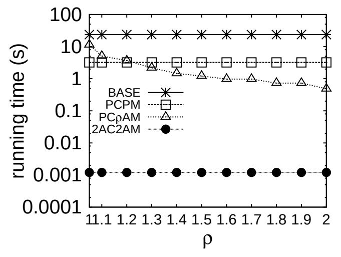


(c) I/O

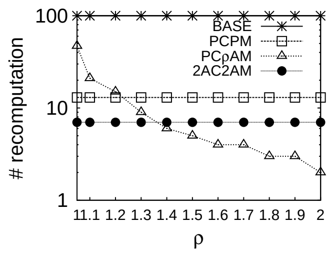


(d) Precision

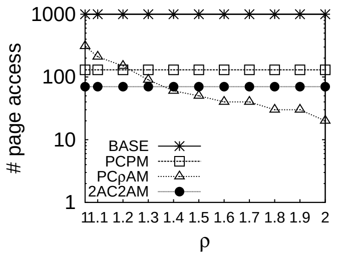
Fig. 13. Effect of λ



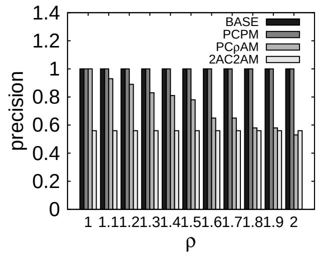
(a) CPU time



(b) kDNN recomputation



(c) I/O



(d) Precision

Fig. 14. Effect of approximation ratio, ρ

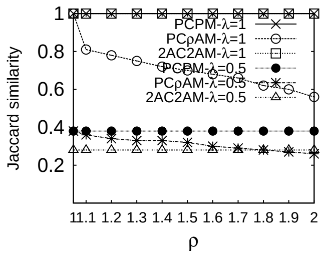


Fig. 15. kNN vs. kDNN

PCρAM produces approximate query answers similar to those produced by 2AC2AM. Its query answer precision is slightly lower as shown in Fig. 14 (d) (although still above 0.5). This is because PCρAM uses a larger safe region than that of 2AC2AM and tends to update the query answer less frequently. As a result, the recomputation frequency and I/O costs of PCρAM are both lower than those of 2AC2AM. However, PCρAM has a much higher kDNN computation cost because its precise kDNN computation has a high time complexity. As a result, its running time is till higher. Therefore, in summary, in application scenarios where ρ is fixed at 1 or 2, PCPM and 2AC2AM should be used. In other scenarios, PCρAM may be used for its flexibility in obtaining query answers of varying approximation ratios.

kNN vs. kDNN. We further validate the effectiveness of the MkDNN query by evaluating the similarity between the kNN sets and the kDNN sets computed under the same default query settings.

We first obtain the kNN sets by setting the distance-diversity trade-off parameter λ to be 1 in PCPM (PCPM-$\lambda = 1$). This way the optimization function is only determined by the spatial distance of the data points to the query point, not the diversity, and hence the algorithm will produce the kNN sets, not kDNN sets. We use these kNN sets as the benchmark for comparing the similarity between the different query answer sets. We then run PCPM at $\lambda = 0.5$ to produce precise kDNN sets (PCPM-$\lambda = 0.5$). Similarly, we run PCρAM at $\lambda = 1, 0.5$ (PCρAM-$\lambda = 1$, PCρAM-$\lambda = 0.5$), and vary ρ from 1 to 2 (the precision is therefore varied from 1 to 0.5). Now the algorithm will produce approximate kNN and kDNN sets. We do the same for 2AC2AM as well (2AC2AM-$\lambda = 1$, 2AC2AM-$\lambda = 0.5$). Its precision cannot be varied, however, because it is designed to return kDNN sets with an approximation ratio of 0.5.

We record the average *Jaccard similarity* between the actual kNN set and the sets obtained as above. Here, the Jaccard similarity between two sets S_1 and S_2 is defined as:

$$similarity(S_1, S_2) = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|}$$

Fig. 15 shows the result. Specifically, PCPM-$\lambda = 1$ are the precise kNN sets. The similarity value is 1. PCPM-$\lambda = 0.5$ are the precise kDNN sets. The similarity to the kNN sets is just 0.4. For PCρAM, its similarity value drops as ρ increases (i.e., less precise answer sets are produced). When $\lambda = 0.5$ the kDNN sets computed are very different from the kNN sets. The similarity values are below 0.4 for all cases tested. In comparison the approximate kNN sets produced by PCρAM at $\lambda = 1$ are more similar to the actual kNN sets (similarity values above 0.5). These approximate kNN sets are still significantly different from the approximate kDNN sets produced by PCρAM at $\lambda = 0.5$. Similar difference is observed on the approximate kDNN sets produced by 2AC2AM-$\lambda = 0.5$, except that the set similarity is unaffected by ρ (note when $\lambda = 1$, 2AC2AM will compute precise kNN sets according to the MaxSumDispersion algorithm). Therefore, we can see that the kDNN sets are significantly different from the kNN sets, which confirms that effectiveness of the MkDNN query to diversity the query result.

7 Conclusion

We formulated the MkDNN query and conducted a comprehensive study. We first proposed two algorithms to reduce the query recomputation frequency, using the prefetching technique and the safe region technique, respectively. Our experiments show that they both can process the MkDNN query effectively. However, due to the high cost of query recomputation, they still lacked computation efficiency. To overcome this limitation, we further proposed an algorithm that computes approximate kDNN sets at recomputation, which successfully reduces the recomputation cost and hence the overall query costs. We conducted a detailed cost analysis for the three proposed algorithms and extensive experiments to evaluate their performance. The results confirmed our cost analysis and the advantages of the proposed algorithms over the baseline algorithm.

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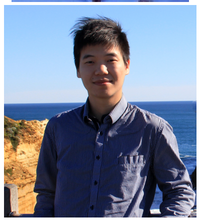

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