Appendix A

Imputation (estimation) of volume growth

Volume growth is estimated for each callipered tree. The estimation is based on the following model for the form height:

$$FH_{pj} = f_{pj} + \beta_p + \gamma_{pj} , \tag{A.1}$$

where FH_{pj} is the true form height for the *j*th tree on the *p*th plot, f_{pj} is the expected form height given a set of values of predictor variables, β_j is a random component common for all trees on the *p*th plot and γ_{pj} is a random tree component (Holm et al. 1979, Ståhl et al. 2014).

The function *f* and the variances of the β and γ components are estimated by means of measured sample trees for a large number of classes of plots and different tree species. In application the value of β_p can be estimated for a plot if there are sample trees on it. The estimator is simply the mean $\hat{\beta}_p$ of the residual $FH_{pj} - \hat{f}_{pj}$ of the sample trees. The precision of this estimator depends on the number of sample trees. The imputed value of the form height is in case there are sample trees on the plot

$$F\hat{H}_{pj} = \hat{f}_{pj} + \varpi_p \cdot \hat{\beta}_p + \sqrt{1-\varpi_p^2} \cdot \tilde{\beta}_p + \tilde{\gamma}_{pj} , \tag{A.2}$$

where ϖ_p is a weight depending on the number of sample trees (and their individual variances according to the model), $\tilde{\beta}_p$ is a random normal number (same for all trees on the plot) and $\tilde{\gamma}_{pj}$ is a random normal number (unique for each tree). Both $\tilde{\beta}_P$ and $\tilde{\gamma}_{pj}$ have zero expectation and their variances are depending on plot and tree characteristics.

If there are no sample trees on the plot the form height is imputed as

$$F\hat{H}_{pj} = \hat{f}_{pj} + \tilde{\beta}_i + \tilde{\gamma}_{pj} \tag{A.3}$$

The random components are motivated by the desire to retain the natural population variation.

Appendix B

Swedish NFI-estimators (Ranneby et al. 1987)

Estimator of totals Y^*_{hst}

$$\hat{Y}^*_{hst} = \frac{\sum_{i=1}^{K} y_i^*}{\sum_{i=1}^{K} a_i} \times A_h = \text{Estimator of stratum total for variable of interest} \tag{B.1}$$

$$V\hat{a}r\left(\hat{Y}^*_{hst}\right) = \frac{A_h^2}{\left(\sum_{i=1}^{K} a_i\right)^2} \times K_{hst} \times V\hat{a}r\left(\hat{Q}^*_{hst}\right) = \text{Variance estimator for } \hat{Y}^*_{hst} \tag{B.2}$$

$$\hat{Q}^*_{hst} = y_i^* - \hat{R}^*_{hst} \times a_i \text{ and } \hat{R}^*_{hst} = \frac{\sum_{i=1}^{K} y_i^*}{\sum_{i=1}^{K} a_i}$$

where

h = stratum*, s* = year of inventory*, t* = type of tract, A_h = total stratum area, a_i = total sample-plot area of tract *i, y_i^** = sample total of variable of interest on tract *i (e.g., productive forestland or total stem volume of Pine on productive forest land), i* = index for tract, *K* = total number of tracts in stratum *h* (the indices *h*, *s* and *t* are for simplicity of text omitted for *y* and *a* in the expression).

Estimator of totals Y^*_{hs}

The estimators based on the two tract types are weighed according to

$$\hat{Y}^*_{hs} = w_{hs1} \cdot \hat{Y}^*_{hs1} + w_{hs2} \cdot \hat{Y}^*_{hs2} = \text{ Estimator of stratum total for a variable of interest,} \tag{B.3}$$

based on both tract types, where the weights are chosen to minimize the standard deviation of

$\hat{Y}^*_{hs}$ (given that $w_{hs1} + w_{hs2} = 1$)

$$\hat{Var}\left(\hat{Y}_{hs}^{*}\right) = w_{hs1}^{2}(a) \times \hat{Var}\left(\hat{Y}_{hs1}^{*}\right) + w_{hs2}^{2}(a) \times \hat{Var}\left(\hat{Y}_{hs2}^{*}\right) \quad \text{(B.4)}$$

From a theoretical point of view the weights should depend on stratum, year and variable of interest. However, only a restricted sets of weights are used (the optimum is "flat" with respect to the weights).

Estimator of totals Y_s^*

$$\hat{Y}_s^* = \sum_{h=1}^{31} \hat{Y}_{hs}^* = \text{Estimator of total of a variable of interest,} \quad \text{(B.5)}$$

where the number 31 is the number of strata (county or sub-county). Note that $\hat{Y}_{hs}^* = 0$ for strata that are of "no interest" (for the moment).

$$\hat{Var}\left(\hat{Y}_s^*\right) = \sum_{h=1}^{31}\left(\hat{Var}\left(\hat{Y}_{hs}^*\right)\right) \quad \text{(B.6)}$$

Estimator of means $\hat{B}^*_{hst}$

$$\hat{B}^*_{hst} = \frac{\hat{Y}^*_{hst}}{\hat{A}^*_{hst}} = \frac{\sum_{i=1}^{K} y_i^*}{\sum_{i=1}^{K} a_i^*} \text{ = Estimator of mean of interest, per area unit,} \tag{B.7}$$

$$\hat{Var}\left(\hat{B}^*_{hst}\right) = \frac{1}{\left(\sum_{i=1}^{K} a_i^*\right)^2} \times K_{hst} \times \hat{Var}\left(\hat{Q}^*_{hst}\right) \tag{B.8}$$

$\hat{Q}^*_{hst} = y_i^* - \hat{R}^*_{hst} \times a_i^*$ and $\hat{R}^*_{hst} = \frac{\sum_{i=1}^{K} y_i^*}{\sum_{i=1}^{K} a_i^*}$

given stratum and year, based on tract type *t* where both numerator and denominator are the total estimators (see above), with the same notations and where y^* = variable of interest (*e.g., stem volume of Pine on productive forest land, privately owned*) and a^* = total area of interest (*e.g., area of productive forest land, privately owned*).

Estimator of means $\hat{B}_{hs}^{*}$

$$\hat{B}_{hs}^{*} = \frac{\hat{Y}_{hs}^{*}}{\hat{A}_{hs}^{*}} = \text{ Estimator of mean of interest per area unit, given stratum and year,} \tag{B.9}$$

based on both tract types, where both numerator and denominator are estimators of totals.

$$\hat{Var}\left(\hat{B}_{hs}^{*}\right) = \frac{\hat{U}_{hs1}^{*} + \hat{U}_{hs2}^{*}}{\hat{A}_{hs}^{*\,2}} \tag{B.10}$$

$$\hat{U}_{hs1}^{*} = \left(w_{hs1}(y)\right)^{2} \times \frac{A_{h}^{2}}{\left(\sum_{i=1}^{K_{hs1}} a_{i}\right)^{2}} \times K_{hs1} \times \hat{Var}\left(\hat{D}_{hs1}^{*}\right)$$

$$\hat{U}_{hs2}^{*} = \left(w_{hs2}(y)\right)^{2} \times \frac{A_{h}^{2}}{\left(\sum_{i=1}^{K_{hs2}} a_{i}\right)^{2}} \times K_{hs2} \times \hat{Var}\left(\hat{D}_{hs2}^{*}\right)$$

$$\hat{D}_{hs1}^{*} = y_{i}^{*} - \frac{w_{hs1}(a)}{w_{hs1}(y)} \hat{R}_{hs1}^{*} \times a_{i}^{*}$$

$$\hat{D}_{hs2}^{*} = y_{i}^{*} - \frac{w_{hs2}(a)}{w_{hs2}(y)} \hat{R}_{hs2}^{*} \times a_{i}^{*}$$

Estimator of means B_s^*

$$\hat{B}_s^* = \frac{\hat{Y}_s^*}{\hat{A}_s^*} = \text{ Estimator of means of interest, per unit area, given year,} \qquad \text{(B.11)}$$

$$\hat{Var}\left(\hat{B}_s^*\right) = \frac{\sum_{h=1}^{31}\left(\hat{U}_{hs1}^* + \hat{U}_{hs2}^*\right)}{\left(\sum_{h=1}^{31}\hat{A}_{hs}^*\right)^2} \qquad \text{(B.12)}$$

where both numerator and denominator are the estimators. The estimator is thus of the "combined ratio type".

Estimator of five year means

Many NFI-estimates are presented for a five period, i.e. five year averages. The estimate of totals is then calculated as the arithmetic mean of the annual estimates within the period:

$$\hat{A}^* = \frac{1}{n}\sum_{s=1}^{n}\hat{A}_s^* \text{ where } n \text{ is the number of years within the period} \qquad \text{(B.13)}$$

$$\hat{Var}\left(\hat{A}^*\right) = \frac{1}{n^2}\sum_{s=1}^{n}\hat{Var}\left(\hat{A}_s^*\right) \qquad \text{(B.14)}$$

And the estimate of five year averages for means are calculated as:

$$\hat{B}^* = \frac{\sum_{s=1}^{n}\hat{Y}_s^*}{\sum_{s=1}^{n}\hat{A}_s^*} \qquad \text{(B.15)}$$

$$\hat{Var}\left(\hat{B}^*\right) = \frac{\sum_{s=1}^{n}\sum_{h=1}^{31}\left(\hat{U}_{hs1}^* + \hat{U}_{hs2}^*\right)}{\left(\sum_{s=1}^{n}\sum_{h=1}^{31}\hat{A}_{hs}^*\right)^2} \qquad \text{(B.16)}$$

Other estimators

Below is a brief description of two other estimators which are not regularly used, but for which procedures are prepared.

- Estimators of change between two occasions
    - The estimator is a weighted mean of the estimated changes based on permanent and on temporary tracts, similar to the estimator (B.3), where $\hat{Y}_{hst}^{*}$ is replaced by the difference between the two occasions.
    - The estimator is the difference of two estimates based on permanent tracts.
- State estimators that make use of data from two or more occasions
    - The estimators are constructed as a weighted mean of estimators based on present permanent and temporary tract data and on the same kind of data from the previous time of inventory, in the Swedish NFI normally five years ago (with proper restrictions on the weights to guarantee unbiasedness). Theoretically these estimators fall under the heading “partial replacement estimators”.

Appendix C

Algorithms for post-stratification

In the post-stratification approach used by the Swedish NFI (Nilsson et al. 2003), a forest variable in a specific stratum is estimated as a weighted average of two independent samples: one based on tracts with permanent plots and one based on tracts with temporary plots. The weight given to a specific variable estimate in each sample is inversely proportional to its variance. Since parts of the tracts could belong to different strata, the within-strata estimators are correlated. This motivates formulas (C.2) and (C.3) below. Formula (C.3) is derived in a way similar to that of the variance of a ratio estimator (e.g., Thompson 1992, Ståhl et al. 2011b). For both permanent and temporary tracts in a NFI region, each forest variable ($\hat{Y}$) is estimated as:

$$\hat{Y} = \sum_{h=1}^{L} \hat{Y}_h = \sum_{h=1}^{L} \left(\frac{A_h}{a} \cdot \frac{\sum_{i=1}^{n} y_{ih}}{\sum_{i=1}^{n} m_{ih}} \right) = \sum_{h=1}^{L} \frac{A_h}{a} \cdot \hat{R}_h \tag{C.1}$$

with $\hat{R}_h = \sum_{i=1}^{n} y_{ih} / \sum_{i=1}^{n} m_{ih}$ and where:

n=total number of NFI tracts within the area

A_h=total area in stratum h, h=1, 2, ..., L

a=the area covered by a field plot

y_{ih}=observed total value for tract i, stratum h, i = 1, 2, ..., n and thus 0 if no part of tract i belongs to stratum h

m_{ih}=total number of field plots in tract i, stratum h, and thus 0 if no part of tract i belongs to stratum h

For the reasons mentioned above the variance is estimated as:

$$Var(\hat{Y}) = \sum_{h=1}^{L} \sum_{k=1}^{L} \frac{A_h A_k}{a^2} \cdot Cov(\hat{R}_h, \hat{R}_k) \tag{C.2}$$

where $Cov(\hat{R}_h, \hat{R}_k)$ is estimated by

$$Cov(\hat{R}_h, \hat{R}_k) = \frac{1}{n \cdot \overline{m}_h \cdot \overline{m}_k} \cdot \frac{\sum_{i=1}^{n} (y_{ih} - \hat{R}_h \cdot m_{ih})(y_{ik} - \hat{R}_k \cdot m_{ik})}{(n-1)} \quad \text{(C.3)}$$

where the mean values $\overline{m}_h$ and $\overline{m}_k$ are calculated on all *n* tracts

FIGURE LEGENDS

Fig. 1. The design of the first Swedish NFI (1923-1929). Delineated areas indicates counties and raster visualizes different distance between belts.

Fig. 2. The design of the Swedish NFI from 1923 onwards (T=temporary, P=Permanent). The squares are tracts (clusters of sample plots) with side-length 300-1800 m, and the circles are sample plots.

Fig 3. Changes in sample plot design from 1983 onwards for permanent (above) and temporary plots (below).

Fig. 4. Outline of the Swedish NFI reporting and analysis system.

TABLES

Table 1. An overview of the design of the Swedish NFI at different time points

Period	**Design**	**Coverage**
1923-29	Sampling of temporary belts 10 m wide	County-wise
1938-52	Sampling of temporary belts combined with temporary plots positioned along the center line of the belt	County-wise
1953-72	Sampling of temporary tracts, i.e. a cluster of plots where the plots are positioned along the sides of a rectangle with varying side length depending on stratum	Whole country, 1/10th annually
1973-82	Sampling of temporary tracts, i.e. a cluster of plots where the plots are positioned along the sides of a rectangle with varying side length depending on stratum	Whole country, 1/5th annually
1983-92	Sampling of temporary and permanent tracts. Permanent tracts re-measured every 5th year.	Whole country, 1/5th annually
1993-02	Sampling of temporary and permanent tracts. Varying re-measurement interval.	Whole country, varying from 1/10th to 1/5th annually
2003-	Sampling of temporary and permanent tracts. Permanent tracts re-measured every 5th year	Whole country, 1/5th annually

Table 2. New and modified variables introduced in the Swedish NFI in 2003 for improved data related to sustainable wood-production.

New or modified variable from 2003	Description	Method
Diameter on thickest branch (below 2 m of height)	Measured on sub-sample trees	Measurement with a slide (vernier) caliper
Presence of abrupt bend	Assessed on sub-sample trees	Measurement
Presence of long bend	Assessed on sub-sample trees	Ocular assessment
Height and crown height measurement	Measured on sub-sample trees	Registered in 0.1 m classes instead of 0.5 m classes
Height of stump	Measured on stumps	Measurement with a ruler
Objective elk-damage inventory (ÄBIN)	Assessment of damage/no damage caused by elk-grazing	Assessed on individual trees on sample plots (r=3.5 m)
Stand damage	Assessment of the type of stand damage including cause of damage for existing stands, or previous stand if ruined due to damage	Assessed on sample plots (r=20 m)
Tree damage	Assessment on sub-sample trees	Assessed on sub-sample trees separately for crown-, stem-, cambium- and root damages

Table 3. New and modified variables introduced in the Swedish NFI in 2003 for improved data on biodiversity and wildlife.

New or modified variable from 2003	**Description**	**Method**
Presence of traces from woodpeckers	Presence/absence registration	Assessment on sub-sample trees and on dead wood objects
Presence of *Polyporus* fungi	Presence/absence registration including species	Assessment on sub-sample trees and on dead wood objects
Assessment of vegetation on dead trees and logs	Assessment of coverage	Assessment on dead wood objects
Assessment of distance to the ground	Measurement of distance to ground	Measurement on logs with a ruler
Additional class of decomposition on dead wood: Newly dead	Assesment of decomposition class	Assessment on dead wood objects
Bark-coverage on dead trees and logs	Assessment	Classes 0-100%
Ant-hills	Presence of ant-hills	Measurement of diameter and height
Large trees and retention trees (abandoned 2008)	Presence of large trees (definition see Fig. 3)	Registration of species and measurement of dbh
Presence of ground- and bottom flora	Short list of species (22 species or species groups).	Performed on 2 sub-plots (r=0.28 m) Height above ground recorded for Lichens. Flowers/berries counted on plants of *Vaccinium Vitis* idea and *Vacc. Myrtillus*.
Crown cover	Vertical projection of tree crowns in %. Over-coverage not considered	All living trees included, bushes not included.

Area of land-use class	To fulfill international reporting where minimum area is often included	Area based on map- and ocular studies registered
New sampling design for measurement of dead wood	Point of germination or thickest end defined as point of inclination	All lying objects included if point of inclination is within the sample plot, i.e. part outside plot included.
Inventory within reserves	Ordinary field work also on plots within National parks and Nature reserves (excluded before 2003)	Identical sampling scheme as for plots outside these areas.
Performed forestry measures registered also on unproductive forestland	Before 2003 only registered on productive forestland	Identical methods to those used for plots located on productive forestland
Registration of cause of felling	If felling of trees are registered on a plot, also cause is registered	Three categories: Normal felling, Clearing after severe damage, Nature conservation
Modified procedures for registration and assessment of tree-layers	Maximum three layers, presence of all occurring tree species in % of total number of stems or basal area (depending on height)	Regardless of how small proportion a tree species amount to, it´s occurrence is indicated registering 1%
Stand-attributes also registered on unproductive forestland	Stand-age, mean height, basal area, tree species composition	Identical methods to those used for plots located on productive forestland
Forest stands suitable for selective cutting	Included as a separate cutting class	-
Number of occurrences of elk - droppings	Counted (r=3.5 m). Before 2003 r=5.0 m	-