Optical Radiometry and Applications

William L. Wolfe

Douglas W. Wolfe

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About the Authors

Douglas William Wolfe graduated from the University of Michigan in 1986 with a degree in mechanical engineering. He spent his career at the Hughes Aircraft Company and Raytheon after the takeover, working on a variety of opto-mechanical projects—often creative and cutting-edge conceptual telescopes. He has enjoyed working in the same field as his father William L. Wolfe, and has more recently enjoyed providing feedback on his books. Doug is a longtime fan of Mathcad as a communication and calculation tool. He encouraged his father to build Mathcad into his next book (this one) and considers it an excellent and powerful tool particularly good for radiometry, as it handles the difficult use of units very, very well. He has enjoyed working on the Mathcad materials and giving occasional advice. It is an irony and a pleasure to advise and criticize one's own father.

William Louis Wolfe received his B.S. in physics from Bucknell University and advanced degrees in physics and electrical engineering from the University of Michigan. He worked as an engineer at what was then the Willow Run Laboratories and later became the Environmental Research Institute of Michigan (ERIM) and as a lecturer in the Electrical Engineering department of the University of Michigan-Dearborn. He then worked for three years at the Honeywell Radiation Center in Lexington, Massachusetts as the manager of the Electro-optics Systems Department. In 1969 he became Professor of Optical Sciences at the University of Arizona, where he created and developed the infrared and radiometry programs and curricula. He was *remoted* to Professor Meritless in 1995. He is a Fellow of the Optical Society of America, now Optica, and served on its board of directors. He is a Life Fellow and was president of SPIE, the International Society of Optics and Photonics.

He has received several honors, including the Gold Medal of SPIE and Most Successful in a Chosen Field from his alma mater. He has been editor or coeditor of several handbooks, including: *The Handbook of Military Infrared Technology*, *The Infrared Handbook*, *The Handbook of Optics*, and *The Optical Engineers Desk Reference*. He has written a dozen books, the most relevant being *Introduction to Radiometry* (SPIE Press,1998) and *Introduction to Infrared System Design* (SPIE Press,1996). He has served on many governmental advisory committees, including the Defense Science Board, Naval Intelligence Advisory Committee, National Bureau of Standards Radiometry Advisory Committee, and the Army Countermine Advisory Committee. He consulted for many corporations, including Raytheon, Grumman, RCA, General Electric, and General Motors. He was an expert witness for about one dozen patent cases.

Last and certainly not least, he guided 50 students to their advanced degrees in optical science.

Throughout this book, any first-person usage ("I") is an assertion by William Wolfe.

Chapter 1
Introduction

This chapter includes descriptions of the scope, history, philosophy, and language of radiometry. It even includes a bit of radiometric *strategery*.

Subsequent chapters include the **theory** of radiation emission and absorption; **normalization** and **accuracy and precision**; and methods of **measurement** of radiometric properties. There is a short discussion of the detectors, sources, and optics that may be used in radiometric instruments. Chapter 6 Applications includes many examples that we hope are both interesting and informative. They are interesting to us, but their real intent is to teach the techniques of radiometry.

1.1 The Scope of Radiometry

The literal meaning of radiometry is the measurement of radiation. Radio from the Latin *radiolis* is ray, and meter from *metrum* is to measure. This meaning of radiomety includes the measurement of radiation but also the measurement of the properties of the components that influence the propagation of the radiation: the emission, absorption, reflection, and transmission properties. Radiometry has also come to include the measurement of temperature by radiometric means. The most extensive use of radiometry by far is the calculation of the sensitivity of many different instruments by determining whether they have enough radiation to accomplish their intended functions. **Optical** radiometry is generally considered those activities in the electromagnetic spectral region from about 0.3 μm to about 20 μm.

Radiometric instruments range from the very simple intrusion detector on our houses to the very sophisticated ballistic missile interceptors and infrared cameras that detect border crossers.

The detector on a house has two simple thermal detectors, a plastic cover, and very simple electronics. It does a fine job of detecting coyotes, javelinas, and bobcats but not cold-blooded snakes. It is described in Section 6.21.

light also obeys the inverse square law of propagation.[12] This is almost self-evident if you consider that the force, whatever it may be, is essentially a point whose sphere of influence—its "force" per unit area—decreases quadratically since the area of a sphere is proportional to the square of the radius. The force remains constant, but the area increases by the square of the distance.

The law of exponential attenuation of radiation in absorbing materials was first enunciated by Pierre Bouguer in 1729,[13] although it is now often referred to as the Beer–Lambert law. Johann Lambert quoted Bouguer in 1760 and stated the law in a slightly different form.[14] Lambert stated that light loss was *proportional to the path length and the intensity.* August Beer said that the transmittance is constant if the concentration and the pathlength are constant at a specific wavelength or spectral interval.[15]

The cosine law of emission and irradiation first stated by Johann Lambert in Ref. 15 states that a uniform radiator or receiver has a flux density proportional to the cosine of the angle of emission or reception. It is called Lambertian. It is better to state that such a surface has a **radiance** that is independent of angle.

Kirchhoff's law of emissivity and absorptivity states that under identical conditions, the efficiency of radiation is the same as the efficiency of absorption. He deduced this in 1860.[16]

The Stefan–Boltzmann law for total blackbody radiation was derived by Josef Stefan based on his experimental studies in 1879[17] and later derived thermodynamically by Ludwig Boltzmann in 1884.[18]

The famous Planck expression for the spectral distribution of blackbody radiation was derived by Max Planck in 1900.[19]

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[12]J. Freely, *Before Galileo: The Birth of Modern Science in Medieval Europe*, Abrams Press (2013).

[13]P. Bouguer, *Essai d'Optique sur la Gradation de la Lumière*, Claude Jombert, Paris, pp. 16–22 (1729).

[14]J. H. Lambert, *Photometria sive de mensura et gradibus luminis, colorum et umbrae*, Eberhardt Klett, Augsburg, Germany (1760).

[15]A. Beer, "Bestimmung der Absorption des rothen Lichts in farbigen Flussigkeiten," *Annalen der Physik und Chemie* **162**(5), 78–88 (1852).

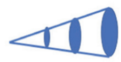
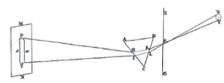
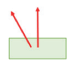
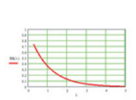
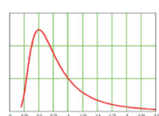
[16]G. Kirchhoff, "Über das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme and Licht," *Annalen der Physik und Chemie* **109**(2), 275–301 (1860).

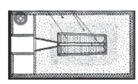
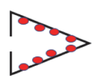
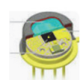
[17]J. Stefan, "Über die Beziehung zwischen der Wärmestrahlung und der Temperatur," *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften: Mathematisch-Naturwissenschaftliche* **79**(3) 391–428 (1879).

[18]L. Boltzmann, "Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie," *Annalen der Physik und Chemie* **258**(6), 291–294 (1884).

[19]M. Planck, "Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum," *Verhandlungen der Deutschen Physikalischen Gesellschaft* **2**, 237–45 (1900).

A visual presentation of these events is in the following charts. The first is a presentation of important advances in optics and optical understanding. The second is an equivalent presentation of the history of radiometric instruments and components.

	Optics	
Visual star maps	129 BC to 1572	
Inverse square law John Dumbleton?	1349	
Spectra, Newton	1672	
Comparator, Steinheil	1835	
Cosine law, Lambert	1729	
Exponential absorption, Bouguer	1729	
Infrared, Herschel	1800	
Ultraviolet, Ritter	1801	
Film, Daguerre	1826	
Photodetector, Becquerel	1836	
$\alpha = \epsilon$, Kirchhoff	1860	
Total radiation, Stefan & Boltzmann	1879	$M = \sigma T^4$
Quantization, Planck	1900	
Photons, Einstein	1905	

	Instruments	
Eyes		
Comparators, Steinheil	1835	
Sperm whale lamp. The Brits	1860	
Incandescent lamp, Edison	1879	
Bolometer, Langley	1880	
Integrating sphere, Sumpner	1882	
Photodetector, Becquerel	1890	
Globar, American Resistor	1925	
Photomultiplier tube, Iams and Salzburg	1934	
Pyroelectric detector, Yeou Ta	1938	
Platinum blackbody, NIST	1974	
Electrical substitution radiometer Boivin, Smith	1974	
Self-calibrated detector Geist and Zalewski	1980	

It works.

The throughput, the $A\Omega$ product on the right, is the same calculated three different ways. Gerry would be pleased.

Another example comes directly from the preparation of this book. It was in the calculation of the sensitivity of the ratio temperature technique (see Section 3.5). The radiant exitances in two spectral bands were calculated in Mathcad. Their values were copied onto an Excel spreadsheet for calculation of the relative differences in the ratios. As a check, these ratios were plotted as a function of temperature. The curves came out with kinks. That is not the way nature behaves. Further checking showed that one or more of the data entries was off by a digit or two. When corrected, the curve was linear and smooth. Mother Nature may be relentless and devastating, but she is not macroscopically kinky.

There are two areas to consider when you think of everything: the environment of the object and the state of the instrument. The object can be indoors or outdoors. If it is outdoors, one must consider the sunshine, moonlight, starlight, general background, and all sorts of reflections as well as the atmosphere through which both the object and the background radiation transmit, as illustrated in Fig. 1.4.1.

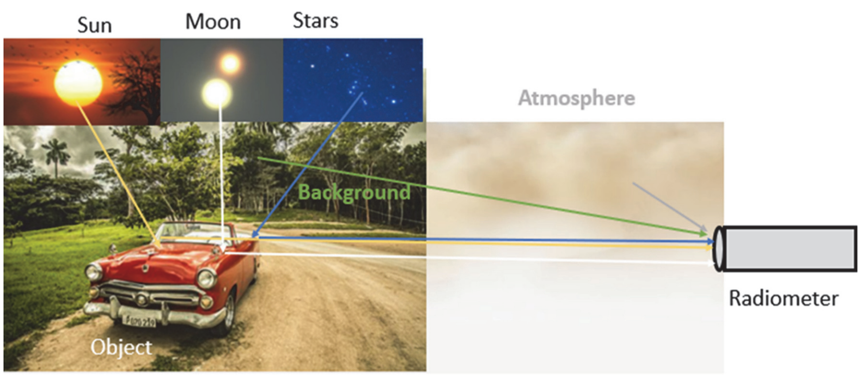


Figure 1.4.1 The outdoor environment.

The indoor environment can also be complicated. Figure 1.4.2 illustrates the situation with the curtains open. There are even more reflections than shown.

Figure 1.4.2 The indoor environment with open curtains.

Even with the curtains drawn, there are many sources and reflections.

Figure 1.4.3 Indoor environment with closed curtains.

In some circumstances, like the multispectral imaging of a hand, these spurious sources can be eliminated by shielding and the close proximity of the hand to the instrument.

A useful memory tip to account for the state of the radiometer and the measurement is to write the radiometric responsivity equation with all of its possible variables like this:

$$\mathfrak{R} = \mathfrak{R}(t,\ T,\ s,\ p,\ RH,\ \lambda,\ \ldots), \tag{1.4.1}$$

where the responsivity is a function of time t, temperature T, polarization s and p, relative humidity RH, and wavelength λ.

Chapter 2
Theory

This chapter describes the theories of emission and absorption, reflection and transmission, and the transfer of radiation. It also includes the vital concepts of accuracy and precision, and the techniques and dangers of normalization, including photometry.

2.1 Emission and Absorption

Emission and absorption are closely related phenomena. It was proven by Gustav Kirchhoff in 1860[1] that the efficiency of emission of any real body is equal to its efficiency of absorption. Our version is given below. A perfect radiator, one that has an emissivity of 100%, is called a blackbody because it also absorbs all light. Many of its properties are also described below. These efficiencies are labeled absorptivity and emissivity in this book and have the symbols α and ε. Kirchhoff's law is that absorptivity equals emissivity under the same conditions.

This is our version of his proof. Assume that there are two spheres of different materials, each encased in a totally reflecting shell, as in Fig. 2.1.1. Assume also that they are in thermal equilibrium and that there is a small hole that connects them. Also assume that the left-hand material has a higher absorptivity than its emissivity; it absorbs more radiation from the right-hand sphere than it emits back. It will soon become warmer and violate not only equilibrium but also the second law of thermodynamics, one version of which states that it is not possible for heat to flow from a cooler body to a warmer one without work being applied. In this case, heat in the form of radiation would flow from the cooler, right-hand body to the warmer one shortly after equilibrium was disturbed.

[1]G. Kirchhoff, "Über das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme and Licht," *Annalen der Physik und Chemie* **109**(2), 275–301 (1860).

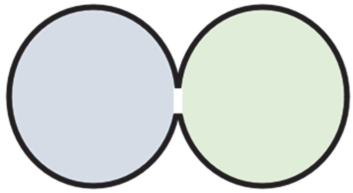


Figure 2.1.1 Kirchhoff spheres.

The result can be extended in two ways. First imagine that a narrowband spectral filter is placed in the aperture so that only radiation in a limited spectral band is transmitted. The same argument applies. The law therefore applies not only for total absorptivity and emissivity but also for spectral quantities. Then imagine that a tube is inserted in the aperture at an arbitrary angle. The argument applies once again. The conclusion then is that absorptivity α equals emissivity ε spectrally and geometrically:

$$\alpha(\lambda, \theta, \varphi) = \varepsilon(\lambda, \theta, \varphi). \tag{2.1.1}$$

This can be described in a more dramatic way. Imagine that the right-hand sphere is in equilibrium with its surroundings. It will continue to heat up the left-hand sphere that absorbs more than it emits. That heat can be used to drive a small motor. The combination of spheres is then a perpetual motion machine. It draws its energy from the surroundings and runs a motor. It is even conceivable that in my backyard in June it could run a lawnmower if I had grass.

As a side note and warning, some authors quote *alpha-over-epsilon* ratios that do not equal one. These are **not** for the **same** conditions. They are usually the absorptivity in the visible and the emissivity in the infrared. Some of the literature also describes gray bodies and spectral bodies. These are, respectively, bodies that have constant absorptivity and those that vary with wavelength. Every material in nature has a spectrally varying emissivity, and every material is gray over a limited spectral range.

In summary, there are values for these two efficiencies that are total and spectral-hemispherical and directional. Techniques for measuring them and references to useful values are in Section 3.5.

The rest of this chapter deals with many aspects of the perfect radiator: the blackbody or Planck radiator. Its emissivity is one and it radiates uniformly in all directions. In its usual form, radiant spectral exitance $M(\lambda,T)$ is the power per unit area emitting uniformly into the overlying hemisphere as a function of its wavelength and temperature:

$$M(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^x - 1)}, \tag{2.1.2}$$

where

$$x = \frac{hc}{\lambda kT} \tag{2.1.3}$$

and h is Planck's constant, c is the speed of light, and k is Boltzmann's constant.

This can also be written in terms of the two radiation constants, c_1 and c_2, as

$$M(\lambda, T) = \frac{c_1}{\lambda^5(e^x - 1)}, \tag{2.1.4}$$

where

$$x = c_2/\lambda T. \tag{2.1.5}$$

The first radiation constant is usually reported as $c_1 = 3.741771852... \times 10^{-16}$ Wm^2. A more useful form for radiometric purposes is 37,417.71852 Wμm^{4}m^{-2}. Sufficient accuracy for most applications is 37418 Wμm^4 m^{-2}. The second radiation constant is usually reported as $1.438776877... \times 10^{-2}$ mK. A more useful form for these purposes is 14,387.76877 μmK. Sufficient accuracy in this case is 14,388 μmK.

The photonic radiant exitance, the number of photons per unit time and area, may be found by simply dividing the expression for radiant exitance by the energy of a photon as

$$M_q(\lambda, T) = \frac{M(\lambda, T)}{hc/\lambda} = \frac{2\pi c}{\lambda^4(e^x - 1)}. \tag{2.1.6}$$

Conversion to exitance as a function of frequency is not quite so simple. $M(\nu,T)$ is **not** equal to $M(\lambda,T)$. They are both distributions. One is flux density **per wavelength** and the other is flux density **per cycle per second**, and frequency does not equal wavelength. But the actual amounts are equal; that is,

$$M(\nu, T)d\nu = M(\lambda, T)d\lambda \tag{2.1.7}$$

so that

$$M(\nu, T) = \frac{M(\lambda, T)d\lambda}{d\nu}. \tag{2.1.8}$$

The speed of light is equal to the product of the frequency and the wavelength, $c = \nu\lambda$, so that

$$\lambda = c/\nu, \tag{2.1.9}$$

The power on the detector can be calculated in the same way. The only reduction in power is the transmission loss of the optical system by reflection and absorption. Therefore, the full-blown equation can be written as

$$P = \frac{\tau_a \tau_o L A_s \cos\theta_s A_o \cos\theta_o}{R^2} = \frac{\tau_a \tau_o L A_o \cos\theta_o A_d \cos\theta_d}{f^2}. \tag{2.4.4}$$

The left-hand term can be used if the properties of the source and the optics are known. The right-hand term can be used if the optics and the detector are known. Equation (2.4.4) can also be written more compactly by indicating projected areas with primes and the product of the transmittances as just τ and in terms of solid angles:

$$P = \tau A'_s \Omega_{os} = \tau A'_o \Omega_{so} = \tau A'_o \Omega_{do} = \tau A'_d \Omega_{od}, \tag{2.4.5}$$

where Ω_{ab} means the solid angle of the first area a, as subtended at the second area b. There is an even more compact way of writing this. It is in terms of the throughput of the optical system. The throughput is also called the A-Ω product as it is the product of the area and the solid angle that the other area subtends. The French term is étendue. There is no accepted symbol for it, and it is difficult to invent one. We have chosen G for "grasp" because this symbol is simple, meaningful, and has no other similar meanings. The most compact form of the fundamental equation of transfer is therefore

$$P = \tau L G, \tag{2.4.6}$$

which is the product of the total transmission τ, the radiance L, and the throughput G.

The throughput—the A-Ω product, the product of the area and the solid angle—is the product of the area and the **opposing** solid angle, the solid angle whose base is the area, as shown in Fig. 2.4.1.

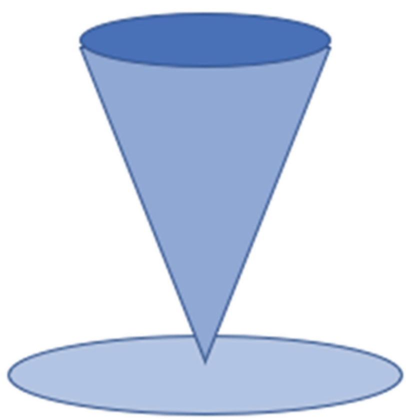


Figure 2.4.1 The étendue.

"No ice cream cones," as my friend, Jim Palmer, once put it.

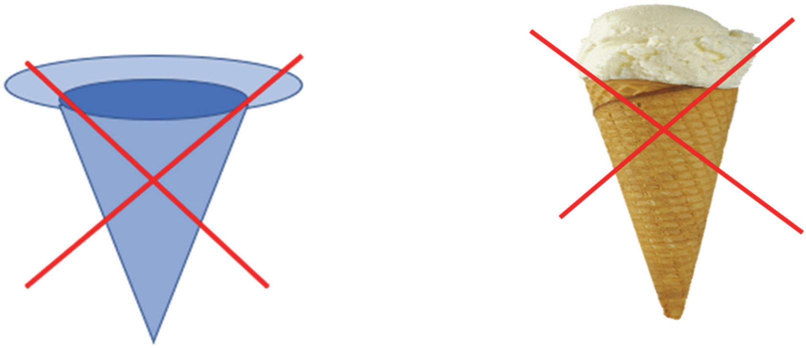


Figure 2.4.2 The *nonétendue*.

In some circumstances, there is no information about the details of the area of the source. It may be too small to be resolved. (These are often called point sources, but they are not geometric points, which are infinitesimal). Then the equation must be modified to eliminate consideration of the source area. This is done by using the radiant intensity of the source *I*, which is its radiance times its area *LA*. Then the power on the detector may be written as

$$P = \tau_a \tau_o I \Omega_{os} = \tau_a \tau_o I A_o / R^2. \tag{2.4.7}$$

Since the distance is not under the control of the investigator, especially astronomers, the component that maximizes the power on the detector is the area of the optical aperture. This is the reason for such large astronomical telescopes. With such a source, it is the weighted average of the intensity that must be used, either from measurements or other data.

If the object is resolved, as with cameras and binoculars, then the applicable equation in system space is

$$P = \tau_a \tau_o L \Omega_{od} A_d = \tau_a \tau_o L \Omega_{do} A_o = \tau L G. \tag{2.4.8}$$

Then the two options for increasing the power on the detector are increasing either solid angle. You can increase Ω_{od} by increasing the aperture area, as with night-viewing binoculars, or you can increase Ω_{do} by decreasing the focal length, as with most cameras.

The solid angle Ω_{do} is often described by the focal ratio *F*, which is also called the relative aperture, the speed, and similar terms. It is the ratio of the focal length to the aperture diameter and is a linear measure of the solid angle. Then the expression becomes

$$P = \frac{\tau_a \tau_o L A_d}{F^2}. \tag{2.4.9}$$

Chapter 3
Measurements

This chapter describes how radiometric properties are measured. These properties include power, emissivity and absorptivity, reflectivity, transmissivity, and temperature.

3.1 Radiation Measurements

Radiation measurements are what most people think radiometry is: a measurement of the power in some spectral region. This measurement is made with a radiometer that is one of two kinds: an electrical substitution radiometer or a comparison radiometer. These are discussed in Section 5.1. Radiation level measurements are probably the simplest of all radiometric measurements.

Comparison radiometers compare the electrical signal from the object of interest to the signal from an internal source. The internal source is a blackbody instrument but has an emissivity less than 1. It is therefore necessary to calibrate the comparison radiometer using a standard blackbody source. None of these sources has a full 100% emissivity, but they may be as good as 99%. See Section 5.3 for more details.

The electrical substitution radiometer (ESR) is a cavity, usually conical, with a thermal detector at its apex, a small aperture in front, and a series of heating coils on its interior surface. In operation, the aperture is opened, and the detector records the level of input radiation. Then the aperture is closed, and the heating coils are activated until the same signal level is recorded. This is the amount of electrical power that is equivalent to the input radiant power, the electrical substitution. Note that the closed cavity is a perfect blackbody.

The most common use of radiometers that measure the level of radiation is in non-destructive testing. The actual temperature is not the key; the fact that there is a local increase in temperature is the key. This increase indicates some sort of abnormality, a bad connection, a chafing component, something bad. Overheating is a sign of trauma, both human and mechanical.

A general rule of thumb is that these radiometers have an uncertainty of 5 to 10%. It is important to get a NIST certification for one that is purchased. Depending on your application, a blackbody calibration source may also be indicated.

Radiometers must be aimed at the object being measured. Ideally, the object fills the field of view of the radiometer. There are two schools of thought about this procedure. The believers insist that the use of any visible optical pointer will contaminate the measurement in the infrared. The skeptics disagree. We present an analysis and a technique for doing it both ways in Section 6.26.

Assume that the radiometer measures a circular area of the object with a diameter of 10 cm and that the object is at 300 K. The pointer uses visible light with a diameter of 1 cm on the object. The radiometer is filtered so that it does not sense visible radiation. Then the pointer must heat the object to a temperature that provides 1% contribution to be minimally significant. We can estimate this based on the Stefan–Boltzmann law:

$$\sigma T_{\mathrm{p}}^4 A_{\mathrm{p}} = 0.01 \sigma T_{\mathrm{r}}^4 A_{\mathrm{r}}, \tag{3.1.1}$$

where the T_{p} indicates the temperature that must be generated in the sample by the pointer and T_{r} is the temperature recorded by the radiometer. The assumed areas have a ratio of 100 to 1; the sigma's cancel, and the pointer must raise the temperature by an amount equal to the object temperature. The time this takes can be estimated based on the specific heat of the object. The equation for the rate of temperature increase, an inversion of the definition of specific heat, is

$$\frac{dT}{t} = \frac{P}{c_{\mathrm{p}} m} = \frac{P}{c_{\mathrm{p}} \rho V}. \tag{3.1.2}$$

Excel shows this for two extremes, steel and plastic objects, and a pointer power of 5 mW. Raising the temperature to a significant level takes much longer than making a measurement—longer than we can wait. Mathcad confirms this.

	A	B	C	D	E	F	G
1			**Radiometer Pointer Heating Rate**				
2				**$dT/t=P/mc_p$**			
3	**Item**	**Symbol**	**Formula**	**Formula**	**Steel**	**Plastic**	**Units**
4					**Value**	**Value**	
5	π	π	known		3.14159	3.14159	#
6	Power	P	assumed		0.005	0.005	W
7	Diameter	D	assumed		1	1	cm
8	Thickness	d	assumed		1	1	cm
9	Specific heat	c_p	known		0.166	1.3	J/gK
10	Density	ρ	known		7.9	0.95	g/cc
11	Melting temperature	T_{melt}	known		1500	422	K
12	Volume	V	$\pi D^2d/4$	$E5xE7^2xE8/4$	1	1	cc
13	Mass	m	ρV	E10xE11	6	1	g
14	Rate	dT/t	P/mc_p	E6/E12/E9	0.0049	0.0052	K/t
15	Time	t	$(T_{melt}-300)$/Rate	$(T_{melt}-300)$/E13	247193	23667	s
16	Time	t		E14/60	4120	394	min
17	Time	t		E16/60	69	7	hr

Pointer Heating Rate

$P := 5\ mW \quad D := 1\ cm \quad d := 1\ cm \quad V := \frac{\pi \cdot D^2 \cdot d}{4}$

$T_{steel} := 1500\ K \quad c_p := \frac{0.166\ J}{gm \cdot K} \quad \rho := 7.9\ \frac{gm}{cm^3} \quad m := \rho \cdot V$

$rate := \frac{P}{c_p \cdot m}$

$time := \frac{T_{steel} - 300\ K}{rate} \quad time = 247193\ s \quad time = 4120\ min \quad time = 69\ hr$

$T_{plastic} := 422\ K \quad c_p := 1.3\ \frac{J}{gm \cdot K} \quad \rho := 0.95\ \frac{gm}{cm^3} \quad m := \rho \cdot V$

$rate := \frac{P}{c_p \cdot m}$

$time := \frac{T_{plastic} - 300\ K}{rate} \quad time = 23667\ s \quad time = 394\ min \quad time = 7\ hr$

This conceptual scheme will accommodate both the believers and the skeptics. It consists of all-reflective fore optics, pointer optics, and appropriate **optics for the radiometer**. There is a flip mirror, as shown in Fig. 3.1.1, that acts just like a single-lens reflex camera. The mirror is in the position shown for **pointing** and moves out of the way for a **measurement**. Is this a radiometer reflex?

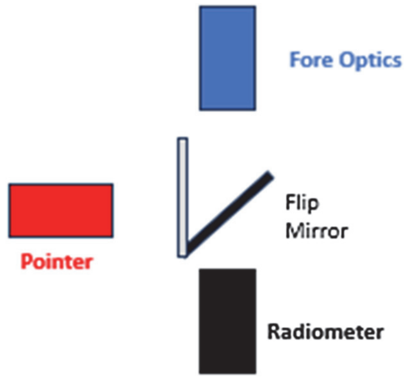


Figure 3.1.1 Pointer concept.

The flip mirror may be replaced by a fixed silicon beam splitter. Silicon is reflective and absorptive in the visible, so it reflects the pointer radiation, and it is almost transparent but partially reflective in the infrared. The transmission loss due to reflection can be approximated with the expression for multiple reflections and the normal reflectivity of silicon based in its refractive index:

$$R_{\text{multiple}} = \frac{1-r}{1+r} = \frac{1-\dfrac{(n-1)^2}{(n+1)^2}}{1+\dfrac{(n-1)^2}{(n+1)^2}} = \frac{2n}{n^2-1}. \tag{3.1.3}$$

The refractive index of silicon is approximately 3.4.[1] The reflection is about 64%. A more accurate treatment is to use the Fresnel equations and 45 deg, 0.79 rad. This treatment gives the same result for an index of 3.4 and changes only 0.5% with refractive index values ranging from 3.4255 to 3.4176.

Silicon *Transmission*

$$n := 3.4255 \quad r := \frac{2 \cdot n}{n^2 - 1} \quad t := 1 - r = 0.362$$

$$n := 3.4176 \quad r := \frac{2 \cdot n}{n^2 - 1} \quad t := 1 - r = 0.36$$

3.2 Reflectivity Measurements

There are several types of reflectivity based on the geometry of the incident and reflected light: specular, hemispheric, and goniometric. Reflectivity measurements are made spectrally as a function of wavelength or in a spectral band.

[1]H. Icenogle, B. Platt, and W. Wolfe, "Refractive indexes and temperature coefficients of germanium and silicon," *Appl. Opt*. **15**, 2348–2351 (1976).

Specular reflectivity, wherein the light comes from a specific direction and is reflected at an angle equal to the incident angle, is measured several ways. The simplest is the substitution method illustrated below. First the test beam is shone directly on the receiver. Then the mirror is interposed and reflects the light on the receiver. In both configurations, the mirror reflects light from the source and from the background. The measurement equation is

$$\frac{V_s}{V_b} = \frac{\mathcal{R}\rho(P_S + P_b)}{\mathcal{R}(P_S + P_b)}, \tag{3.2.1}$$

where the $\mathcal{R}$'s represent the measuring instrument responsivity and the P's represent the power on the instrument from the source and the background. The voltage ratio is a correct measure of the reflectivity only if the background power on the measuring instrument is the same in both configurations. One way to do this is to arrange the field stop of the instrument so that only the source can be seen. The source area is determined by the entrance window, which is the image of the field stop. This is shown in Fig. 3.2.1 along with stray radiation, which can be prevented or controlled with an enclosure. Figure 3.2.2 shows the arrangement with the test mirror in place.

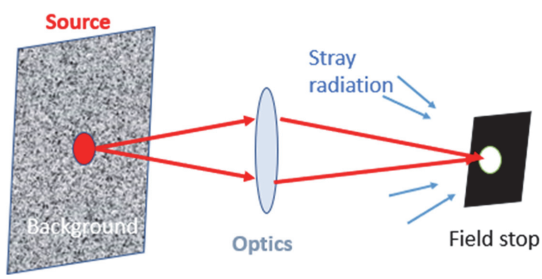


Figure 3.2.1 Substitution measurement before measurement.

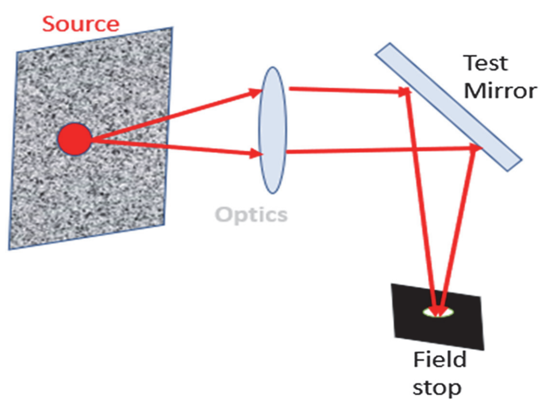


Figure 3.2.2 Substitution measurement with test mirror.

Chapter 4
Components

This chapter contains descriptions of radiometric instrument components: detectors, sources, and optics. There are far too many of these to be included in detail. Descriptions of specific commercial items can be found on the Internet.

4.1 Detectors

The performance of detectors is described by two main properties, their responsivity and their sensitivity. The responsivity is usually designated by a script $\mathcal{R}$. It is the ratio of the output to the input. The input is usually power. The output is volts or amps or number of electrons. We will use power and volts. The sensitivity is some form of output related to the noise. For a specific case, it can be the signal-to-noise ratio per input power, that is, the responsivity divided by the noise, $\mathcal{R}/N$. The reciprocal of $\mathcal{R}/N$ is the noise equivalent power (NEP), or the power that results in a signal equal to the noise. The sensitivity S of detectors depends on both their area and their electrical bandwidth: the square roots of both. Accordingly, Bob Jones defined **detectivity** as the signal-to-noise ratio divided by the input power (the inverse of *NEP*), and he normalized with the square roots of the detector area A and the electrical bandwidth B and called it **specific detectivity**.[1] Thus, detectivity D is

$$D = \frac{S}{NP} = \frac{\mathcal{R}}{N} = \frac{SNR}{P}. \tag{4.1.1}$$

Specific detectivity D^* is the normalized version of detectivity:

$$D^* = D\sqrt{AB} = \frac{\sqrt{AB}}{NEP} = \frac{SNR\sqrt{AB}}{P}. \tag{4.1.2}$$

[1]R. C. Jones, "Quantum efficiency of photoconductors," *Proc. IRIS* **2**, 9 (1957); R. C. Jones, "Proposal of the detectivity D^{**} for detectors limited by radiation noise," *J. Opt. Soc. Am.* **50**, 1058 (1960).

Chapter 5
Radiometers

This chapter describes radiometers and ways to calibrate them.

5.1 Radiometers

Radiometers are instruments that measure the amount of radiation on their apertures. Some operate in the X-ray spectrum, some in the microwave region, and the ones of interest here operate in the optical region, which we define to be from about 0.4 μm to about 20 μm. They vary in method of measurement, size, popularity, and accuracy.

The Crookes radiometer, shown above, is probably the best known and least accurate.[1] It is sold in just about every planetarium. It consists of a

[1]W. Crookes, "On attraction and repulsion resulting from radiation," *Philosophical Transactions of the Royal Society of London* **164**, 501–527 (1874).

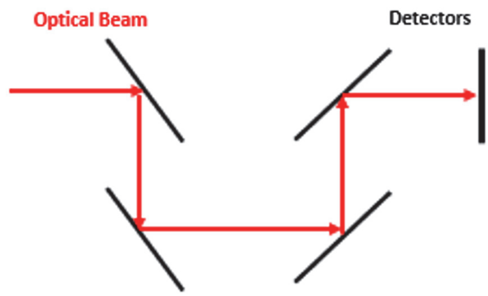


Figure 5.1.3 Coudé detector arrangement.

As far as we have been able to determine, no such instruments are commercially available. They should not be difficult to make, but we leave the details to the practitioner.

5.2 Calibration

All radiometers except ESRs need to be calibrated. Their methods of calibration vary widely and are based largely on the intended application. They are often calibrated using a blackbody source, but sometimes a laser or even a forehead is used.

The mantra for making radiometric measurements in Section 5.1 was

Think of Everything.

The mantra for calibration is

Calibrate like You Measure.

This means that the calibration source should have the same spectral radiance as the unknown, and the backgrounds should be the same. The ideal calibration would be to simply substitute the unknown for the known *in situ*, but that is almost never possible. Do the best you can.

Calibration techniques are logically divided into two categories: those that overfill the detector and those that underfill it. The point source technique is one of the latter. It employs a small source at the focus of a collimator to illuminate the radiometer. The radiometer focuses that source on the detector or field stop. This arrangement is shown in Fig. 5.2.1. The radiometer senses all of the radiation from the source (except for any transmission losses of the collimator, which we assume have been accounted for).

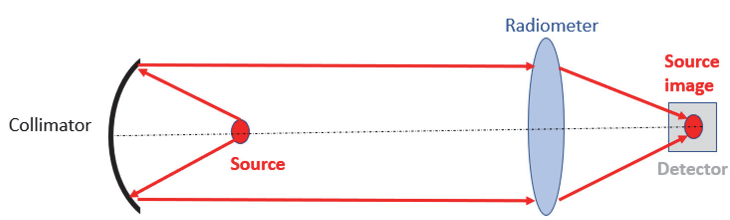


Figure 5.2.1 Point source calibration.

If the source overfills the detector, the ratio of the image of the source area to that of the detector must be included. This requires knowledge of the image area that may not be uniformly irradiated. The extended source arrangement is shown in Fig. 5.2.2. Use it only if you must.

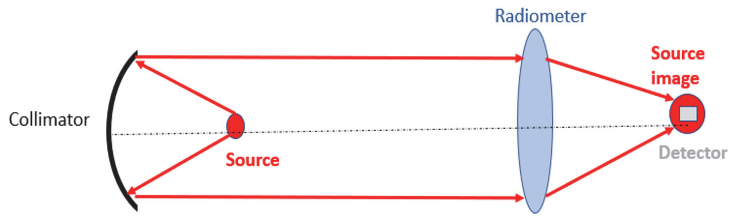


Figure 5.2.2 Extended source calibration.

One can also use a source with no optics, as shown in Fig. 5.2.3. The source must be Lambertian, emitting the same radiance in all directions. That radiance is M/π, where M is the source radiant exitance. Then the irradiance on the radiometer is the radiance times the solid angle it subtends at the source, $M\Omega/\pi = MA_r/\pi R^2$, where A_r is the radiometer aperture area. The irradiance responsivity is the output electrical signal divided by this expression. If the radiance is not isotropic, then its distribution must be calibrated.

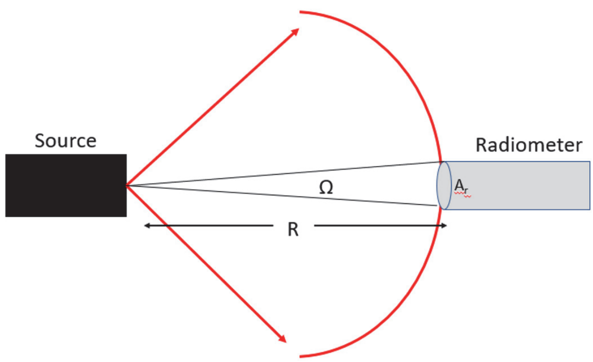
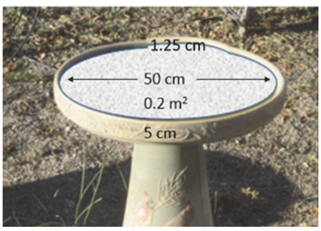


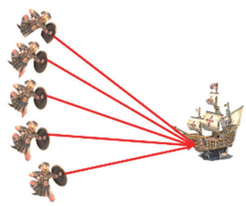
Figure 5.2.3 Calibration without optics.

Here are some examples that may help clarify the concept of *calibrate like you measure*.

Calibrate a remote thermometer by using real foreheads (Fig. 5.2.4), preferably with people who have both normal and elevated temperatures. Then transfer that calibration to a set of secondary sources (Fig. 5.2.5).

Chapter 6
Applications






This chapter includes a variety of applications that require radiometric calculations to establish their viability. Each example is described and then calculated in either Excel or Mathcad or both. In some cases, commercial products are mentioned, but only as examples; no endorsement or "*undorsement*" is intended.

We hope that the applications are sufficiently inclusive. We have personal experience with many of them and consider some fascinating, but they are all meant to illustrate radiometric calculations and are examples to help solidify the previous explanations. The assumed values of some of the quantities are as realistic as possible, but the purpose is not to obtain a realistic solution. It is to illustrate a process.

The design process for most of these radiometric instruments is the same. And it is the same as the analysis process: calculate the geometry, then the dynamics, then the sensitivity. The reasoning behind this is that the geometry will determine the approximate requirements of the optical system. The dynamics will determine the required noise bandwidth, which is a component of the radiometric sensitivity calculations.

The procedure is to first calculate the area to be covered, the resolution needed, and the object distance. From these, calculate the angular resolution and the field of view. Then calculate the timeline to get the required bandwidth. Use the approximations of optical performance if necessary and change any of the above values if necessary to reach a practical configuration. Then calculate the radiometric performance using the appropriate sensitivity

6.1 The Sun is My Doing[2]

The sun is the source of all our energy, our being and our doing. The *solar constant* is the radiant flux density on the Earth from the sun. It has been measured, and it can be calculated. It is a good example of radiometric calculations. The term solar constant is used for the total irradiance from the sun at the Earth above its atmosphere. It is not a constant; it varies with time and place. A more precise specification is total solar irradiance, but *solar constant* has come to be the accepted term. The current average exo-atmospheric value is now 1360.8 Wm^{-2} according to satellite measurements made by NOAA's *SORCE* project.[3] The sun can be approximated as a 5800 K blackbody with an average temperature of 5772 K, and ranging from 4400 K to 6000 K.[4] It has a radius of 696,330 km. The Earth is 149,600,000 km from the sun. We can assume that the sun is a point source at its center. Then by the inverse square law, the flux density on the Earth is the solar irradiance on the sun surface times the square of the ratio of the solar radius to the distance from the center of the sun to the Earth. The geometry is shown in Fig. 6.1.1.

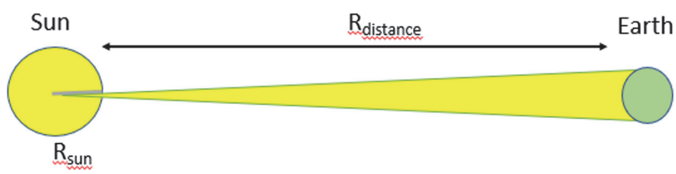


Figure 6.1.1 Sun–Earth geometry.

The calculation is shown in Excel. We adjusted the temperature until it agreed with the reported value since the temperature of the photosphere is not known exactly.

The first row is the title. The next row is the equation: the geometric factor squared times the Stefan–Boltzmann expression. The next row is the expression for the factor. Then, the titles. The first operative row (row 5) is the solar radius. The next is the orbital distance. Then the Stefan–Boltzmann constant and the adjusted temperature. The next three lines make the calculation: the solar blackbody exitance, the geometric attenuation factor, and the resultant exo-atmospheric solar constant. The result agrees with the reported value because we adjusted the temperature.

[2]With apologies to Marguerite Steen, *The Sun is My Undoing*, Viking Press (1941).
[3]Total irradiance data, noaa.gov.
[4]Photosphere, wikipedia.org.

	A	B	C	D	E	F
1			**E6.1.1. Solar Constant**			
2			**$E=f^2\sigma T^4$**			
3			**$f=R_{sun}/(R_{sun}+R_{distance})$**			
4	**Item**	**Symbol**	**Equation**	**Equation**	**Value**	**Units**
5	Solar Radius	Rsun	known		696330	km
6	Orbital distance	Rdistance	known		149600000	km
7	Stefan Botzmann constant	σ	known		5.670E-08	Wm-2K-1
8	Solar Temperature	T	assumed		5782.5	K
9	Solar Exitance	M	σT^4	E7*E8^4	6.34E+07	Wm-2
10	Factor	f	$R_{sun}/(R_{sun}+R_{distance})$	E5/(E5+E6)	4.63E-03	#
11	Solar constant	E	Mf^2	E9*E10^2	1360.8	Wm-2

That was the calculation in Excel. The Mathcad calculation using the same temperature and geometry is below. The result is the same, as it must be. The Mathcad calculation is simpler and shows that the constants in Excel have proper units.

M6.1.1 Exo–Atmospheric Solar Constant

$$T := 5782.5\ K \qquad M := \sigma \cdot T^4$$

$$R_{sun} := 696330\ km \qquad R_{distance} := 149600000\ km$$

$$f := \frac{R_{sun}}{R_{sun} + R_{distance}}$$

$$E := M \cdot f^2 \qquad E = 1360.8\ \frac{W}{m^2}$$

A better measure for almost all radiometric calculations is the **solar spectral irradiance**. Figure 6.1.2 shows the results of Excel calculations compared to the data reported by Thekaekara that is quoted in *The Infrared Handbook.*[5] The data points around 1 μm are questionable, in part because the curve is not smooth. The sun is essentially a blackbody, and its irradiance above the atmosphere should follow a smooth curve. The data at longer wavelengths follow a smooth curve that agrees quite well with a blackbody curve of 5750 K found by trial and error. It was noted above that an exact agreement could be made with a blackbody temperature of 5782.5 K, which is reasonably close to 5750 K (half a percent different). Another such fit yielded a geometric attenuation factor of 2.17×10^{-5} compared to 2.15×10^{-5} based on the geometry reported above.

[5] A. LaRocca, "Natural Sources," in W. Wolfe and G. Zissis, Eds., *The Infrared Handbook*, U.S. Government Printing Office (1978).

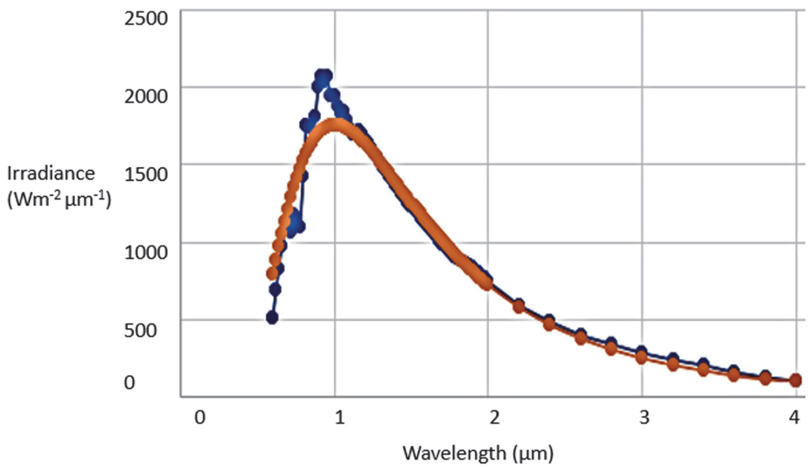


Figure 6.1.2 Solar spectral irradiance comparison.

As the saying goes, "You pay your money, and you take your choice," but for most calculational efforts involving the solar irradiance, we would use a 5780 K blackbody (or maybe 5800 K) and an attenuation of 2.15×10^{-5}. As you make these calculations involving the solar spectral irradiance, remember that its value varies with the solar cycle, the seasons, the time of day, the location, and weather conditions.

The solar spectral irradiance at the surface of the Earth is less than 1360.8 Wm^{-2} due to atmospheric transmission losses. It is approximately[6] 1000 Wm^{-2}. A representative vertical atmospheric transmission curve is shown in Fig. 6.1.3. The curve was created by MODTRAN® for four very different moisture conditions as indicated. The only slight difference among these curves is in the peaks for high humidity. There is only about a 1% difference between the transmisssions for 10% humidity as in Tucson, Arizona or 100%, as in Memphis, Tennessee. The solar spectral irradiance can be modeled quite well without the use of MODTRAN, as shown in Fig. 6.1.4 in blue and in Mathcad M6.1.2 and the description that follows it.

[6]Solar constant, wikipedia.org.

6.5 One Lonely Photon

The human eye is an amazing optical instrument. It can guide us during the brightness of noon in the desert and on dark nights when there is only starlight. It can even detect a single photon. The calculation of this fact is an interesting exercise in radiometry and photometry—even "*photonometry*."

There have been several investigations of how many photons we need for detection. The most recent and probably most definitive concluded that there was a significant probability that we can detect a single photon.[17] But even that exhaustive treatment has come into question.[18]

W. T. Walsh in his classic book on photometry reports that the eye can detect as little as 10^{-4} trolands.[19] He further explains that a troland is *the retinal illumination produced by a surface having a luminance of one candela per square meter when the pupil area is one square millimeter.*[20] A candela is a unit of luminous intensity with units of lumen per steradian. The eye has a focal length of 24 mm, an aperture of 1 mm^2, and a retinal ending diameter of 2 μm. Mathcad does not recognize trolands, so we need to enter the calculation in terms of lumens, steradians, and meters. We use his luminance sensitivity L_{sens}, the luminous efficacy η of the longest wavelength in our tables, and the values provided above for the eye. The result is essentially one photon for an integration time of 0.1 s.

M6.5.1 Walsh Photon Calculation

$$L_{sens} := 10^{-4}\ \frac{cd}{m^2} \qquad \eta := 2.55\ \frac{lm}{W} \qquad L := \frac{L_{sens}}{\eta}$$

$$\lambda := 8\ \mu m \qquad D_r := 2\ \mu m \qquad f := 24\ mm$$

$$A_o := 1\ mm^2 \qquad A_r := \frac{\pi \cdot D_r^2}{4} \qquad G := \frac{A_o \cdot A_r}{f^2}$$

$$P := L \cdot G \qquad P = \left(2.139 \cdot 10^{-19}\right)\ W$$

$$U := \frac{h \cdot c}{\lambda} \qquad N := \frac{P}{U} = 8.614\ \frac{1}{s}$$

$$t := 0.1\ s \qquad n := N \cdot t = 0.861$$

[17]J. N. Tinsley, M. I. Mlodtsov, R. Prevedel, et al, "Direct detection of a single photon by humans," *Nature Communications* **7**, 12172 (2016).

[18]MIT Technology Review, The Thorny Question as to Whether Humans Can Detect a Single Photon, technologyreview.com.

[19]J. W. T. Walsh, *Photometry*, Dover, p. 75 (1958).

[20]ibid., p. 59.

6.13 Over There, Over There[31]

We are all familiar with laser pointers. I remember paying $200 for one about 30 years ago. But now they are only a few dollars, and just about every speaker uses one. The typical use of a pointer is by a lecturer in a classroom or lecture hall. The lecturer stands 5 to 10 ft from the screen; the audience is 10 to 100 ft away. The screen is highly reflective and highly diffuse (or it would not be very good). We present the Mathcad results below in watts, lumens, and photons for the front and back of the room.

The first operative line initiates the range of ranges R, the distances the viewers are from the screen, the laser power P, the spot diameter D_s, the wavelength λ, and the screen reflectivity ρ. The next line calculates the beam spot area A_s, the irradiance E on the screen, the reflected radiance L, the solid angle of the spot as seen at the eye, and the irradiance at the eye. Then the irradiances at the front and back of the room (1 and 10 m) are calculated. We know from experience in the lecture hall that such a spot can be seen, but it is not clear that a fraction of a microwatt is detectible with the naked eye. That is why we also make the calculation photometrically in lumens. The irradiance is converted to illuminance by multiplying by the luminous efficacy η at 0.77 μm, 0.02 lmW^{-1}. The two illuminances are 30 and 0.3 μlux. The irradiance is converted to photon irradiance by dividing it by the energy of a photon, symbolized by u in Mathcad. The two photon flux densities are 6×10^{15} and 6×10^{13} photons per second per square meter.

[31] Thank you, George M. Cohan, for the great patriotic song of two world wars, "The Yanks are Coming. . . til its over over there."

M6.13.1 Laser Pointer

$$R := 1\,m, 2\,m..100\,m \qquad P := 5\,mW \qquad D_s := 1\,cm \qquad \lambda := 0.77\,\mu m \qquad \rho := 0.95$$

$$A_s := \frac{\pi \cdot D_s^2}{4} \qquad E := \frac{P}{A_s} \qquad L := \frac{\rho \cdot E}{\pi} \qquad \Omega(R) := \frac{A_s}{R^2} \qquad E_{eye}(R) := L \cdot \Omega(R)$$

$$E_{eye}(1\,m) = 1512\,\frac{\mu W}{m^2} \qquad E_{eye}(10\,m) = 15.12\,\frac{\mu W}{m^2}$$

$$\eta := 0.02\,\frac{lm}{W} \qquad E_v(R) := \eta \cdot E_{eye}(R)$$

$$E_v(1\,m) = (3.024 \cdot 10^{-5})\,lx \qquad E_v(10\,m) = (3.024 \cdot 10^{-7})\,lx$$

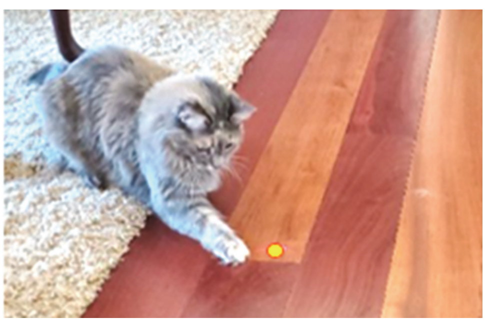
$$u := \frac{h \cdot c}{\lambda} \qquad E_q(R) := \frac{E_{eye}(R)}{u}$$

$$E_q(1\,m) = (5.861 \cdot 10^{15})\,\frac{1}{m^2 \cdot s} \qquad E_q(10\,m) = (5.861 \cdot 10^{13})\,\frac{1}{m^2 \cdot s}$$

The human eye has an aperture diameter of 2 to 4 mm, depending on the light level. For these assumptions, that is about 10^{-12} lumens, which we know is enough.

The problem—the danger—with laser pointers is when they are aimed directly at people. A 5 mW, collimated laser will put 5 mW on one's eye. That is the threshold of damage,[32] but fortunately that threshold is not often reached. One reference reports an average of 20 to 30 incidents per year in all of Canada.[33] But keep your head down and your eye on the prize! If you are the lecturer, keep the laser pointed at the screen.

Be careful with your cat. The solid angle of a cat's eye near its "prey" is about a million times larger. Then the eye irradiance is about 0.02 Wm^{-2} and close to the danger zone for humans. It could be less for cats. Reflect the laser beam off an object that is darker than a white screen and keep it moving. That's the fun, anyway.



[32]Are Lasers Pointers Strong enough to Cause Eye Damage? nvisioncenters.com.

[33]S. S. Qutob, K. P. Feder, M. O'Brien, et al., "Survey of reported eye injuries from handheld laser devices in Canada," *Canadian Journal of Ophthalmology* **54**, 548–555 (2019).