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Investigation of Biaxial Bending of Reinforced Concrete Columns Through Fiber Method Modeling

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The analysis and design of reinforced concrete columns with biaxial bending are difficult because a trial and adjustment procedure is necessary to find the inclination and the depth of the neutral axis that satisfies equilibrium conditions. This study addresses the problem of accurately predicting the behavior of a reinforced concrete column with biaxial bending through fiber method modeling in order to be able to establish its capacity at the ultimate stage. The fiber method has been found to be an effective method in predicting the flexural response of structural members, especially when bending moments and axial loads dominate the behavior. In implementing the fiber method, Bazant's Endochronic theory was used as a constitutive model for concrete while the Ciampi model was used for steel. The effects of different structural parameters were considered in establishing the interaction surfaces. Numerical analyses of square reinforced concrete columns with symmetrical reinforcement were conducted. The strength of concrete considered varied from 21 MPa (3,000 psi) to 62 MPa (9,000 psi). The result of the fiber method modeling agreed well with some available experimental data. The development of interaction diagrams for the biaxial bending of column sections provides structural designers with an alternative way to analyze and design such column sections.

INTRODUCTION

Generally, the column is the most critical part of a building, bridge, or any structural skeletal frame system. A failure of one of these columns could lead to disastrous damage. These columns are usually loaded by biaxial bending, aside from axial compression or tension. However, most of the designs for reinforced concrete columns is based on unidirectional bending. This is understandable because most of the available design charts are for the unidirectional bending of reinforced concrete columns. These design charts can be extended to biaxial bending through the use of the "load contour method" and "reciprocal load method" developed by Bresler (1960). It is upon this premise that this research aims to investigate the behavior of reinforced concrete columns subjected to biaxial bending and to develop design charts for this kind of loading.

Most of the design charts available today are only for the uniaxial bending of columns. The development of design charts for biaxial bending of column sections will provide structural designers with an alternative way to analyze and design such column sections. This will not only make the design work easier but will also increase accuracy and, in turn, provide greater safety to the structure. In the course of developing the design charts, a better understanding of the behavior of biaxially-loaded columns will be achieved. The computer program that will be developed may be used as instructional material to help both students and practicing professionals understand and visualize what happens to a column subjected to biaxial bending.

Review of Related Literature

Over the past years, different approximation methods for the analysis and design of biaxial bending of reinforced concrete columns have been used by structural designers. The most popular among these are Bresler's (1960) Load Contour Method and the Reciprocal Load Method. Shown below is Bresler's equation:

$$(Mz / Moz)^{\alpha} + (My / Moy)^{\alpha} = 1 \qquad \text{(Eq. 1)}$$

Where Mz = actual moment about z-axis
My = actual moment about y-axis
Moz = moment capacity about the x-axis under unidirectional bending
Moy = moment capacity about the x-axis under unidirectional bending

However, Nilson (1997), in his book entitled "Design of Concrete Structures", states the following: "Although the load contour method and reciprocal load method are widely used in practice, each has serious shortcomings. With the load contour method, selection of the appropriate value of the exponent α is made difficult by a number of factors relating to the column shapes and bar distribution."

According to Park and Paulay (1975), the study of biaxial bending over the past years may be classified into the following: (a) methods of superposition; (b) methods of equivalent uniaxial eccentricity; and (c) methods based on approximation of shape interaction surface. Moran (1972) discussed simplified methods of superposition to reduce the inclined bending to bending about the major axes, thus allowing the uniaxial bending approach for the case of symmetrical reinforcement. This method has been used in the code of Valenzuela. In the 1968 Spanish Code, an approximate analytical expression was adopted in order to be able to determine the equivalent uniaxial eccentricity of a section. Aside from Bresler's popular methods, Pannell (1963), Furlong (1961), and Meek (1963) also made suggestions for the shape of the interaction surface. In the case of Weber (1966), he produced a series of design charts for square columns by linear interpolation between bending about the major axis and bending about a diagonal. A major drawback of these previous studies is the problem of finding an accurate model for the stress-strain relationship of concrete.

A new and simpler way, as compared to the Finite Element Method, is to study the behavior of reinforced columns which may have been pioneered by Kaba and Mahin (1984). They presented the concept of the fiber method in their refined modeling of reinforced concrete columns for seismic analysis. The fiber method was found to be an effective method in predicting the flexural hysteretic response of reinforced concrete members, especially when bending and axial load dominate the behavior. Their work though was limited to the uniaxial bending of columns. As an extension, this research will utilize the fiber method and extend its formulation to biaxial bending, and ultimately produce design charts.

Statement of the Problem

The analysis and design of column sections with biaxial bending is difficult because a trial and adjustment procedure is necessary to find the inclination and depth of the neutral axis that satisfies equilibrium conditions. This study addresses the problem of accurately predicting the behavior of column section with biaxial bending in order to able to establish its capacity at the ultimate stage. In particular, the specific problems that were tackled in this study are the following:

1. How to extend the fiber method formulation in order for it to be applicable to the biaxial bending of reinforced concrete columns.
2. Encode the formulation into an efficient computer program.
3. What will be the effect of structural parameters in the ultimate capacity and performance of a column subjected to biaxial bending?
4. What is the appropriate value of a to best represent the interaction surface?
5. Find ways to construct design charts for biaxially loaded columns.

SCOPE AND LIMITATION

The study is limited to the numerical analysis of reinforced concrete columns with square cross-section only. The investigation will be limited to sections with symmetrical and uniform reinforcement. The concrete strength range that will be considered is envisioned to be in the range of 21 Mpa (3,000 psi) to 62 Mpa (9,000 psi) and the range of the steel ratio is from 1% to 3% . The slenderness effect will not be considered at the moment, but will be subsequently addressed in future research.

METHODOLOGY

The first phase of the study is the development of a computer program that can simulate the behavior of reinforced concrete columns subjected to biaxial bending. The developed computer program is then used to perform numerical experiments and analyses. The methodology of the research is in accordance with the different tasks that have to be performed in order to achieve the development of the Design Charts. The tasks or activities were generally divided into the following:

a. Formulation of the necessary equations and formulas to extend the uniaxial fiber method in order for it to be applicable to the biaxial bending of reinforced concrete columns.
b. Creation of a computer program to implement the formulated equations.
c. Testing and verification of the accuracy of the computer program.
d. Performing of numerical experiments to determine the effect of some structural parameters.
e. Gathering and collating test results.
f. Regression analysis of the results of numerical experiments.
g. Making the Design Charts.

In general, the research work is about creating a computer program capable of implementing the desired numerical experiments in order to get the necessary information for making the Design Charts for reinforced concrete columns subjected to biaxial bending.

THEORETICAL FRAMEWORK OF THE STUDY

In practice, many reinforced concrete columns are subjected to bending about both major axes simultaneously, especially the corner columns of buildings. A typical interaction diagram for biaxially loaded column is shown in Figure 1. Case (a) and Case (b) are the uniaxial bending in the two principal directions, y and z. The interaction curve represent the failure envelop for different combinations of the axial load and bending moments. An analysis and design of a reinforced concrete column with biaxial bending such as in Case (c) is difficult because a trial and adjustment procedure is necessary to find the inclination and depth of the neutral axis satisfying equilibrium conditions. If the depth and inclination of the neutral axis is determined, the corresponding interaction curve can be easily established. By solving the interaction curve for different values of λ, the failure surface for biaxial bending may be constructed. Any combination of N, My, and Mz falling inside the failure surface may be considered safe. It would look like that the construction of the interaction surface is an

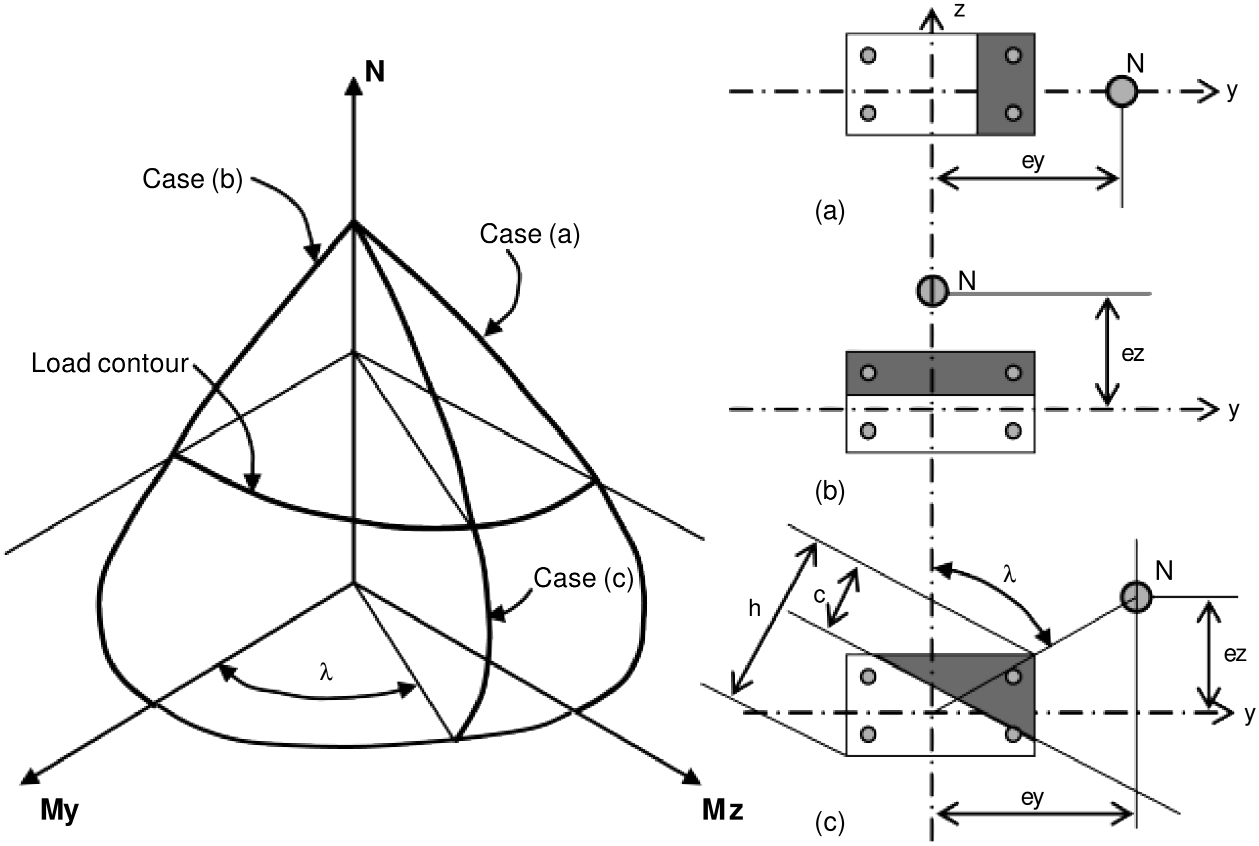

Figure 1. Typical 3D Interaction Diagram of RC Column

extension of the uniaxial bending. The load contour curve on the My-Mz plane may be used to express the failure surface for a constant axial in terms of the uniaxial bending. However, the difficulty is that the neutral axis is not perpendicular to the direction of eccentricity.

This study proposes the use of the fiber method in clarifying the behavior of these columns. The advantage of this method is that there is no need to determine the inclination or depth of the neutral axis. The fiber method can be defined as a simplified one-dimensional finite element method analysis. The discretized element is treated as a fiber with resistance only effective in the direction of its length. The fiber method is accomplished by dividing the column into segments along the member axis, with the segments further subdivided into concrete fibers and steel fibers. By considering the uniaxial stress-strain relationship of each fiber, the average forces of the section can be calculated by summing up the resisting forces of all fibers in a particular section. A typical modeling by fiber method is shown below in Figure 2. The discretization of fibers will be extended to the z-axis in order to be able to simulate the biaxial bending resistance of the section.

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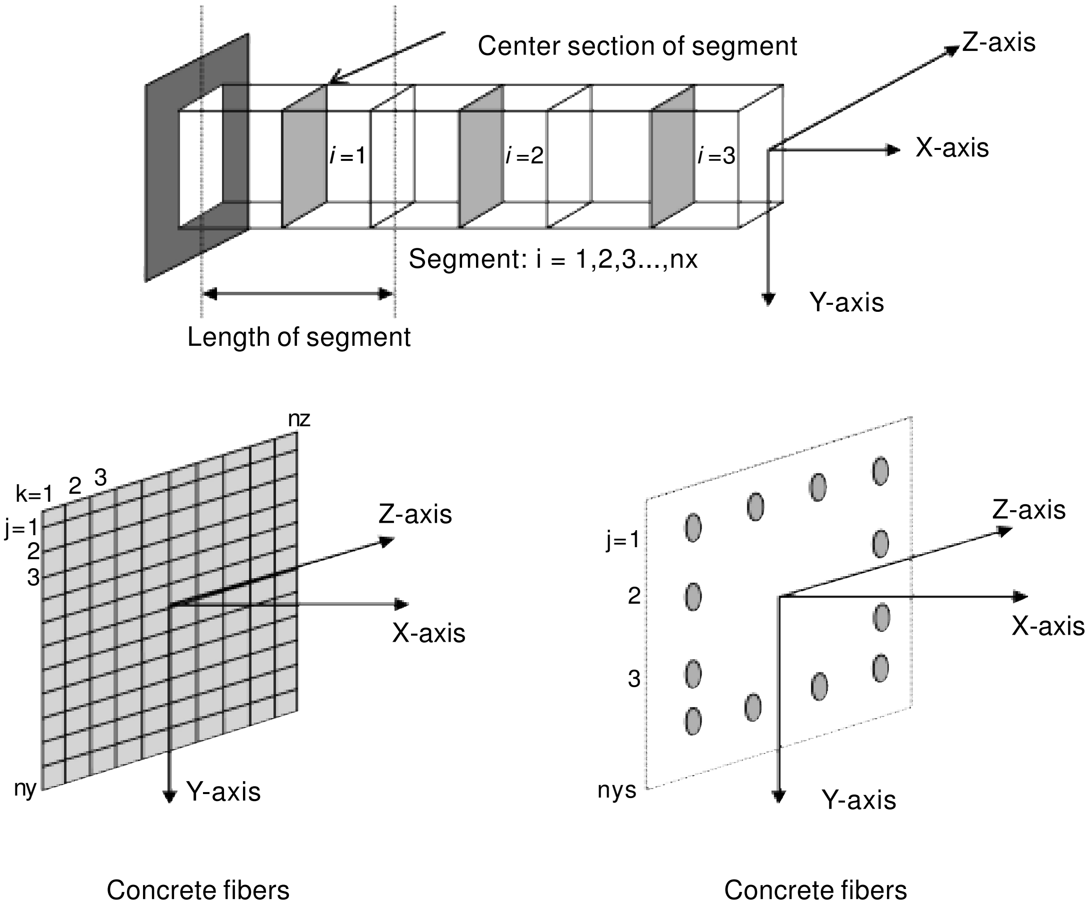


Figure 2. Fiber Modeling of a Reinforced Concrete Column

$$\Delta Nxi = \sum_{j=1}^{ny}\sum_{k=1}^{nnz} (Aijk)(\Delta\sigma ijk)$$

$$\Delta Myi = \sum_{j=1}^{ny}\sum_{k=1}^{nnz} [(Aijk)(\Delta\sigma ijk)(Zik) + (Eijk)(Izijk)(\Delta\phi yi)] \qquad \text{(Eq. 2)}$$

$$\Delta Mzi = \sum_{j=1}^{ny}\sum_{k=1}^{nnz} [-(Aijk)(\Delta\sigma ijk)(Yik) + (Eijk)(Izijk)(\Delta\phi zi)]$$

where N = axial load
My = moment about y-axis
Mz = moment about z-axis
A = cross-sectional area of the fiber
σijk = fiber stress
Eijk = modulus of elasticity of fiber
Izijk = moment of inertia of fiber
ϕyi = curvature of section about y-axis
ϕzi = curvature of section about z-axis
i = index counter of segment along x-axis
j = index counter of fiber along y-axis
k = index counter of fiber along z-axis

Although, a uniaxial stress is assumed in each fiber, the effect of the confining effect on concrete is incorporated by calculating the stress that develops in the hoops and applying it as an average lateral stress to the core concrete. In essence, therefore, a 3-dimensional stress-strain behavior of concrete is still captured.

The incremental analysis procedure making use of the initial stiffness method is adopted. The governing matrix equation is:

$$\{\Delta Pi\} + \{\Delta Pi''\} = [Ki]\ \{\Delta\delta i\} \qquad \text{(Eq. 3)}$$

where $\{\Delta Pi\}$ = the force vector of a particular section "i" = $\{\Delta Ni, \Delta Myi, \Delta Mzi\}^T$

$\{\Delta Pi''\}$ = the unbalanced force vector of a particular section "i" = $\{\Delta Ni'', \Delta Myi'', \Delta Mzi''\}^T$

$\{\Delta\delta i\}$ = the deformation vector of a particular section "i" = $\{\Delta\varepsilon i'', \Delta\phi yi'', \Delta\phi zi''\}^T$

[Ki] = stiffness matrix of section "i"

The member column force vector $\{\Delta P\}$ is related to the section force vector $\{\Delta Pi\}$ through the connectivity matrix [Bi], that is, $\{\Delta Pi\}= [Bi]\{\Delta P\}$. Following the concept of virtual work method, the unbalanced force vector of the member column is obtained as:

$$\{\Delta P''\} = [K] \sum_{i=1}^{nx} \int_0^L [Bi]^T [Ki]^{-1} \{\Delta Pi''\}\, dx \qquad \text{(Eq. 4)}$$

where [K] = member stiffness matrix.

Usually, the biaxial bending of a column is described in terms of eccentricities of the axial load with respect to the x and y axes. The moment about the z-axis is Mz = N ey and the moment about y-axis is My = N ez. The eccentricities give us the direction of the axial load N with respect to the x-axis, i.e., λ = arctan (ey/ez). However, the implementation of the fiber method would be more efficient if the applied moments are expressed in terms of the lateral force Q. We can easily verify that Mz = Qy (L/2) and My = Qz (L/2), where L = height of the column. Also, the components of Q are Qz = Q Cos θ and Qy = Q Sin θ, hence, the angle θ can be expressed as: θ = arctan (Mz/My). Notice that λ and θ are identical angles.

In implementing the loading scheme, the direction of loading will be expressed in terms of θ since it points to the direction where Q will be applied. The loading is simulated by first applying the axial load. After the axial load is applied, the column is forced to deform in a biaxial manner, which is done by prescribing a displacement at an angle θ in the y-z plane. This is similar to performing a push-over loading in a particular direction, θ, in the y-z plane. Therefore, the matrix may be simplified into the form:

The accuracy of the fiber method is critically dependent on the constitutive law of concrete and steel. In this study, Bazant's Endochroni theory (1980) will be used as a constitutive model for concrete and the Ciampi (1982) model for steel. The Endochronic theory is chosen because it gives an accurate prediction of the stress-strain relationship of concrete over a wide range of strength, from normal to high strength concrete. It can simulate as well the confined strength of concrete, thus the effect of hoops or ties can be considered. Furhermore, it considers both the degradation of bulk modulus and shear modulus. The Ciampi model also has been tested extensively to give a very realistic and accurate stress-strain relationship of steel.

Through the use of the fiber method, the interaction surface of a reinforced concrete column under biaxial bending will be established. Regression analysis will be used to identify the exponent value of α. Consequently, different structural parameters will be considered to come up with a practical design chart.

$$\begin{Bmatrix} \Delta N \\ \Delta Q \end{Bmatrix} = \begin{bmatrix} K_{F11} & K_{F12} \\ K_{F21} & K_{F22} \end{bmatrix} \begin{Bmatrix} \Delta\delta_v \\ \Delta\delta_f \end{Bmatrix} \qquad \text{(Eq. 5)}$$

BIAXIAL ANALYSIS USING THE FIBER METHOD MODEL

The developed fiber method model for biaxial bending is applied in the analysis of reinforced concrete columns under biaxial bending. As the strain and stresses of concrete may be monitored in implementing the fiber method, the strength capacity of the column may be determined. The strength capacity may be computed in terms of the yielding of steel or attainment of the ultimate strain of concrete.

Comparison with Experiment Results

To ascertain the correctness of the fiber method model, it is validated by comparing its results with data obtained from physical experiment. The experiment conducted by Kitajima, *et al.* (1992) is the source of the experimental data that is used in validating the develop fiber method model. Specifically, the column specimen named ST4591B was chosen as the basis of comparison since it was loaded in a biaxial manner. The lateral force Q was applied at an angle of 45 degrees from the y-axis. Shown in Figure 3 are the dimensions and reinforcement details of the specimen. Enumerated in Tables 1 and 2 are the structural properties of the specimen and the material properties, respectively. Further analyses using the fiber method were done using the given conditions, specifically the lateral load applied at 45 degrees from the y-axis.

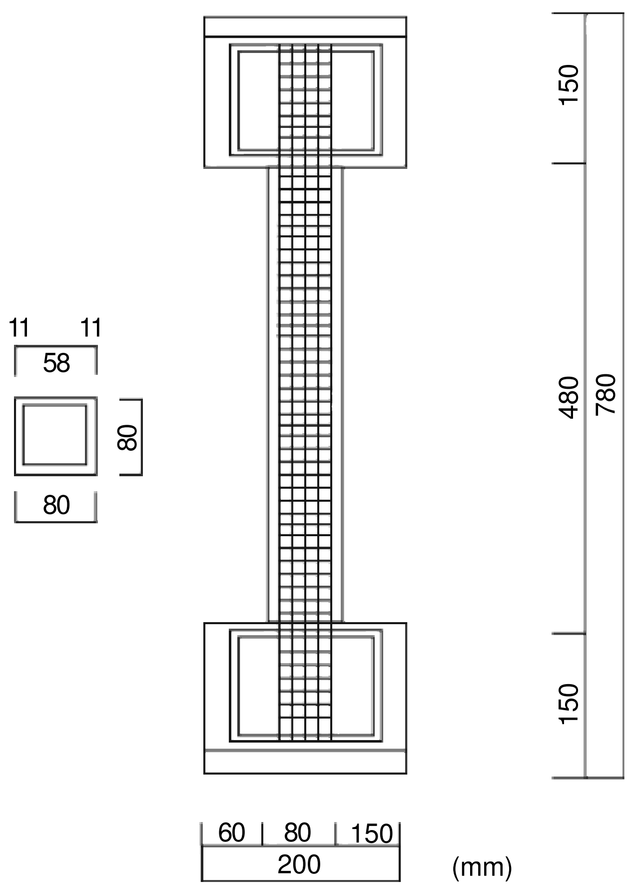


Figure 3. Dimension and Reinforcement Details of Specimen ST4591B

Table 1. Structural Properties of Specimen ST4591B

Shear Span M/QD	Main Steel Reinf. Ratio (Ps %)	Shear Reinf. Ratio (Pw %)	Axial Force (kgf)
3.0	12-D3 (1.49)	1.4ϕ - @ 10 (0.39)	2330

Table 2. Material Properties of Specimen ST4591B

Concrete		Reinforcing Steel		
fc' (kgf/cm²)	Ec (kgf/cm²)	Size	fy (kgf/cm²)	Es (kgf/cm²)
261	2.17×10^5	D3	4485	2.04×10^6
		1.4ϕ	4370	—

Shown in Figure 4 is the plot of the monotonic loading of ST4591B, as predicted, using the developed fiber method. It was plotted against the hysteresis curve obtained from the experiment. It can be seen that the prediction using the fiber method follows the contour of the failure envelop of the hysteresis curve. This means that the prediction of the fiber method agrees well with the experiment results.

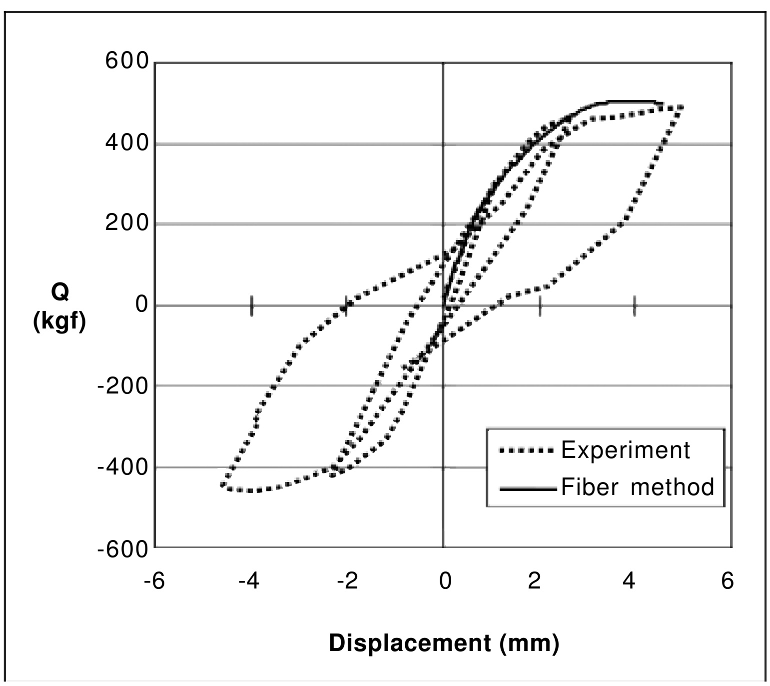


Figure 4. Comparison of Fiber Method Prediction with the Hysteresis Curve of ST4591B

To test further the accuracy of the fiber method, its predicted strength capacity is compared with the experiment result and with ACI methods, as shown in Table 3. Qy is the lateral force corresponding to the first yielding of the reinforcement. Qu, on the other hand, is the lateral force when the strain of concrete reaches the ultimate strain defined by ACI, i.e., $e_u = 0.003$. It can be seen that the fiber method predicted well the ultimate lateral force Qu when the tensile strength of concrete is considered. The fiber method predicted Qu to be 704.2 kgf while Qu in the experiment is 725.5 kgf. This is only a 2.94% conservative difference. It is also noted that the column reinforcement yielded first before the ultimate condition is reached. However, when the concrete tensile strength is neglected, the predicted ultimate force Qu (using the fiber method) is lower but agrees well with the computation using the ACI methods of assuming a rectangular stress block and a parabolic stress-strain curve for concrete in the compression zone.

Table 3. Comparison of Fiber Method Prediction with Experiment and Other Methods

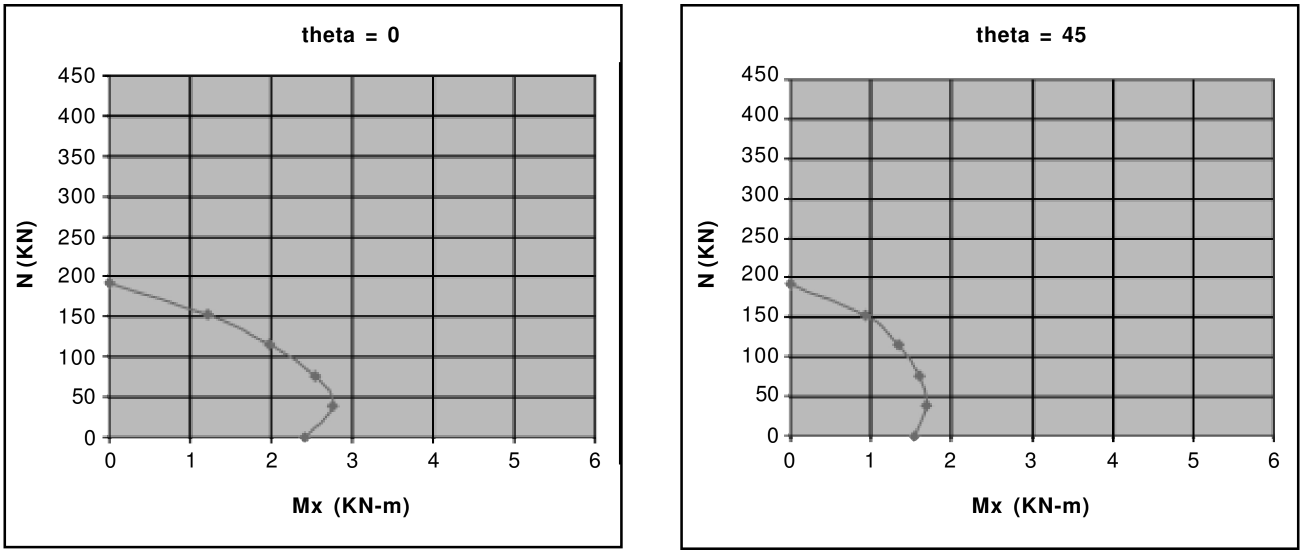
	Experiment	Fiber Method		ACI	
	(ST4591B)	w/ conc. tensile strength (ft) considered**	neglecting concrete tensile strength	rectangular stress block	parabolic stress-strain curve
Qy (kgf)	—	631.4	553.4	—	—
Qu (kgf)	725.5	704.2	646.3	657.7	623.3

** ft = 15.16 Kgf/cm^2

Parametric Analysis

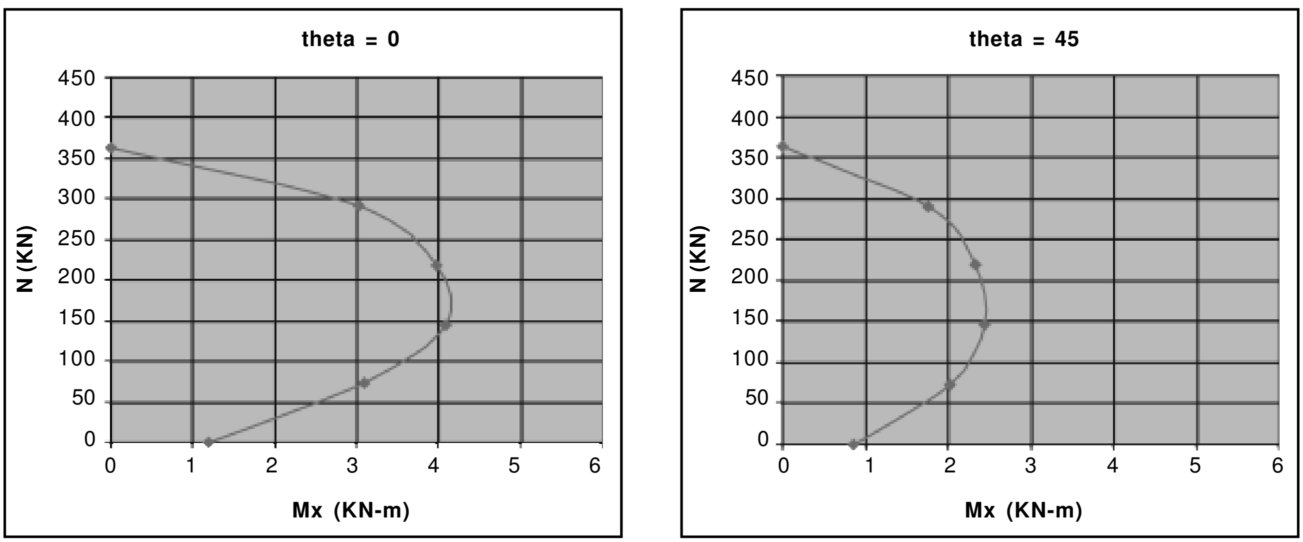
A parametric study was done to determine the effects of varying some structural parameters. The structural parameters varied were the axial force (N), the amount of main reinforcement (Ps), and the compressive strength of concrete (fc'). These parameters were varied based on the usual values used in design. The steel ratio was varied from 1% to 3% and the concrete strength was from 3,000 psi to 9,000 psi. The axial force is varied by varying the axial force ratio (N/No) from 0 to 80%. (Note: No = 0.85fc'Ag + Ps Ag fy, where Ag=gross cross-sectional area of the column) The other structural parameters were maintained as similar to the specimen ST4591B. The analysis is done by varying the angle q and calculating for the Qu and Qy. The lateral and vertical displacements are simultaneously calculated, hence, the ductility ratio can also be determined. However, this paper focuses on the ultimate strength capacity Qu.

Shown in Figure 5 are example interaction diagrams obtained from the analyses. This is obtained by calculating Qu for a given N. The area bounded by the curve is the safe load combinations of the bending moment and axial load. It can be seen that the area covered by the curve greatly increased when the strength of concrete is increased. Also, it can be seen that the area covered by the curve was reduced when Q is directed at an angle of θ=45 degrees from the y-axis. The amount of steel reinforcement also affects the area covered by the curve, i.e., as Ps is increased, the area also increases. However, the effect of the concrete strength is more pronounced. The interaction diagrams for other combinations of structural parameters have been easily established. Design charts can be easily established by making the ordinate and abscissa of the diagram dimensionless. This can be done by changing N to N/Ag and changing M to M/(Ag h), where h=total depth of the column.



a.) High steel ratio, Low concrete strength (Ps = 3%, fc' = 3,000 psi)

Figure 5 – a. Interaction Diagrams Based on the Fiber Method Model Output

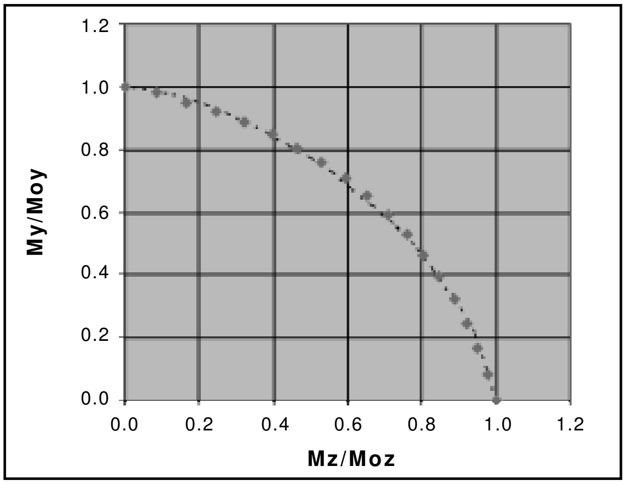


b.) Low steel ratio, High concrete strength (Ps = 1%, fc' = 9,000 psi)

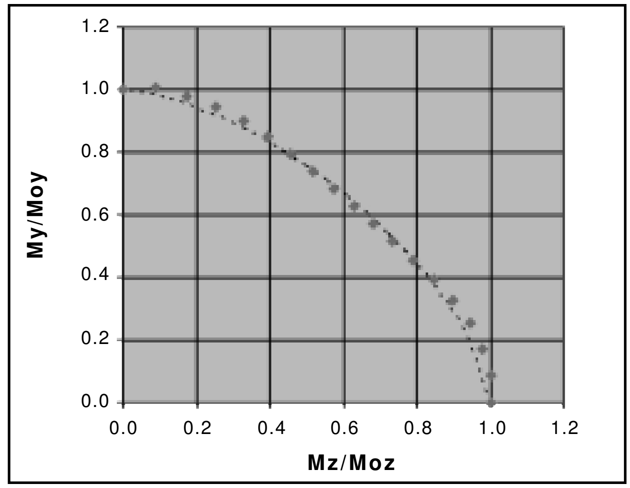
Figure 5 –b. Interaction Diagrams Based on the Fiber Method Model Output

In Figure 6, the interaction contours at different constant axial force ratios (N/No) are shown. The bending moments are normalized with the corresponding unidirectional bending of the columns. It can be seen that the ultimate bending moment decreases as θ approaches 45 degrees. The equation of the contour is determined using Equation 1 through regression analysis. It is assumed that $\alpha_1=\alpha_2=\alpha$ since the column section is square and the reinforcement are symmetrically distributed. The dotted line is the best fit curve for the given fiber method data.

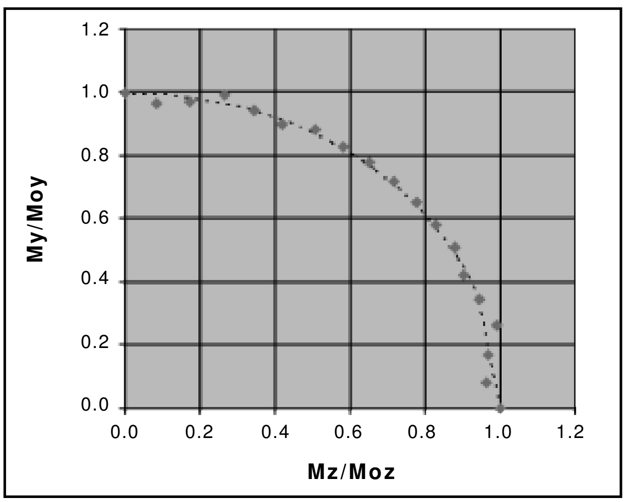
Figure 7 is a bar chart showing the plot of α for the different structural parameter combinations. At this point, there is no clear trend of α with respect to the structural parameters. This may be an indication that Equation 1 may not be the best approximation for the interaction



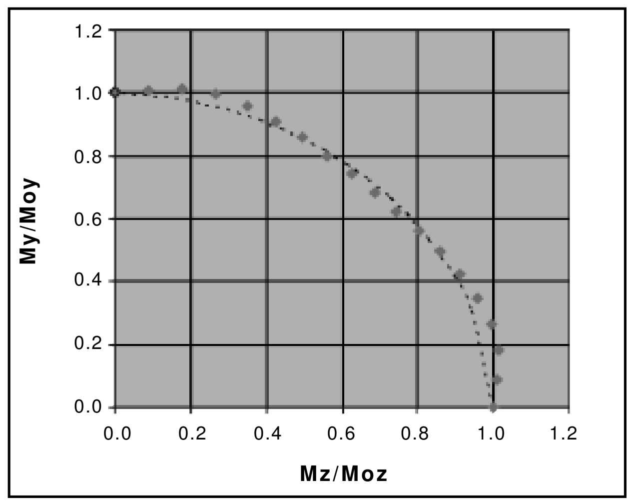
a) Ps=2%, fc'=3,000psi, N/No=0,a=1.593



b) Ps=2%,fc'=3,000psi,N/No=20%,a=1.547



c) Ps=2%,fc'=3,000psi,N/No=60%,a=1.934



d) Ps=2%, fc'=3,000psi, N/No=80%,a=2.064

Figure 6. Load Contours Obtained by Fiber Method Modeling for Different Axial Force Ratio

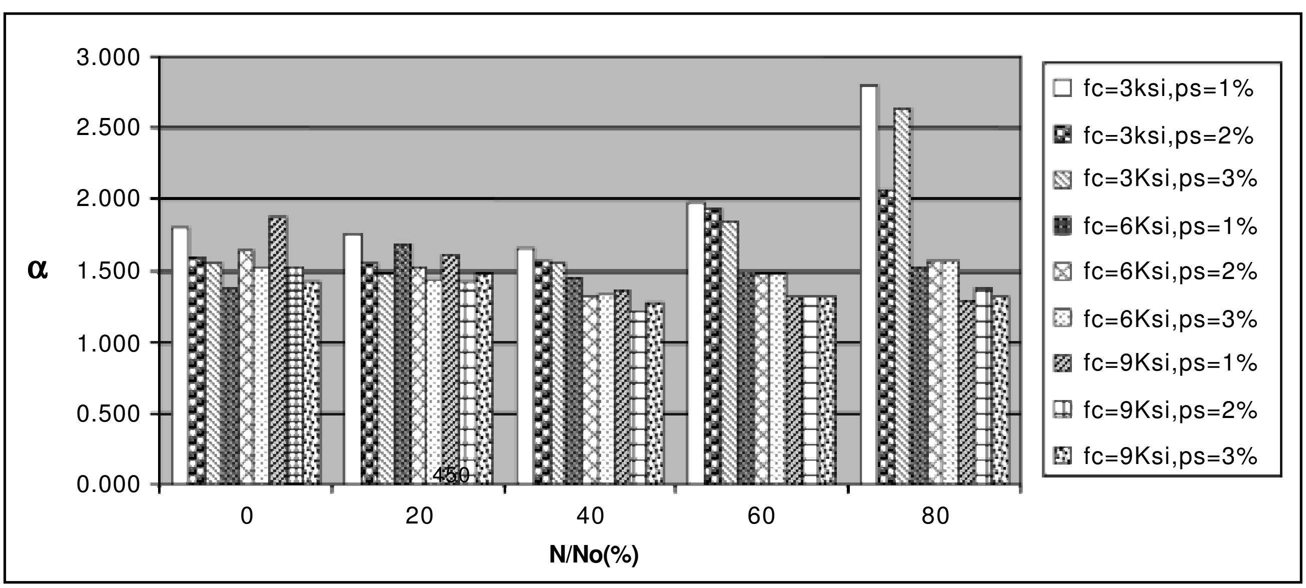


Figure 7. Calculated Values of Exponent α for Different Structural Parameters

contours. However, although not conclusive, it may be observed that the value of α tends to decrease as the concrete strength decreases for a particular N/No. In Table 4, the values of a obtained from regression analysis are enumerated. The value of the computed a ranges from 1.221 to 2.794. Take note that it would be more conservative to use a smaller value of α, such as α=1.2, which is a little bit higher than the a=1.15 which is popularly used. However, it is clear that a higher value of α may be used depending on the N/No.

Table 4. Values of Exponent a Obtained from Regression Analysis

Axial Force Ratio	Concrete strength fc' = 3,000 psi			Concrete strength fc' = 6,000 psi			Concrete strength fc' = 9,000 psi		
(N/No)	Ps = 1%	Ps = 2%	Ps = 3%	Ps = 1%	Ps = 2%	Ps = 3%	Ps = 1%	Ps = 2%	Ps = 3%
0	1.800	1.593	1.552	1.377	1.632	1.526	1.868	1.517	1.420
20%	1.756	1.547	1.482	1.686	1.520	1.434	1.617	1.416	1.474
40%	1.650	1.560	1.544	1.448	1.318	1.332	1.354	1.221	1.277
60%	1.981	1.934	1.840	1.495	1.478	1.472	1.314	1.312	1.319
80%	2.794	2.064	2.640	1.527	1.565	1.567	1.291	1.377	1.312

SUMMARY AND CONCLUSION

The application of the fiber method model presented by Kaba and Mahin (1984) is extended to deal with reinforced concrete columns subjected to biaxial bending. Comparison with experimental data showed that the model performed well and made an accurate prediction of the ultimate strength and yielding strength of the column. It may be concluded that accurate interaction diagrams (N-M curves) and interaction contours (My/Moy-Mz/Moz curves) may be constructed using the fiber method analysis. The parametric analysis resulted to the values of a in the range from 1.22 to 2.79. It may be concluded that a value of a=1.2 may be conservatively adopted for the current range of the structural parameters considered.

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