

Calibration of Hall sensors in three dimensions

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IMMW13
SLAC, May 19-22, 2003



Outline:

- Old methods not good enough for precision better than 10^{-3}
- Full 3D scan necessary
- Machine to do it
- 3D sensor with 10^{-4} precision
- Results
- Future developments

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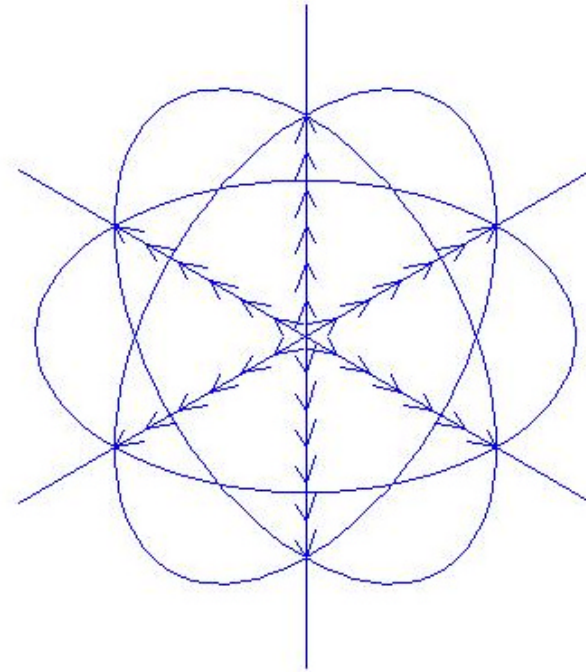
How to calibrate a 3D magnetic sensor with three more or less orthogonal Hall probes?

Correct for:

- Non-linearity
- Non-orthogonality
- Temperature dependence of sensitivity
- Planar Hall Effect (PHE)
- etc

Many tricks available:

- Main-axes calibration
- invert field: main signal changes sign, PHE not, prop. to $B^2 \sin 2\phi$
- Rotate over xy, xz and yz axes : PHE + non-orthogonality
- Search for 0 Volt position with special balance
- Two hall probes back to back to cancel PHE
- etc

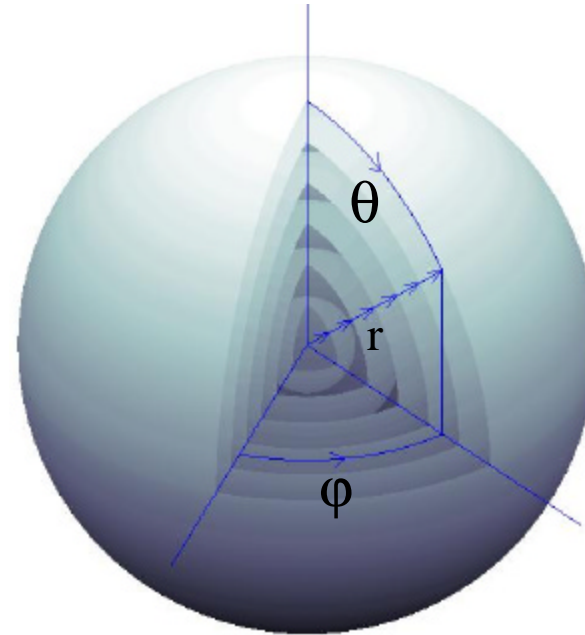


We tried: cubes with 6 hall probes

1. Main axes calibration for x, y and z, -2.4 T to 2.4 T, step 0.1 T
2. Rotation about xy, xz and yz axes at 0.5, 1.0 and 1.5 T, step 15 deg

Result:

Measurements at arbitrary positions \Rightarrow max. error of order 10^{-3} in $|B|$



Solution = full 3D scan:

Rotate sensor over two orthogonal axes in constant homogeneous field , θ and ϕ should be measured very precisely

Repeat for several field strengths and temperatures, $|B|$ and T should be measured very precisely

Decompose the Hall-voltage in orthogonal functions: spherical harmonics for θ and ϕ , Chebyshev polynomials for $|B|$.

Spherical harmonics $Y_{lm}(\theta, \varphi)$

Rotation at constant $|\mathbf{B}|$: $V_{Hall}(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l c_{lm} Y_{lm}(\theta, \varphi)$

$$c_{lm} = \int Y_{lm}^*(\theta, \varphi) V_{Hall}(\theta, \varphi) dO$$

$$\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \cdot Y_{l'm'}^*(\theta, \varphi) Y_{lm}(\theta, \varphi) = \delta_{l'l} \delta_{m'm} \quad Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\varphi}$$

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

$$l=0 \quad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$l=1 \quad \begin{cases} Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi} \\ Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta \end{cases}$$

$$l=2 \quad \begin{cases} Y_{22} = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2\theta e^{2i\varphi} \\ Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\varphi} \\ Y_{20} = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2\theta - \frac{1}{2} \right) \end{cases}$$

How do Y_{lm} scale from one $|B|$ to another?

Solid harmonics $r^l Y_{lm}(\theta, \varphi)$

$$r^l Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} (l+m)!(l-m)!} \sum_{p,q,r} \frac{1}{p!q!r!} \left(-\frac{x+iy}{2}\right)^p \left(\frac{x-iy}{2}\right)^q z^r$$

$$p+q+r=l, p-q=m$$

is power series $\sum_{p,q,r} c_{pqr} x^p y^q z^r$

$$r^1 Y_{10}(\theta, \varphi) = z \quad \text{Main component}$$

$$r^2 Y_{22}(\theta, \varphi) = xy, x^2 - y^2 \quad \text{Planar Hall effect}$$

$$r^3 Y_{30}(\theta, \varphi) = 2z^3 - 3(x^2 + y^2)z \quad \text{Non-linearity}$$

$$r^3 Y_{32}(\theta, \varphi) = xyz, (x^2 - y^2)z$$

In decomposition of Hall voltage Y_{lm} coefficients scale more or less with $|B|^l$

$$V(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l c_{lm}(r) r^l Y_{lm}(\theta, \varphi)$$

Decompose $c_{lm}(r)$ in Chebyshev polynomials

$$\int_{-1}^{+1} \frac{T_i(x)T_j(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0 & i \neq j \\ \pi/2 & i = j \neq 0 \\ \pi & i = j = 0 \end{cases}$$

$$\begin{array}{ll}
 T_0(x) = 1 & -1 \leq x \leq 1 \\
 T_1(x) = x & \\
 T_2(x) = 2x^2 - 1 & \\
 T_3(x) = 4x^3 - 3x & \\
 T_4(x) = 8x^4 - 8x^2 + 1 & \\
 \dots & \\
 T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) & n \geq 1
 \end{array}
 \quad
 \begin{array}{l}
 T_n(x) = 0 \quad \text{at} \quad x = \cos\left(\frac{\pi(k - \frac{1}{2})}{n}\right) \quad k = 1, 2, \dots, n \\
 \text{Extremes: } T_n(x) = \pm 1 \quad \text{at} \quad x = \cos\left(\frac{\pi k}{n}\right) \quad k = 0, 1, \dots, n
 \end{array}$$

Rotate Y_{lm} to symmetry axes of hall probe:

$$Y_{11} = 0 \text{ (only } Y_{10} = \cos\theta \text{), } Y_{22} = \sin^2\theta \sin 2\varphi \text{ (PHE)}$$

Parameters of calibration at fixed $|B|$ and T per hall probe:

Y_{lm} coefficients in symmetry frame (64-3 for $l = 7$)

Direction of symmetry axes (2)

Rotation around symmetry axes (1)

**Calibration at 0.25 - 0.5 - 0.75 - 1.0 - 1.25 - 1.5 Tesla
and 16 - 18 - 20 - 22 - 24 deg. Celsius**

Interpolate with Chebyshev polynomials and reduce:


For $0.5 \cdot 10^{-4}$ precision is needed (3 x Siemens KSY44) :

about 40 +14 for Y_{lm} per hall probe (about 15 Y_{lm} per (B,T)-point)

36+3 for angles per sensor card

Total: ~ 200 parameters per sensor card

many are constant, common or dependend, can reduce more

Highest order Chebyshev needed: $|B| = 5$ 
 $T = 2$

Maximum number of calibrations needed = 5B x 2T

3B x 2T realistic, 2B x 1T +1 simplified T maybe possible

Difference for 3x Siemens KSY44 at 1.5 T , 20 °C in old method and new method

Old method = only main axes calibration
 $B \Rightarrow B_z$

New method = 3D scan

Only symmetrical components

$Y_{lm} \ m=0 \Rightarrow$ PHE off

no change of symmetry axes, simulated data with measured parameters

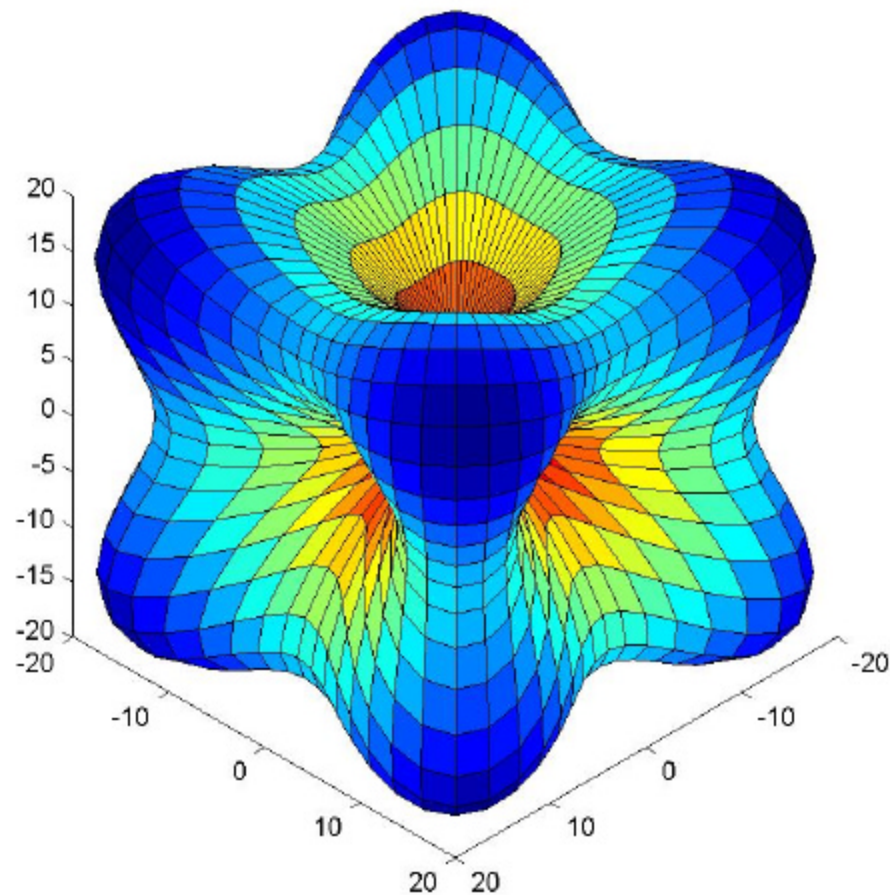
Plotted in figure: $|B_{old} - B_{new}|(\theta, \varphi)$

Color scale = $|B_{old}| - |B_{new}|$

Blue = -31 Gauss, red = 0

At calibration axes error is zero, increases to 2 ‰ off axes.

Difference at 1.5 Tesla

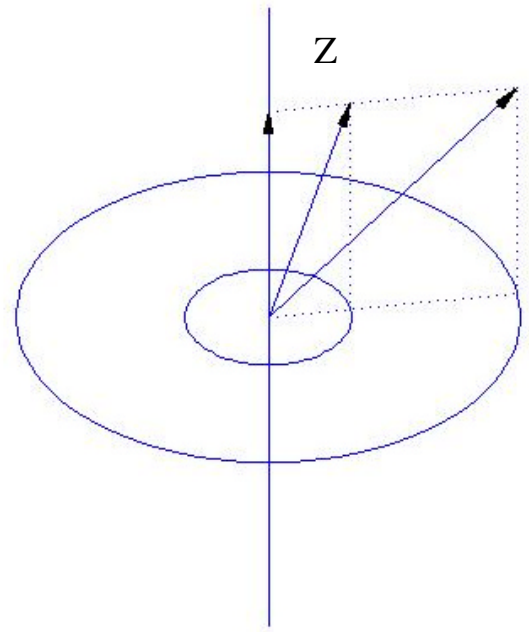


Scales in Gauss

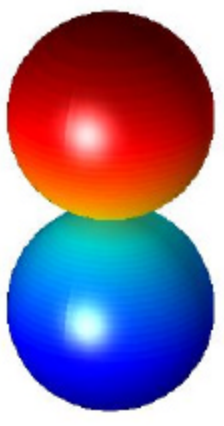
The sensitivity of a Hall probe depends not only on B_z , but also slightly on $|B|$
 $B \Rightarrow B_z$ not allowed

Symmetry axes move with B and T

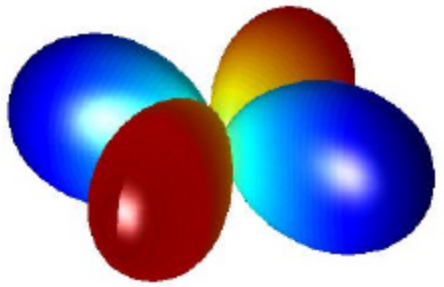
In addition there are $m \neq 0$ terms
 Higher order terms increase with $\approx |B|^l$
 Planar Hall effect not exactly $\propto B^2$



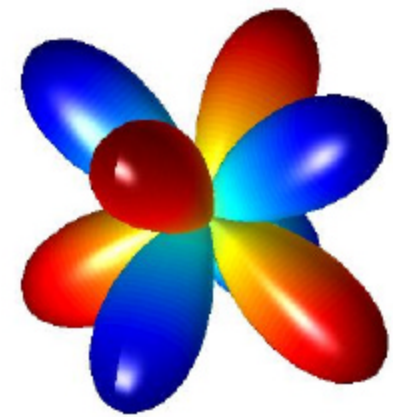
These B-vectors give all different Hall voltages.
 Not to be confused with PHE, this effect is symmetrical around the z-axis



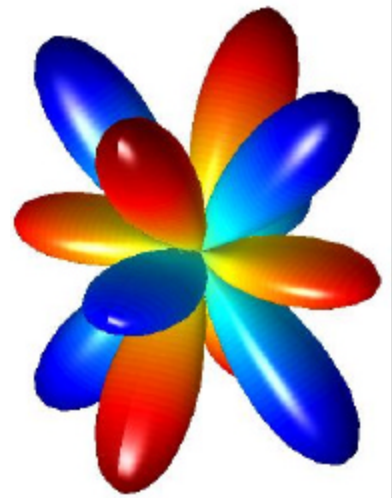
Y_{10}
 $\text{Cos}(\theta)$



Y_{22}
 planar Hall effect



Y_{32}



Y_{42}

Calibrator

Need θ and φ very precise $< 10^{-4}$ on both
Difficult to realize with encoders



Use three more or less orthogonal coils on rotating platform in constant homogeneous field
Sample coils with delta-sigma modulator
Continuous rotation on 2 axes, 2 rev/min on main axes, sample rate = 15 /sec

Adjust integrated coil signal to get
$$\sqrt{B_x^2 + B_y^2 + B_z^2} = 1$$

Correct for:

Self-induction, non-orthogonality, offset ADC, start position, time constants
= 5 x 3 parameters

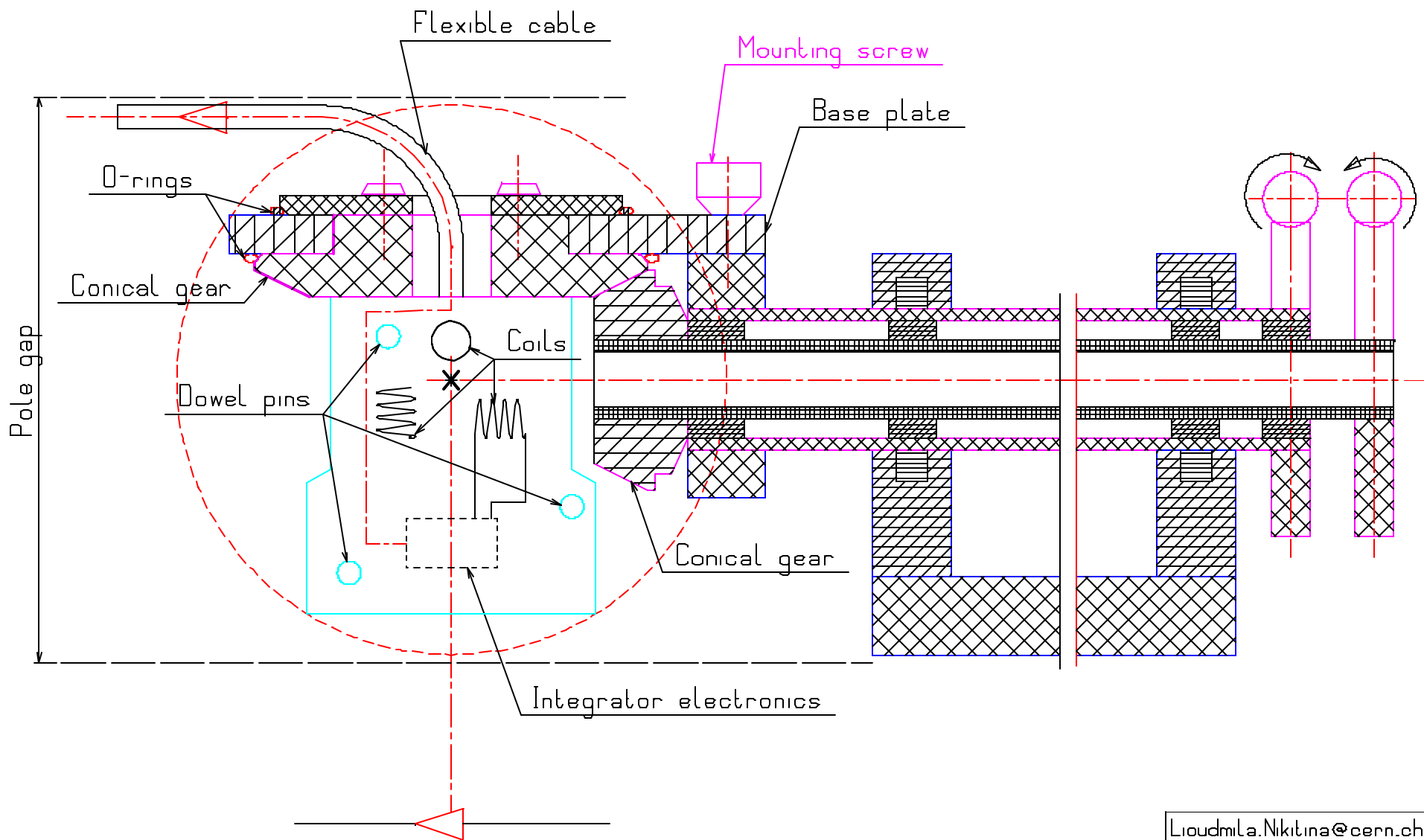
Method works because the integrated coil signal contains only Y_{lm} terms with $l \leq 1$
Not possible with Hall probes because of non-linearity, PHE, etc.

- Take mean of left and right turn against parasitic self-inductions
- # of turns of outer axes \approx order of Y_{lm} extractable

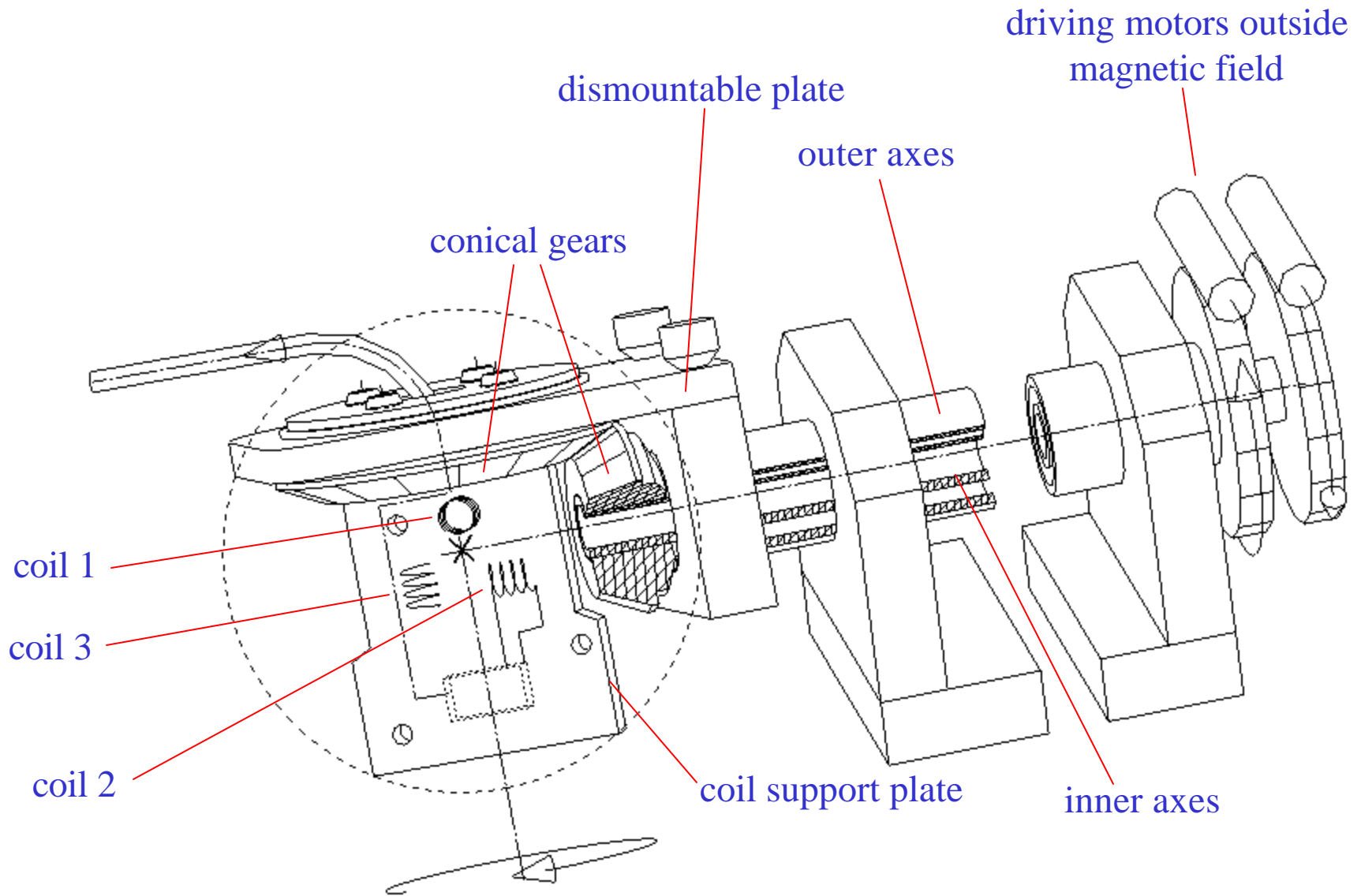
Calibrator

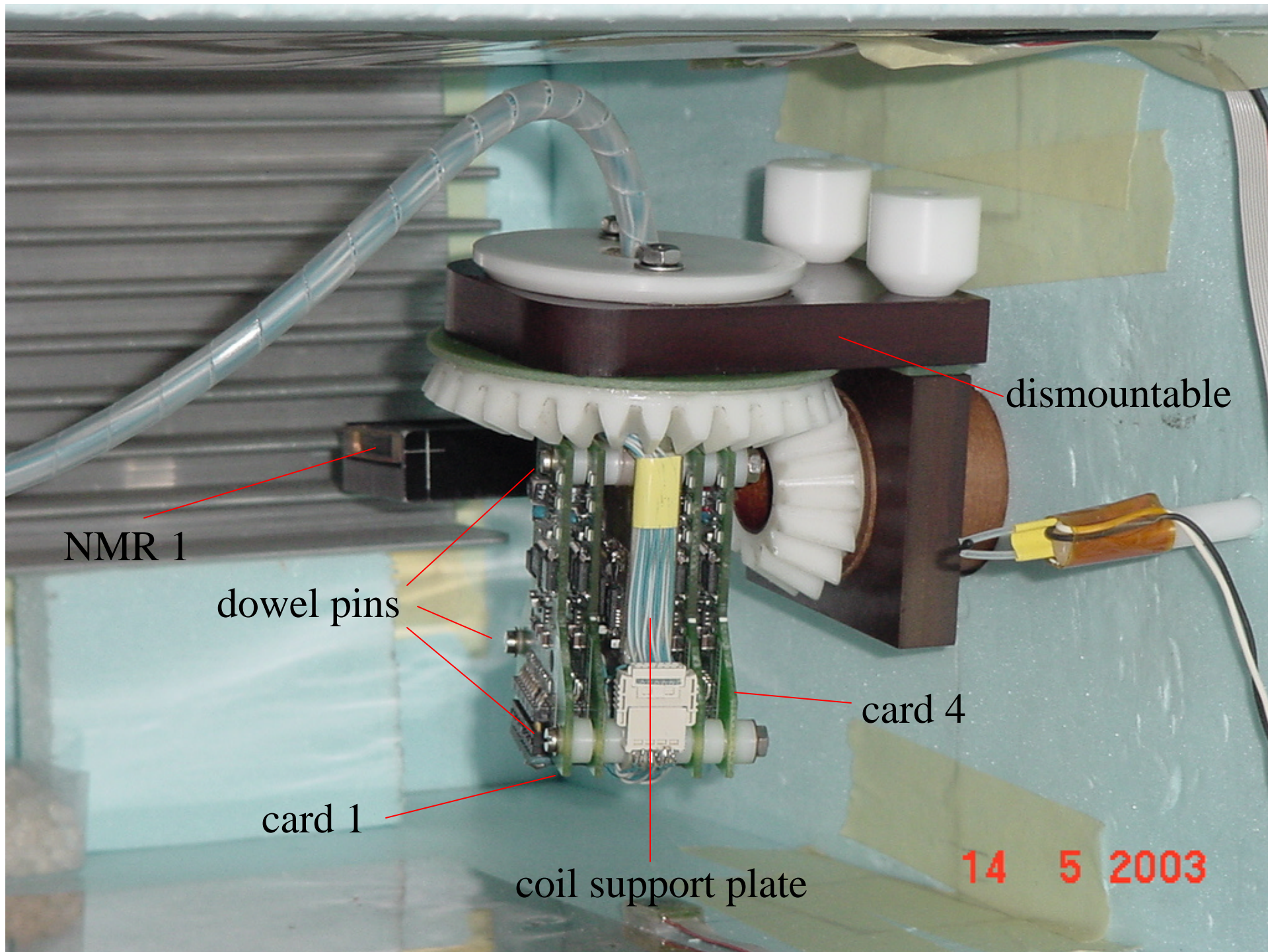
- With every coil (3x) sample also one Hall probe or thermistor is measured
- $|B|$ measured by 2 NMR's at 10 and 25cm from center
- Temperature controlled by Peltier element + ventilator
Temperature stability ± 0.02 deg.C
- Support plate with coils and sensor cards easily removable from rest of the device
Place for 4 sensor cards, fixed with dowel pins, precision ≈ 0.01 mm
- Cable winds by main axis, but unwinds by secondary axes:
At end of calibration cable has made only one turn
- Typical calibration takes 6 turns of main axes = 3 min

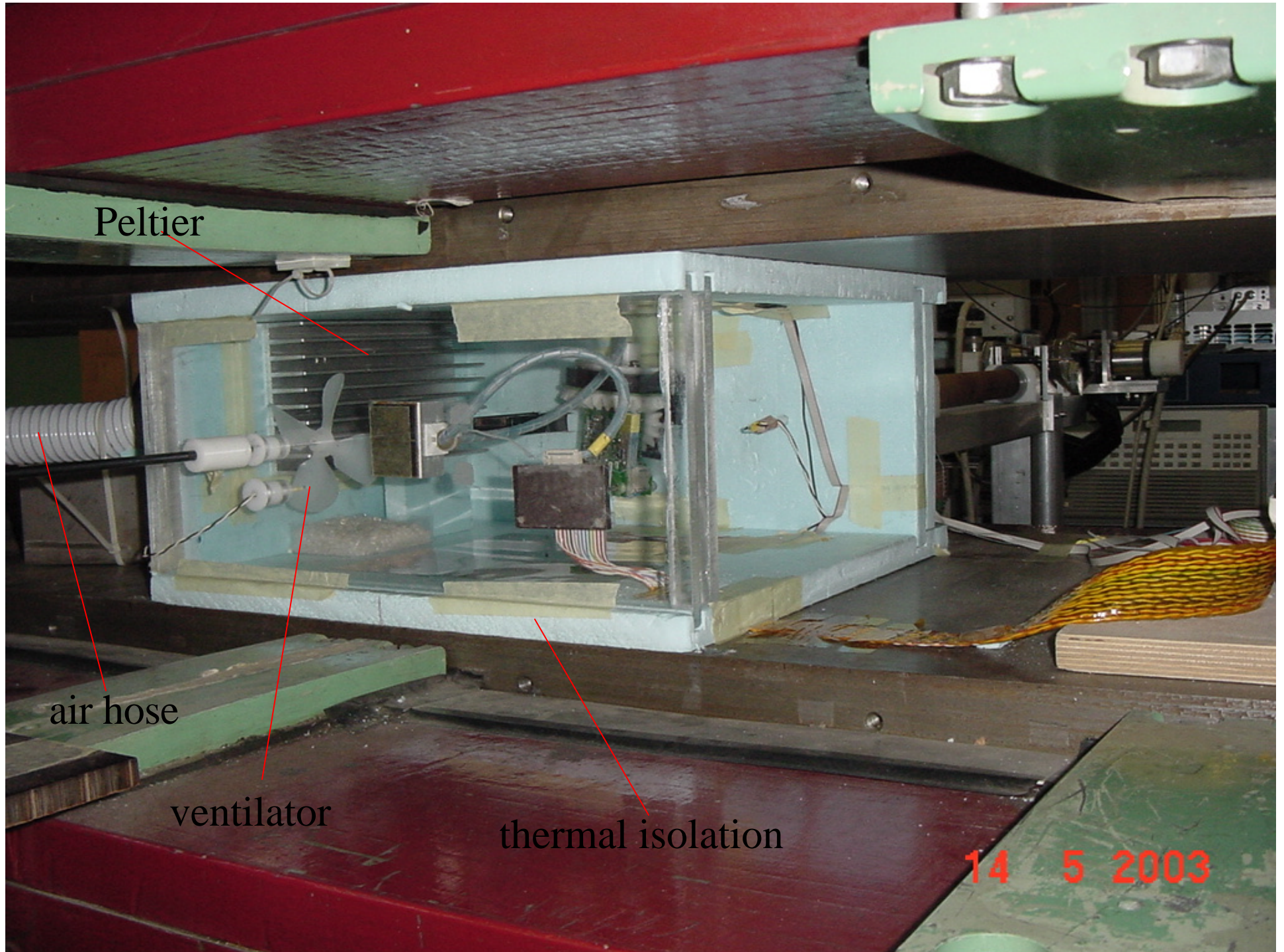
Simple and cheap device, vibrations point of care



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DATE: 20-FEB-2002
EUCLID: DR_BR2
CDD:

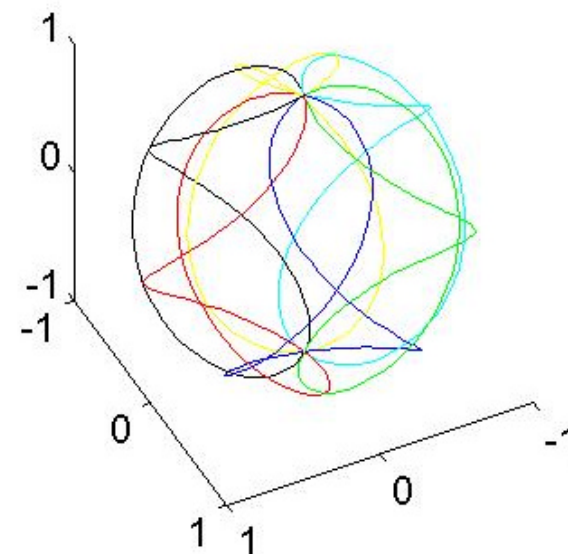
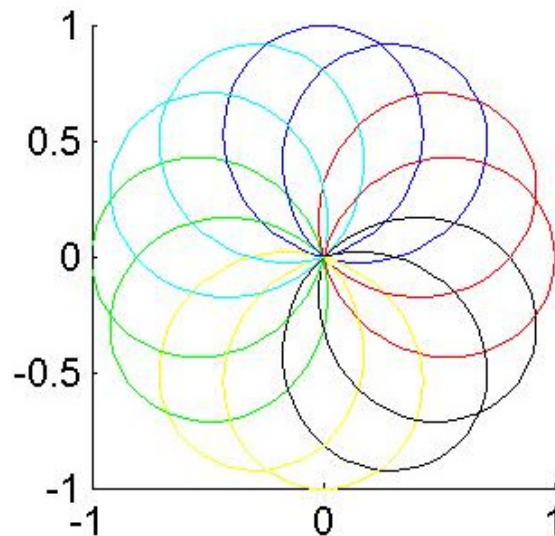
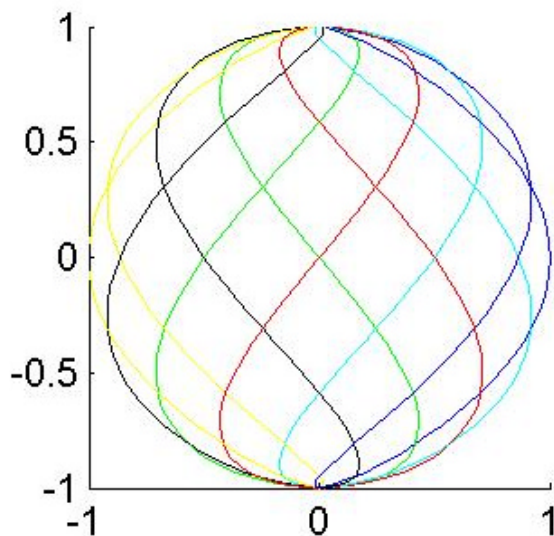








Trajectory of B-vector on calibrator



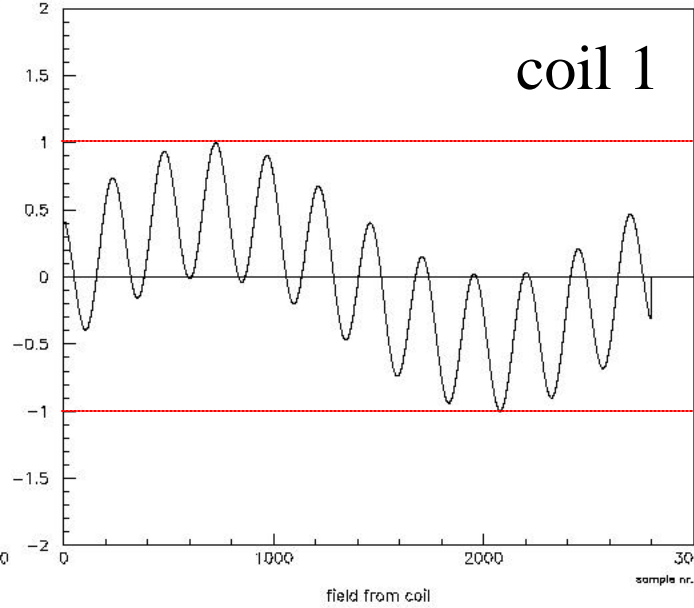
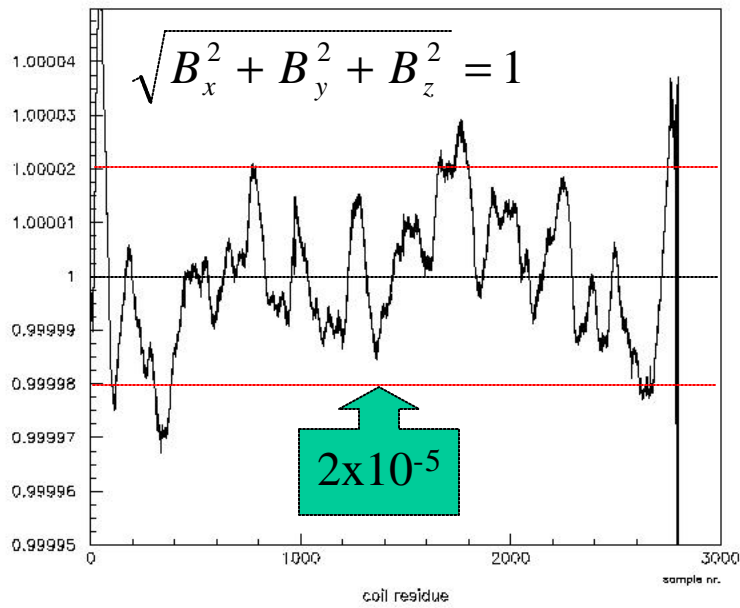
Each 2π turn of main axes
has different color

Outer axes 6 turns
Inner axes 5 turns

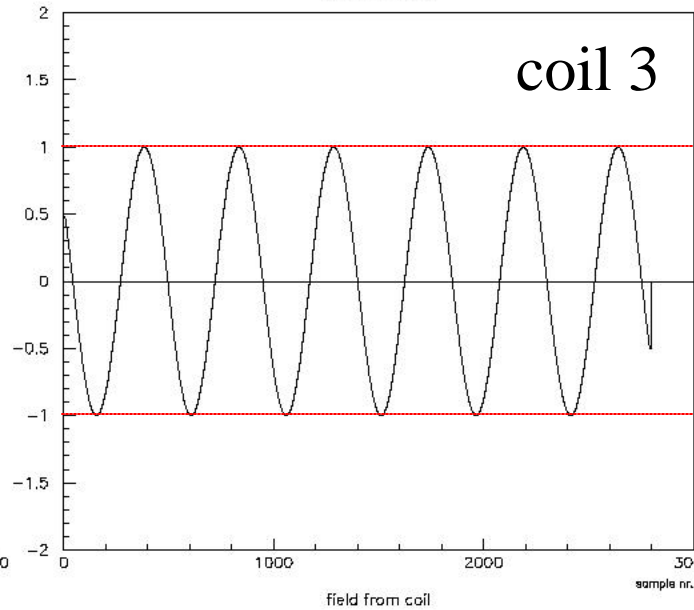
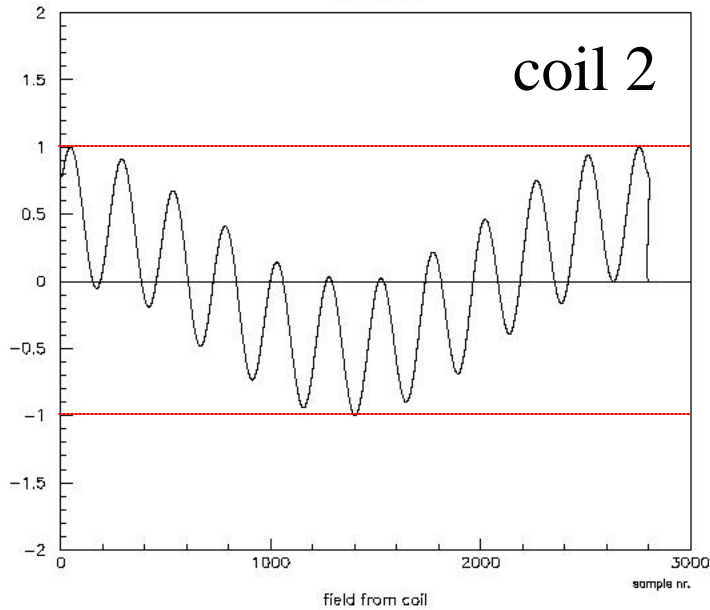


Full coverage of unit sphere
Regular movement, no error build up

Results of coil fit



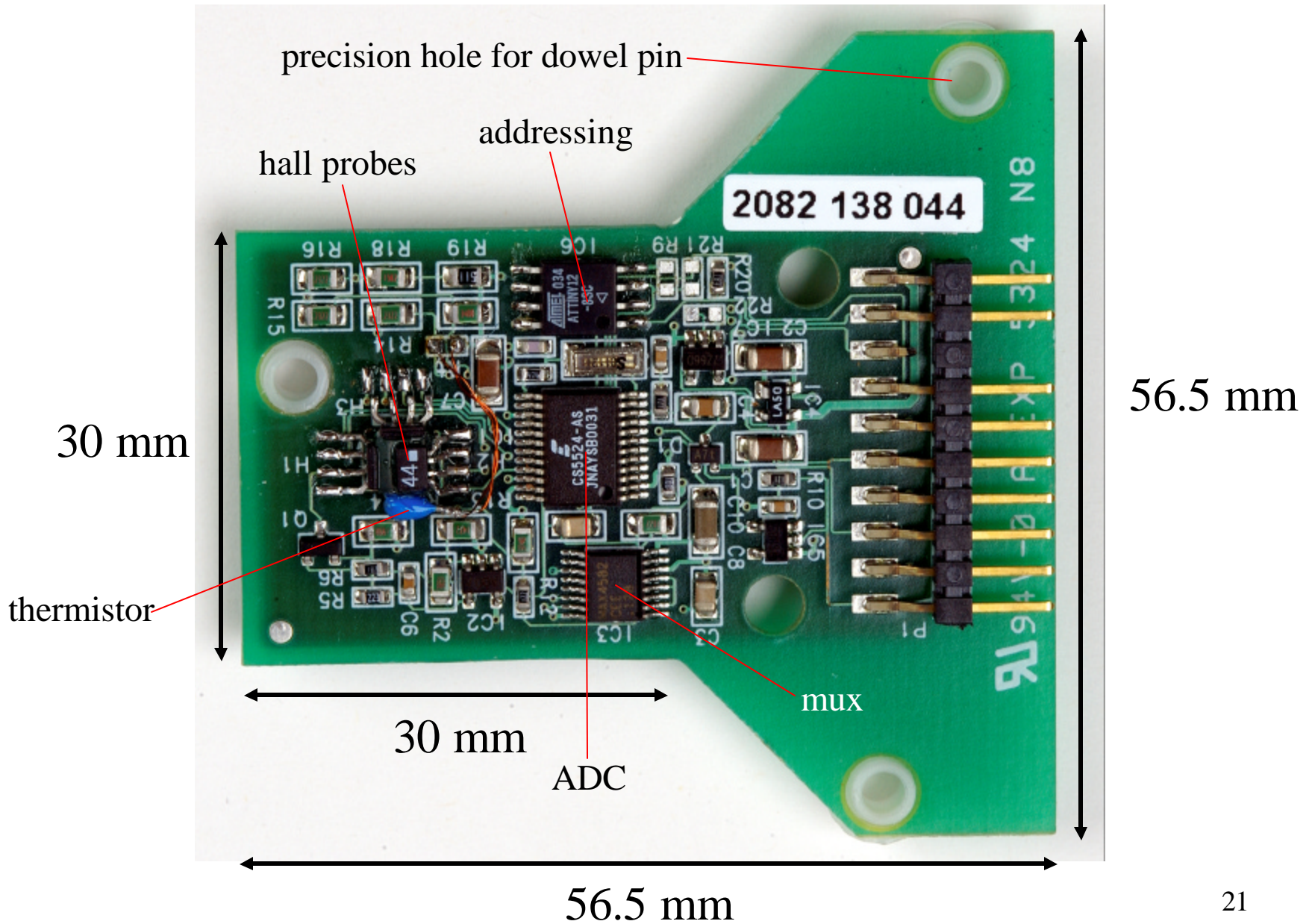
→ Horizontal scale =
time in 1/15th sec.

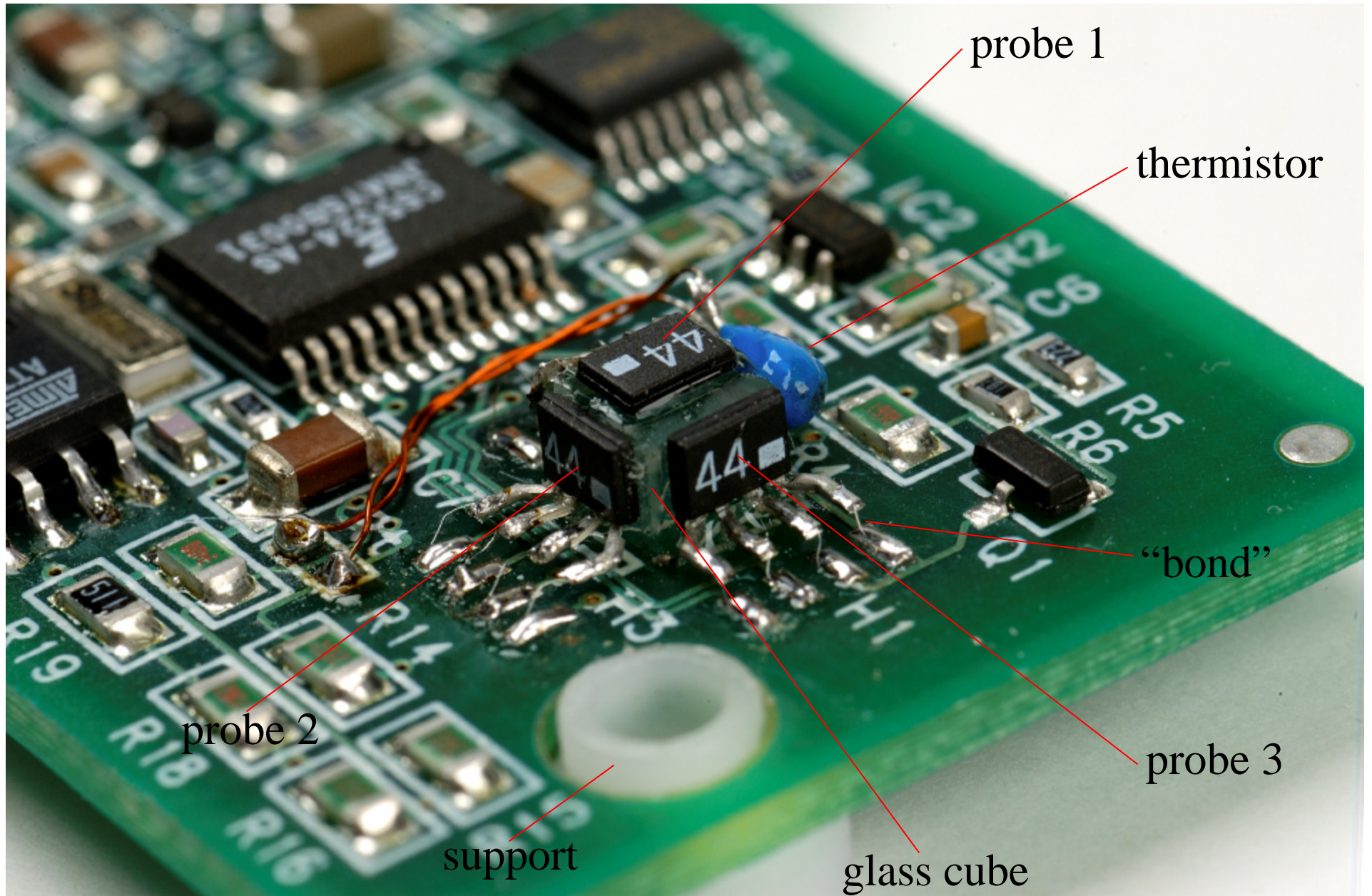


3D magnetic sensor card

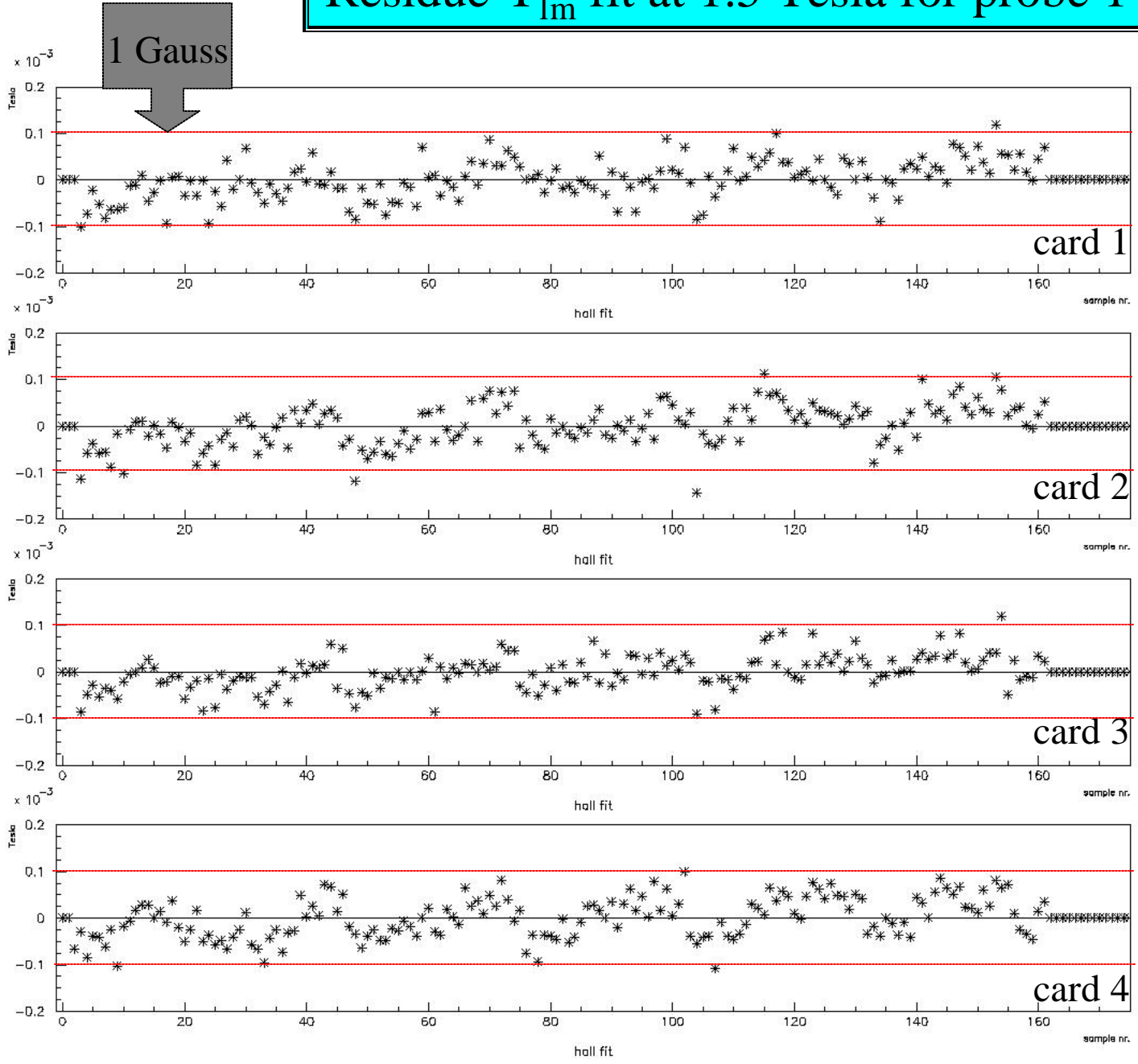
Prototype designed and build by NIKHEF Amsterdam for ATLAS detector at CERN-LHC p-p collider
J.T. van Es (design), J. Kuijt et al.

- Small card containing all analog electronics \Rightarrow electronics in same field as Hall probes, calibrated together
- 3x Siemens KSY44 glued on glass cube, 0.03 mm connection wires “bonds”
- Hall current 230 μ A \Rightarrow small heat dissipation ($I_{\text{nom}} = 5\text{mA}$)
- ADC: 24-bits delta-sigma modulator
- Thermistor connected to cube, no thermostat
- Ref voltage ADC and hall current derived from same voltage source
Offset cal + full scale cal \Rightarrow electronic sensitivity depends only on a few precision resistances
- Calibration circuit for thermistor
- Precision holes to fit on calibrator’s and experiment’s dowel pins
- Addressable: 127 cards on one serial bus





Residue Y_{lm} fit at 1.5 Tesla for probe 1

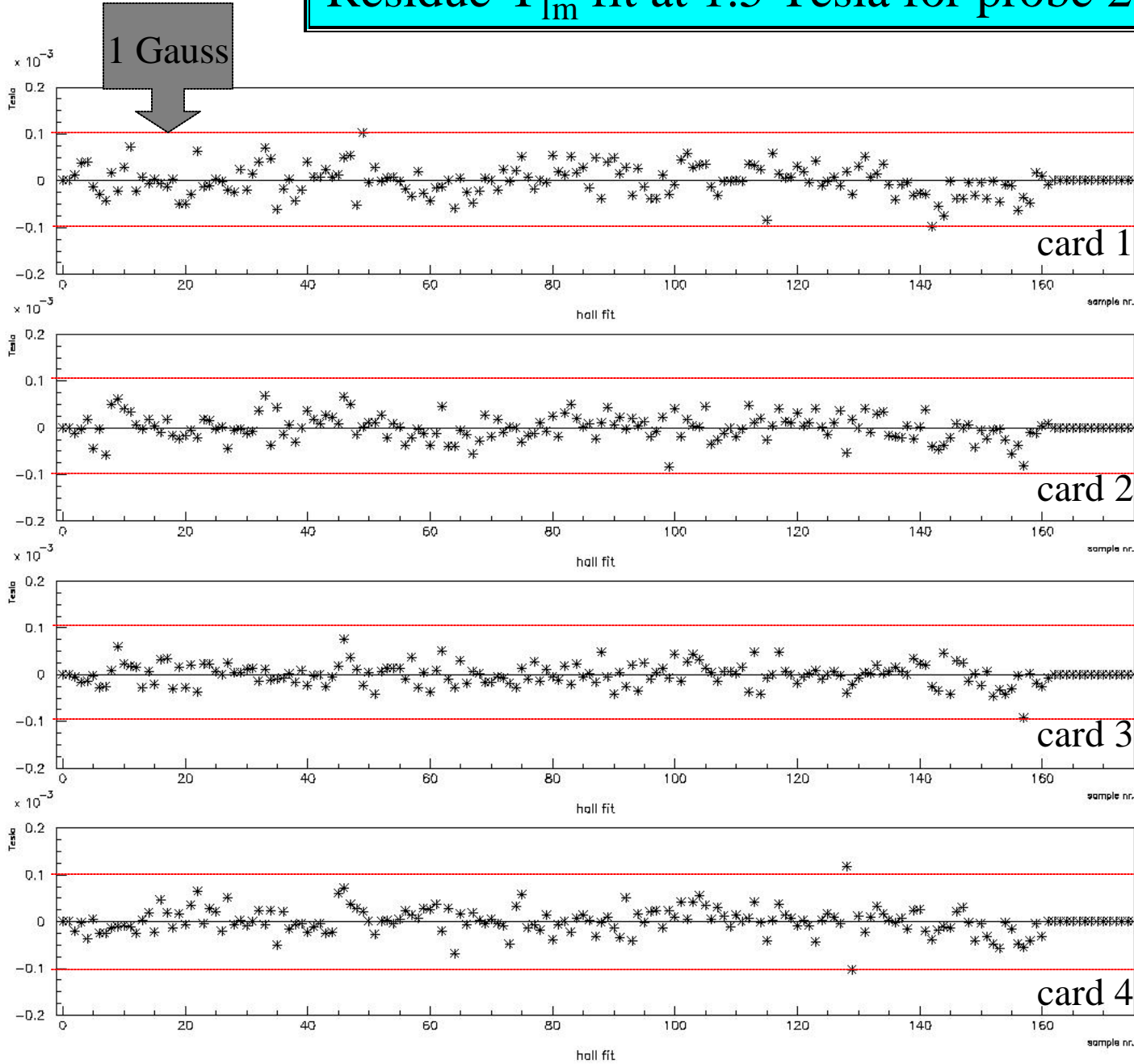


probe n sees same field as coil n

Horizontal scale = time in 16/15th sec.

Vertical scale = Tesla

Residue Y_{lm} fit at 1.5 Tesla for probe 2

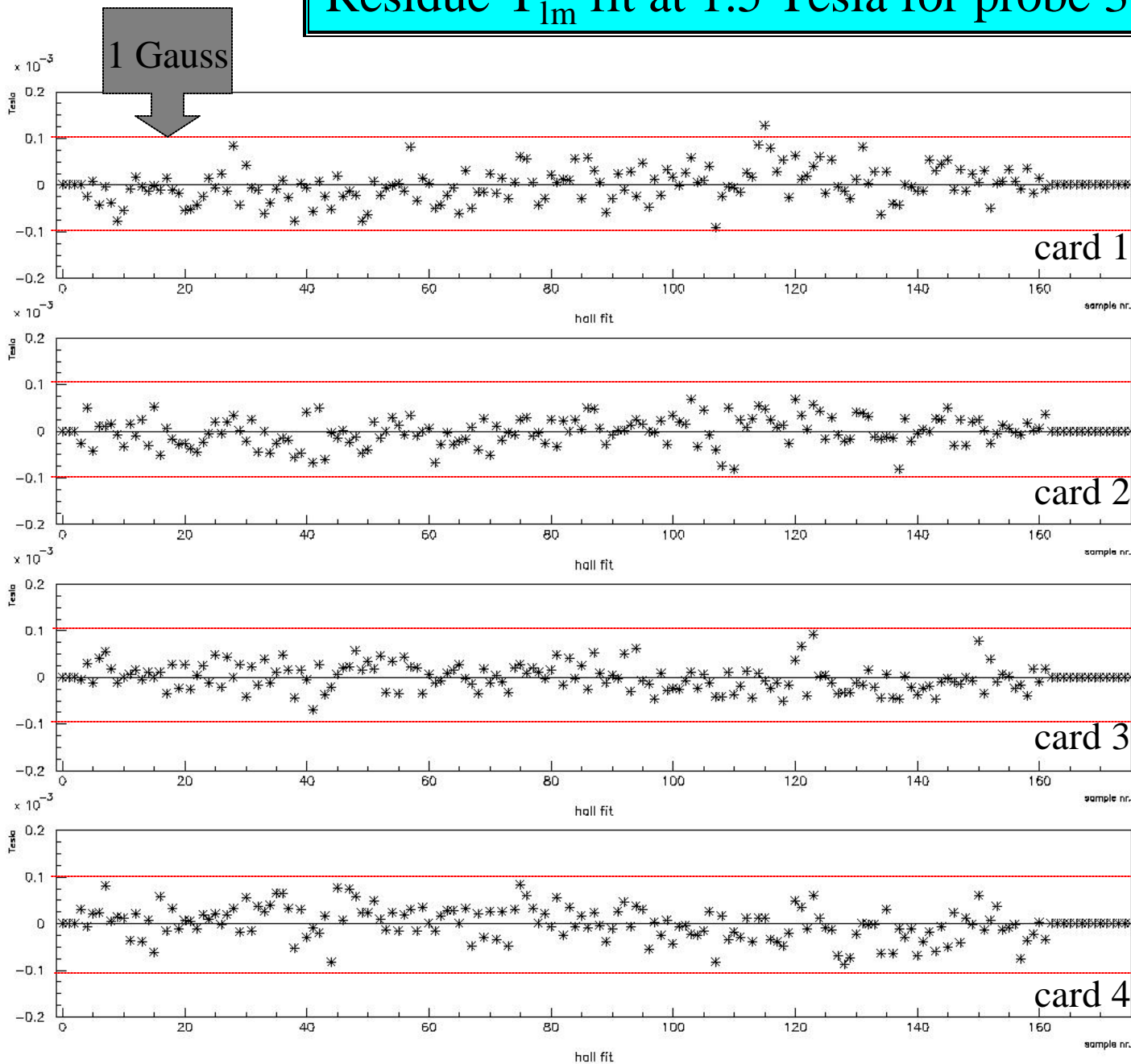


probe n sees same field as coil n

Horizontal scale = time in 16/15th sec.

Vertical scale = Tesla

Residue Y_{lm} fit at 1.5 Tesla for probe 3

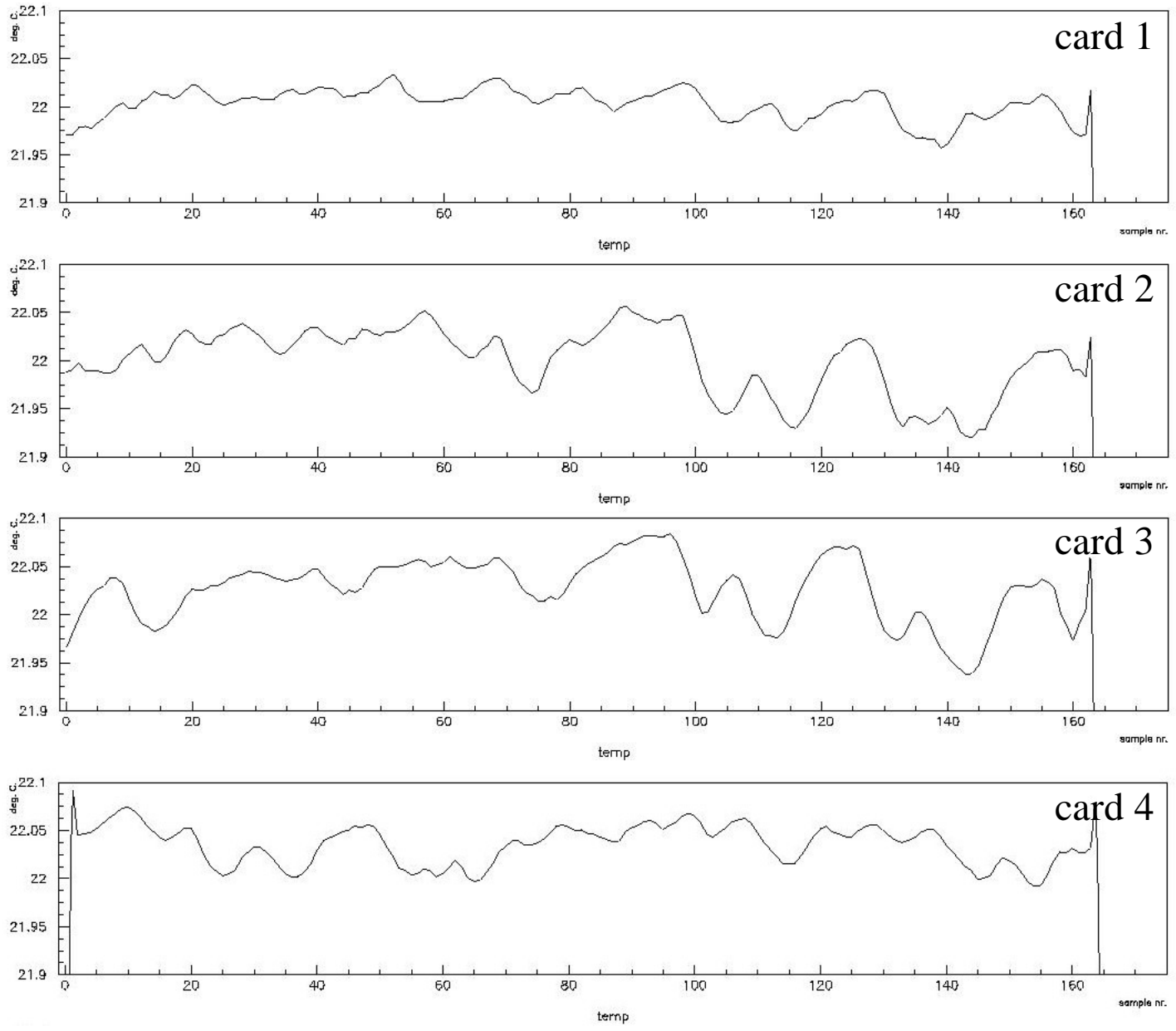


probe n sees same field as coil n

Horizontal scale = time in 16/15th sec.

Vertical scale = Tesla

Temperature during calibration (not corrected for)



← 0.2 deg. C.

→ Horizontal scale = time in 16/15th sec.

↑ Vertical scale = deg. Celsius

sensitivity of Hall probes changes with $\sim 3 \times 10^{-4} / \text{deg.}$

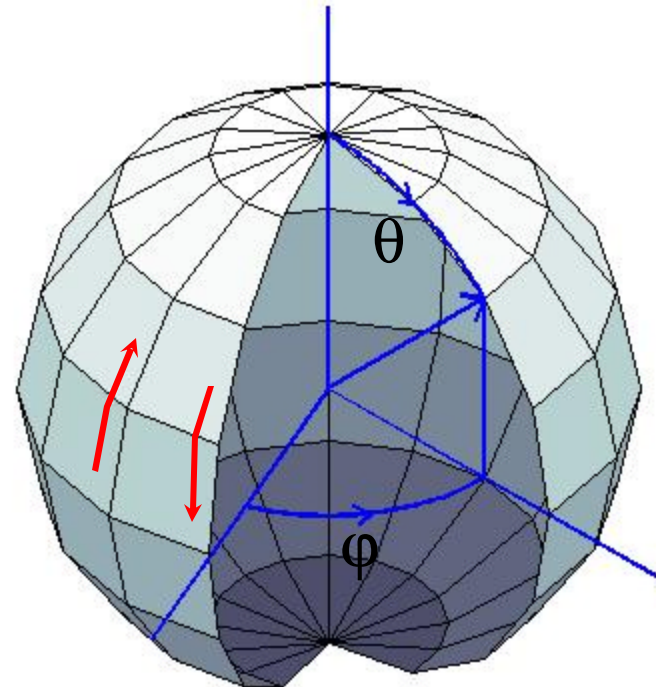
Reconstruction of B

Measurement of 3 x V_{hall} and
T at θ , $\varphi = n \times 22.5$ degree

No movement during measurement

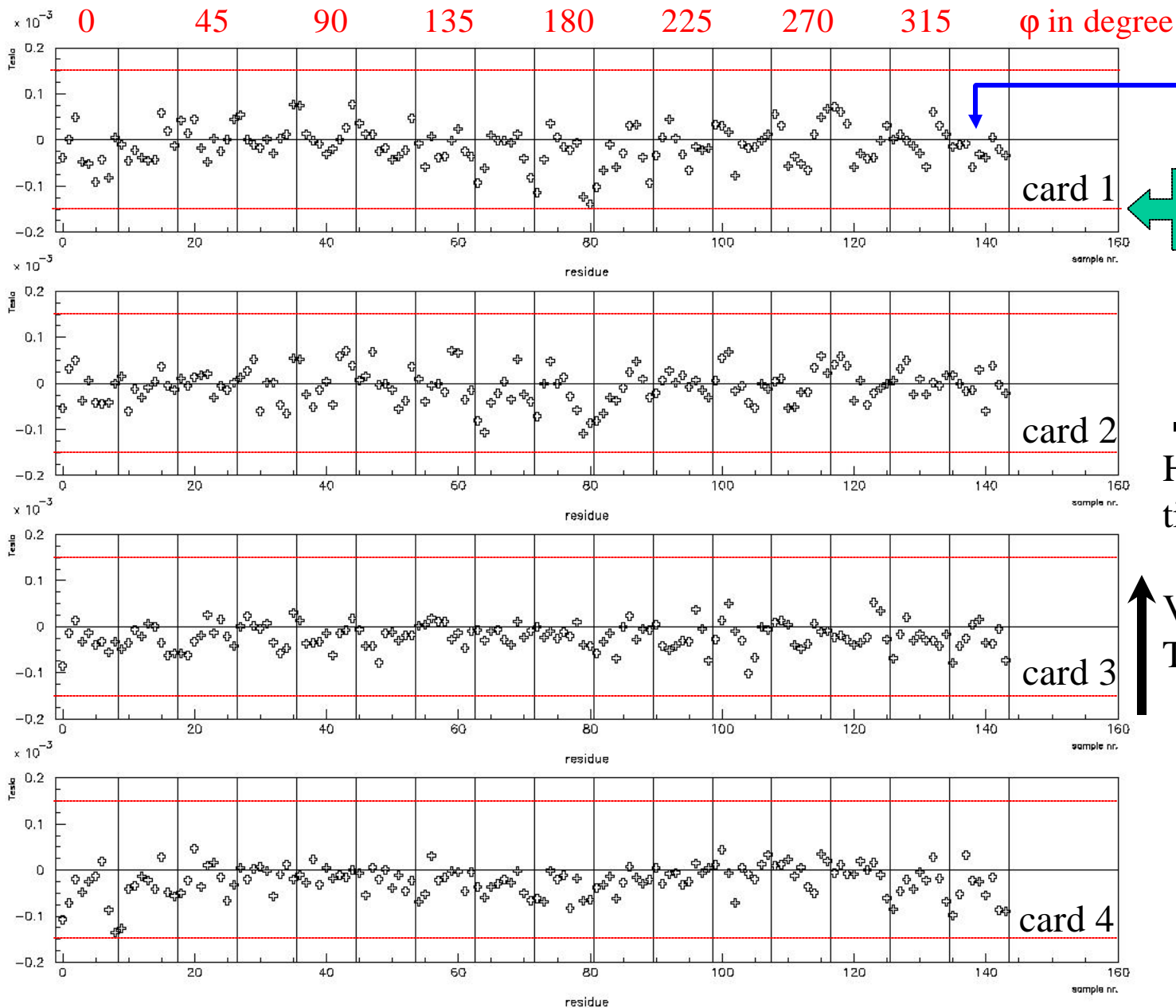
θ alternating in inner loop,
 φ in outer loop

Reconstruction of B_x , B_y , B_z :
Solve 3 nonlinear equations
with 3 unknowns, using
parameterization of calibration data



Tests at several B and T

$|B|_{\text{reconstructed}} - |B|_{\text{NMR}}$ at 1.5 Tesla and 20 deg. C.



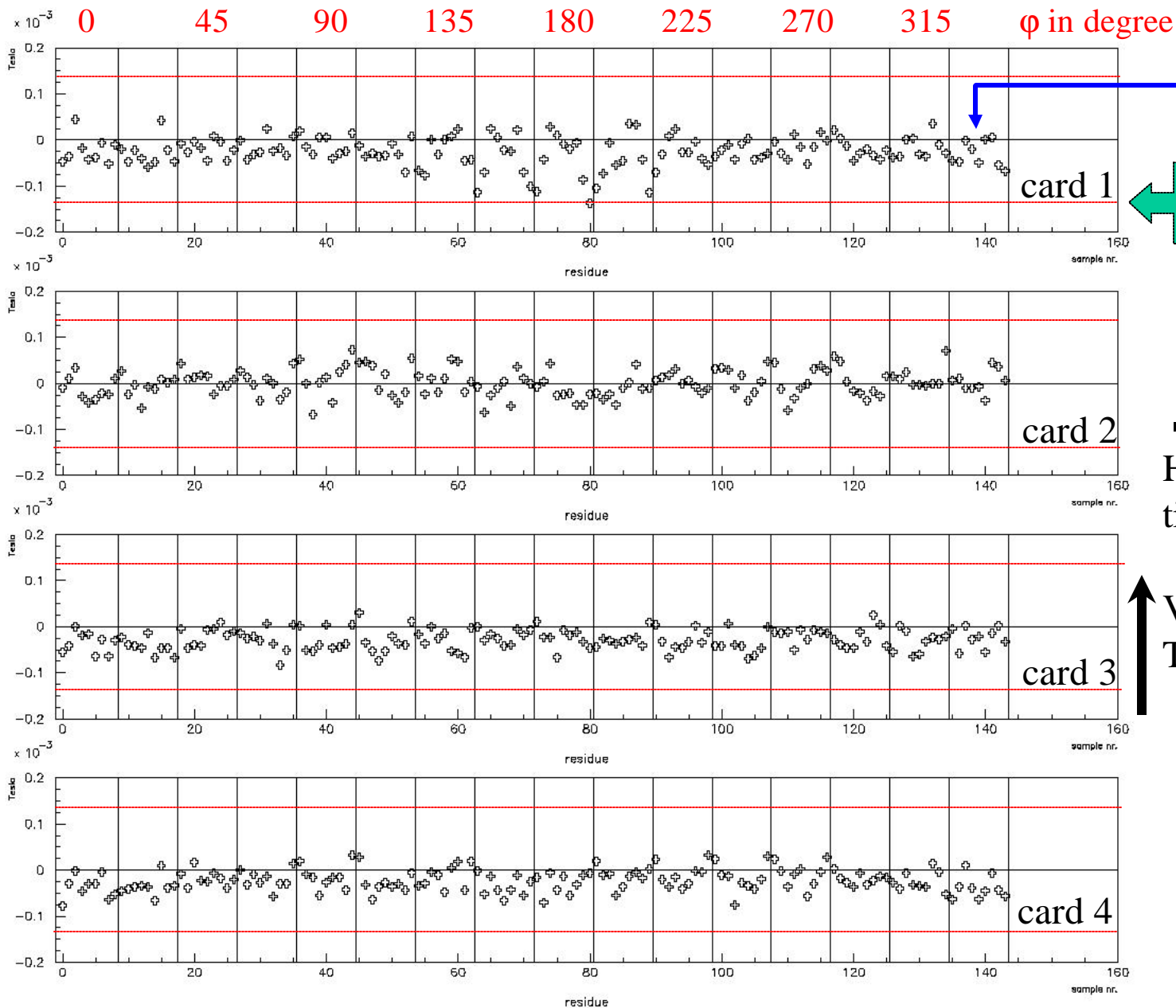
$\theta: n \times 22.5 \text{ deg.}$
 $n = 0 - 8$

10^{-4}

Horizontal scale =
 time in 16/15th sec.

Vertical scale =
 Tesla

$|B|_{\text{reconstructed}} - |B|_{\text{NMR}}$ at 1.375 Tesla and 21 deg. C.



$\theta: n \times 22.5 \text{ deg.}$
 $n = 0 - 8$

10^{-4}

Horizontal scale =
 time in 16/15th sec.

Vertical scale =
 Tesla

$|B|_{\text{reconstructed}} - |B|_{\text{NMR}}$ at 0.625 Tesla and 21 deg. C.



$\theta: n \times 22.5 \text{ deg.}$
 $n = 0 - 8$

10^{-4}
 Horizontal scale =
 time in 16/15th sec.

Vertical scale =
 Tesla

Analysis programs tested with simulated data:

Simulated calibration at fixed $|B|$ and T :

Generate position of coil/Hall sensor, same trajectory as real calibration
coil signal = $L d\Phi/dt$, hall signal from Y_{lm} decomposition of real calibration



residue in coil fit $\sqrt{B_x^2 + B_y^2 + B_z^2} = 1$ of order of machine precision
find back same Y_{lm} decomposition within 0.5×10^{-5}

Simulated B reconstruction at arbitrary B and T :

Hall signal generated with parameterization of calibration



Difference between generated and reconstructed data of order of machine precision

Mean Time Between Calibrations (MTBC)

Main source of instability is a change in offset and angles between the Hall probes

Need “bonded” Hall probes glued on glass cubes

Offset and angles measured over 1 month period



For a precision of $\leq 10^{-4}$: MTBC ≥ 1 month, except for an offset calibration (zero field measurement) for some probes
tests continue

Results wouldn't change with one collective rotation of Hall probes

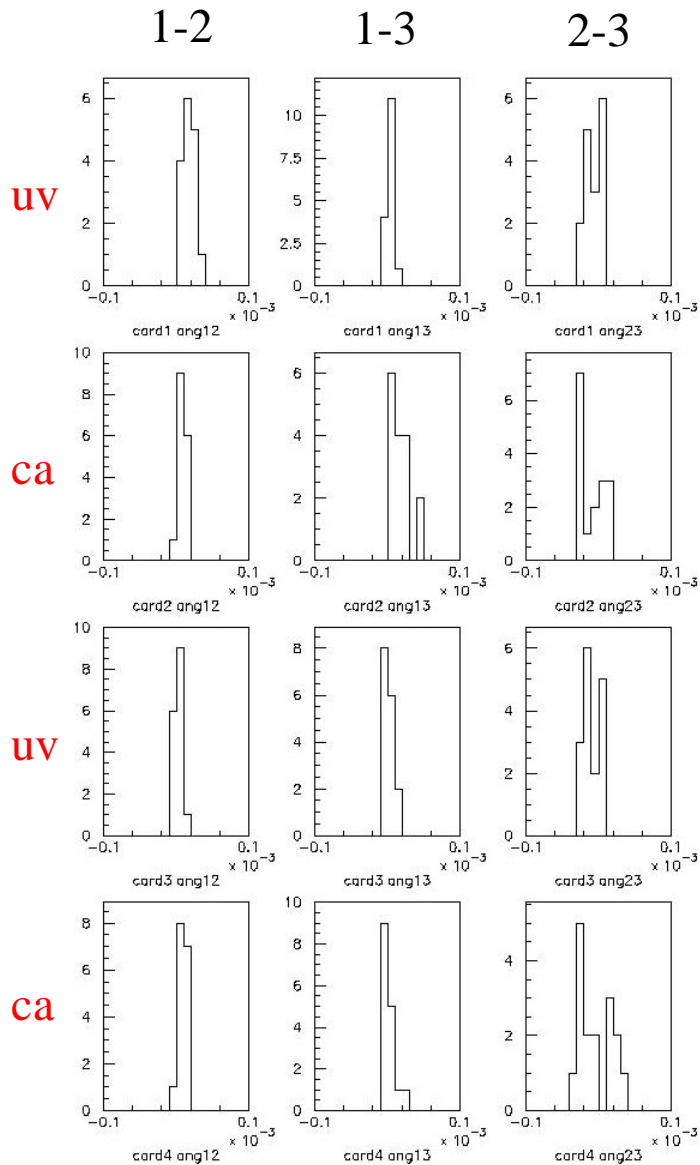


Need confirmation with an absolute measurement

angles between probes

Period of one month

offset



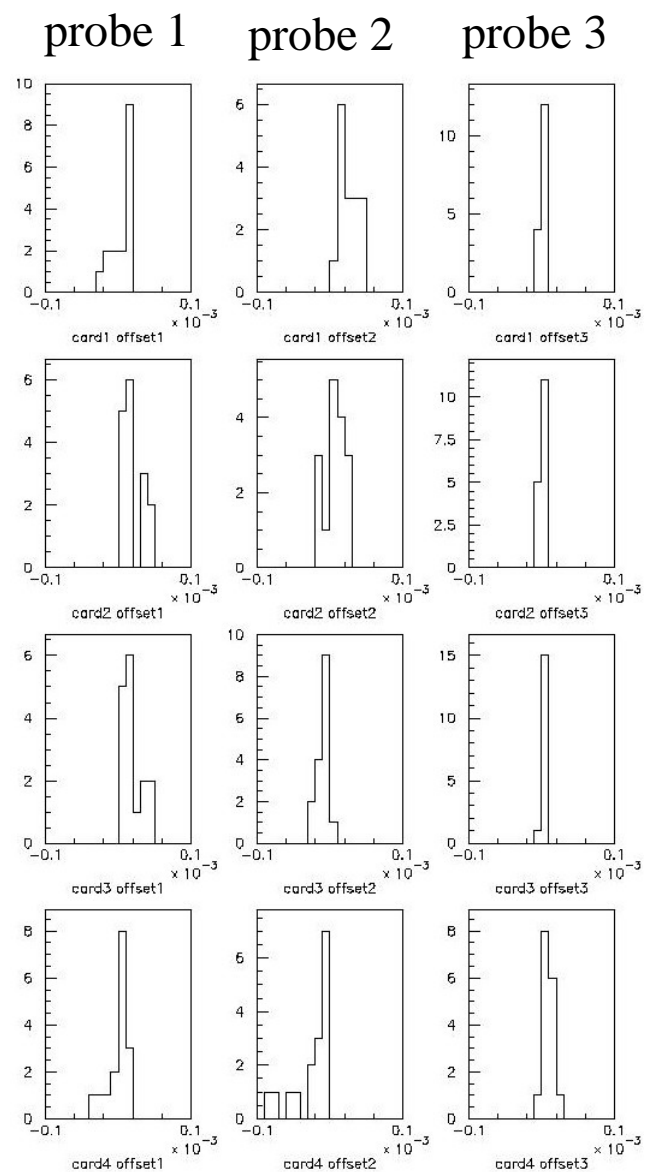
horiz. full scale $\pm 10^{-4}$ rad

card 1

card 2

card 3

card 4



horiz. full scale equiv. of ± 1.5 Gauss

Next

NIKHEF- Amsterdam

- Finish ATLAS 3D-sensor + calibrator for mass production. Make high (4T) field version
- Work on miniature version of calibrator for LHC magnet (40 mm diam.) for 9 Tesla
- Optimize calibrator (precision 10^{-5} seems possible)
Do some absolute measurements
- 3D one-chip sensors:
Small dimensions, almost point like measurements
Decomposition of input $V_{\text{Hall}} \Rightarrow$ direct temperature of chip
Angles between x, y, z channels stable
Spinning Hall current \Rightarrow offset more stable
- Research collaboration with industry sought

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