LINEWIDTH MEASUREMENT OF DIODE LASERS

Dr. Steffen Schmidt-Eberle, Application Scientist Quantum Technologies, TOPTICA Photonics AG

The spectral purity of a laser is one of its core features. It is however non-trivial to find quantities which fully characterize this spectral purity. In this paper we discuss two linewidth definitions which TOPTICA uses to characterize the frequency stability of its diode lasers. These quantities reflect the two conceptual sources of phase or frequency noise in diode lasers.

SOURCES OF PHASE AND FREQUENCY NOISE

The fundamental limit for the linewidth of any laser is given by the Shawlow-Townes [1] expression which describes the impact of phase changes due to spontaneous emission on the laser linewidth. For diode lasers this limit depends e.g. on the laser´s resonator geometry, properties of the semiconductor and laser power. This limit manifests itself as a white frequency noise plateau and will therefore be called white noise limit in the following.[1] Because this noise depends on fundamental properties of the laser it is referred to as intrinsic noise.

Any frequency noise which adds to the intrinsic noise is summarized as technical noise. Most generally, these are contributions introduced from the environment like vibrations or acoustic noise and on longer time scales temperature and air pressure drifts. For this type of noise, the technical realization concerning mechanical stability and isolation from the environment is key to low noise.

TWO DEFINITIONS OF LASER LINEWIDTH

The difference between intrinsic and technical noise becomes more apparent when considering the power spectral density (PSD) of the laser´s frequency noise [2]. The PSD of the frequency noise is the absolute square of the Fourier transform of the time-dependent frequency deviation $\tilde{\nu}(t)$ of the laser from its central frequency ν_0. It contains all typically relevant spectral properties of the laser[2]. Figure 1 shows an example of the frequency PSD of a DL pro with narrow linewidth option at 725 nm. The frequency PSD is largest for small Fourier frequencies and decreases until it converges into a white noise plateau at higher frequencies. This plateau corresponds to the white noise limit caused by intrinsic noise, while the noise at lower frequencies which exceeds this plateau corresponds to technical noise. In the following we will present two linewidth definitions which represent the two different types of noise.

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[1] For semiconductor lasers this limit is further increased by a coupling between intensity and phase noise, caused by a dependence of the refractive index on the carrier density in the semiconductor [2], first explained by Charles H. Henry [3]. The strength of this coupling is described by the linewidth enhancement factor α [4] and the white noise limit is increased by a factor of $(1+\alpha^2)$.

[2] Of course, the absolute square of the Fourier transform neglects the phase spectrum of the signal. For most applications however, this phase spectrum is not of importance.

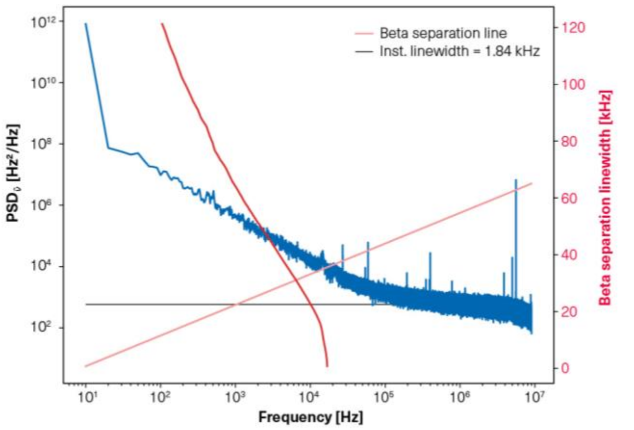

Fig. 1: Power spectral density (PSD) of the laser´s frequency noise (blue curve). The black, horizontal line indicates the white noise level caused by intrinsic noise. The orange line is the beta separation line. Only components of the frequency PSD which lie above this line contribute to the beta separation linewidth. The red line is the beta separation linewidth as a function of the lower integration limit.

First, we define the *instantaneous linewidth*, which mainly describes the intrinsic noise evident at high Fourier frequencies

$$\Delta\nu_{inst} = \pi \cdot PSD_{white}, \tag{1}$$

where PSD_{white} is the white noise level of the PSD in Hz²/Hz. PSD_{white} is obtained by fitting to the data a power law function of the form $y(f) = af^{b} + c$, where the fitting constant c is interpreted as PSD_{white}[3]. Empirically we see that most spectra are approximated well by this type of fitting function. $\Delta\nu_{inst}$ is the full width at half maximum (FWHM) linewidth of a hypothetical laser with a frequency PSD given by $PSD_{\tilde{\nu}}(f) = PSD_{white}$ for all f. Such a laser yields a Lorentzian line shape. This definition is TOPTICA´s core definition of the laser linewidth and is referred to in laser and diode specifications which can be found on TOPTICA´s website and the laser diode stock lists.[4]

Second, we define the *beta separation linewidth*, which is based on the beta separation line introduced by Di Domenico et al. [5]

$$\Delta\nu_{\beta} = \sqrt{8\ln(2)A}, \tag{2}$$

where A is the surface under the portions of $PSD_{\tilde{\nu}}(f)$ that exceed the beta separation line [5]

$$A = \int_{f_0}^{\infty} H\left(PSD_{\tilde{\nu}}(f) - \frac{8\ln(2)\,f}{\pi^2}\right) PSD_{\tilde{\nu}}(f)df, \tag{3}$$

with $H(x)$ being the Heaviside unit step function ($H(x) = 1$ if $x \geq 0$ and $H(x) = 0$ if $x < 0$) and f_0 being a lower integration limit. This quantity is a FWHM linewidth which approximates the linewidth of the actual power spectrum of the laser´s electric field with an integration time of $T_0 = 1/f_0$. For many diodes TOPTICA can provide the beta separation linewidth upon request for various lower integration limits. For the example in *Fig. 1* only noise below approximately 20 kHz contributes to this FWHM linewidth, while noise above this threshold contributes to the pedestal of the line profile only.

Thus, while the instantaneous linewidth is a measure for frequency stability at high Fourier frequencies, the beta separation linewidth describes the frequency stability at low Fourier frequencies. Therefore, in the context of laser locking, the instantaneous linewidth is connected to the residual noise after locking because locking bandwidths are typically limited to several MHz. The beta separation linewidth on the other hand indicates how well the laser can be stabilized within the locking bandwidth. It is generally true that the quality of the locked laser depends on the level of instability of the unlocked laser.[5] Also if the beta separation linewidth is on the order of several MHz, an actuator capable of changing the laser frequency by several MHz is required for frequency stabilization.

[3] The value of $\Delta\nu_{inst}$ that we obtain in this way has to be considered as a typical value and not as a specification, as the exact value depends on many parameters like e.g. diode current, temperature, wavelength, etc.

[4] For some diodes and lasers yet another linewidth definition is used which will be discussed in the section "Comparison to other linewidth definitions". This definition is still used for diodes for which no measurement of the frequency PSD has been performed yet.

[5] Of course, the linewidth of the stabilized laser depends on many other properties of the lock setup, like the reference and locking scheme, the bandwidth of the PID controller, etc. A detailed introduction to laser locking can be found at https://www.toptica.com/application-notes/phase-and-frequency-locking-of-diode-lasers/.

Another interesting quantity is the frequency at which the frequency PSD crosses the beta separation line. In order to achieve significant linewidth reduction a feedback loop with a bandwidth larger than this crossing frequency is required. TOPTICA can also provide this number upon request.

MEASUREMENT OF THE POWER SPECTRAL DENSITY

The most straightforward way to obtain the PSD of the frequency noise is to overlap two lasers with identical noise properties at slightly different central frequencies and record the beat signal as a function of time using a photodiode.[6] From this signal the time-dependent relative frequency fluctuations between the two lasers can be obtained. The absolute square of the Fourier transform of the time-dependent frequency fluctuations is the PSD of the relative frequency noise between the two lasers.[7] However, in most cases only one laser is available. Therefore, TOPTICA measures the PSD of the frequency noise using a delayed self-heterodyne detection setup which is sketched in *Fig. 2*. This method was used to obtain the blue curve in *Fig. 1*. Laser light is separated into two parts of similar power using a non-polarizing beam splitter (NPBS). One part is delayed using a 20 m long fiber, which corresponds to a delay of 100 ns. The other part is frequency shifted by 40 MHz using an acousto-optic modulator (AOM). Both beams are recombined using a second NPBS. The beat signal at 40 MHz is recorded using a photodiode (PD) driven by a low noise power supply.

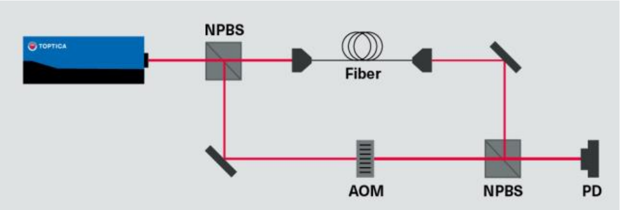

Fig. 2: Optical setup for delayed self-heterodyne detection. This setup is used to measure the frequency PSD of diode lasers. It is based on an asymmetric interferometer. The optical path length difference between the arms is 20 m. This asymmetry is used to transform frequency fluctuations of the laser into phase fluctuations.

Instead of evaluating the signal in the frequency domain using a spectrum analyzer, the time-dependent beat signal is recorded using an oscilloscope. From this signal the time-dependent interferometric phase noise $\varphi(t)$ can be calculated using the Hilbert transformation [6]. The absolute square of the Fourier transform of $\varphi(t)$ yields the power spectral density (PSD) of the interferometric phase noise. The relation between this quantity and the PSD of the laser frequency noise is given by [7]

$$PSD_{\varphi}(f) = \frac{4}{f^2} \sin^2(\pi f \tau_0)\, PSD_{\tilde{\nu}}(f), \qquad (4)$$

where τ_0 is the relative delay between both interferometer arms. Thus the interferometer can be used as a laser frequency detector whose sensitivity is largest for frequencies $f \to 0$ and which has zero sensitivity for frequencies fulfilling $f = \frac{N}{\tau_0}$, where N is an integer number. For a fiber length of 20 m the first frequency of zero sensitivity is at $f = 10$ MHz. Therefore, the interferometric phase signal allows to extract the full PSD of laser frequency noise down to frequencies limited by the overall integration time of the interferometric phase signal up to the first frequency of zero sensitivity at 10 MHz. This can be intuitively understood by considering the fact that even slow laser frequency drifts will cause interferometric phase drifts if both interferometer arms have different length. This sensitivity of the phase to laser frequency drifts is more pronounced the larger the path length difference between the arms is. This intuition coincides with the sensitivity expression extracted from eq. (4) for frequencies $f \to 0$ which is given by $(2\pi\tau)^2$, which shows that the sensitivity is larger the longer the relative delay between both arms is.

[6] A spectrum analyzer can be used to obtain the power spectrum of the laser field from this signal. The FWHM linewidth obtained from the spectrum analyzer trace corresponds to twice or $\sqrt{2}$ times the linewidth of a single laser if the spectrum is Lorentzian or Gaussian, respectively.

[7] The PSD of the frequency noise of a single laser is half the PSD of the relative frequency noise.

COMPARISON TO OTHER LINEWIDTH DEFINITIONS

Like described in the section before, the most straightforward way to obtain the laser linewidth is to overlap two lasers with identical noise properties and record the beat signal using a spectrum analyzer. The FWHM linewidth obtained from the spectrum analyzer trace corresponds to twice or $\sqrt{2}$ times the linewidth of a single laser if the spectrum is Lorentzian or Gaussian, respectively. The linewidth obtained using this method corresponds to the beta separation linewidth where the lower integration limit corresponds to the integration time of the spectrum analyzer.

However, as in many cases there is no second identical laser available, a large number of methods have been developed to obtain the linewidth without the use of a second laser. We will discuss one other very popular way to determine the laser linewidth which is also based on delayed self-heterodyne detection like described above but with a much longer fiber. The original idea was to use a fiber so long that the corresponding relative delay exceeds the coherence time of the laser by far [8]. Thus, the two fields superimposed on the second NPBS are assumed to be uncorrelated. Therefore, the interferometric beat corresponds to a beat between two identical but uncorrelated lasers. This method is not applicable for most of TOPTICA´s diode lasers because their coherence time is so large that many kilometers of fiber would be necessary. But very long fibers have the problem that they easily collect environmental disturbances like vibrations and acoustics.

A more advanced method is based on fibers of moderate length, typically on the order of 1 km. The method takes into account partial coherence of the two paths [9]. Knowing the form of the PSD of the laser frequency noise, it is possible to directly extract the laser linewidth by fitting to the laser power spectrum a function which is derived from the form of the laser frequency PSD[8]. The drawback of this method is that the fit is typically not sensitive to noise components below Fourier frequencies of $f_0 = 1/\tau_0$ [10]. For a fiber length of 1 km this corresponds to $f_0 = 200\ \mathrm{kHz}$. It is important to note that this limit cannot be pushed to lower frequencies by increasing the measurement time. In this sense the linewidth obtained from this method rather corresponds to an instantaneous linewidth as it only takes into account high-frequency components. However, in most cases it does not reproduce the instantaneous linewidth like defined in this paper because the frequency $f_0 = 200\ \mathrm{kHz}$ cannot be understood as a sharp cut-off. Noise at frequencies below but close to f_0 are suppressed but still have a significant contribution. Therefore, the linewidth estimation obtained by fitting a spectrum analyzer trace can be considered an upper bound of the instantaneous linewidth described above. For some diodes TOPTICA specifies this upper bound as a linewidth estimation instead of the instantaneous linewidth. This used to be TOPTICA´s primary definition for short term linewidths. For reasons of consistency, therefore, in all relevant documents, this value is denoted as “typical short term linewidth [5µs]” [9].

CONCLUSION

The instantaneous and beta separation linewidths presented in this paper therefore are preferable for several reasons. Most importantly, they are independent of the details of the measurement setup, e.g. the length of the fiber. Beyond that, for the method presented in this paper, a fiber length of 20 m is sufficient, which is less sensitive to environmental disturbances. And last, no assumptions about the form of the laser frequency PSD are required in the first place.

[8] The power spectrum of two partially correlated beams typically yields a delta peak at the AOM frequency and a sinusoidal envelop with a periodicity given by the delay between the two interferometer arms. The modulation depth of the sinusoidal envelop is a measure for the coherence and thus for the linewidth of the laser. It is this modulation depth which is typically being fitted and which gives rise to the laser linewidth.

[9] In all relevant documents, the * behind the linewidth value denotes that this value is used instead of the instantaneous linewidth.

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