

## Q-factor enhancement of MEMS Resonators with Ditetragonal Prism shaped Phononic Crystal (DTP-PnC)

Temesgen Bailie Workie<sup>1†</sup>, Ting Wu<sup>1</sup>, Jing-Fu Bao<sup>1</sup>, Ken-ya Hashimoto<sup>1,2</sup>

<sup>1</sup>School of ESE, Univ. of Elec. Sci. and Tech. of China, Chengdu China; <sup>2</sup>Chiba Univ., Chiba Japan

### 1. Introduction

Controlling and manipulating the propagation of elastic waves through a material has been of great scientific and technological interest for decades [1]. In this regard, study of the ways to be used in trapping elastic waves leaking from the resonating structure of MEMS resonators through the supporting structures has got our attention as the quality factor(Q) could be boosted in doing so. For this cause, taking the property of phononic crystals in providing acoustic bandgaps which can prevent the elastic wave propagation [2][3] of frequencies in the bandgap range as an advantage, designing ultra-wide bandgap PnCs and deploying it in the supporting structures could prevent energy leakage through the supporting structures.

In this study, we propose a novel DTP-PnC structure with ultra-wide bandgap, low filling fraction as well as relatively small in size and deployed array of DTP-PnC in the supporting structure of a MEMS resonator to improve Q-factor and other important parameters like the motional resistance. All FEM simulations in this study are done using COMSOL Multiphysics.

### 2. Phononic crystal design and simulation

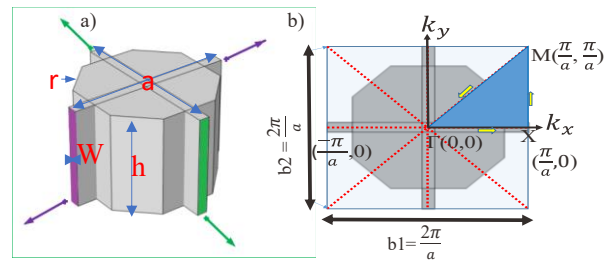
Wave propagation in anisotropic material based 2D PnCs can be analyzed by using the equation of motion of the displacement vector written as [4][5]:

$$\rho(\mathbf{r}) \frac{\partial^2 u_i(\mathbf{r}, t)}{\partial t^2} = T_{ij,j}(\mathbf{r}) \dots \dots \dots (1)$$

$$T_{ij}(\mathbf{r}) = C_{ijkl}(\mathbf{r}) u_{kl}(\mathbf{r}, t) \dots \dots \dots (2)$$

Where  $T_{ij}$  is the stress tensor,  $u_i$  is the displacement components  $u_x$ ,  $u_y$  or  $u_z$ ,  $r$  is the position vector ( $r = x(x, y), z$ ),  $t$  is time,  $\rho$  is position dependent mass density and  $C_{ijnm}$  is elastic stiffness tensor.

Taking material properties and geometric parameters as dominant factors in determining the bandgap of PnCs in to consideration, we have designed a novel PnC structure with dimensions specified in Fig.1a by using silicon (single-crystal anisotropic) material. Analysis is done on the square reciprocal lattice as shown in Fig.1b. Bloch-Floquet periodic boundary conditions along the repetition direction of the PnC structure is set (Fig.1a) such that, violet and green surfaces represents an infinite

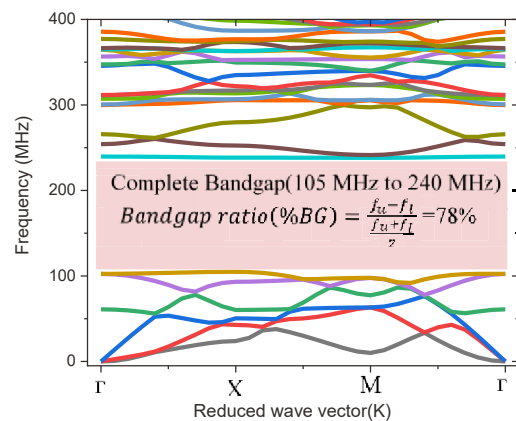


**Fig.1** Illustration of a) Floquet boundary conditions applied to a PnC unit cell with diementions  $a=16\mu\text{m}$ ,  $r=3\mu\text{m}$ ,  $h=10\mu\text{m}$  and  $w=3\mu\text{m}$ , b) Primitive unit cell with its first IBZ

number of repetitions in the x-direction and y-direction respectively. The discrete form of the eigenvalue equations in the unit cell [3] given by Eq. (3) is solved by sweeping wave vectors  $\mathbf{k}(k_x, k_y)$  along the boundary of first irreducible brillouin zone(IBZ) to get the dispersion relation of the PnC.

$$(K - \omega^2 M)u = 0 \dots \dots \dots (3)$$

Where  $K$ ,  $M$ ,  $\omega$  and  $u$  are the global stiffness matrix, global mass matrix, angular frequency, and vector of nodal displacements respectively.



**Fig.2** FEM simulated Band structure of DTP-PnC

### 3. Transmission Characteristics of the PnC

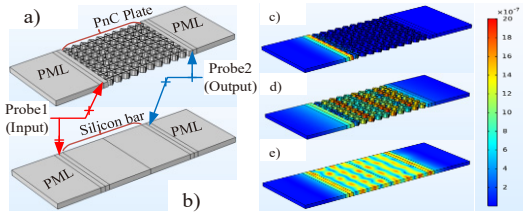
Transmission characteristics of the PnC is simulated and compared with that of a silicon bar used as a transmission medium by using the setup given Fig.3 a and b. Transmission spectrum  $S_{21}$ (dB) is given by [6]:

$$T = S_{21}(\text{dB}) = 10 \log_{10} \left( \frac{P_0}{P_i} \right) \dots \dots \dots (4)$$

Where  $P_0$  is output power and  $P_i$  is input power. Fig.3c reveals inhibition of elastic wave propagation

Corresponding to: baojingfu@uestc.edu.cn  
temesgenbailie@yahoo.com

when the frequency is in the bandgap range of the DTP-PnC.



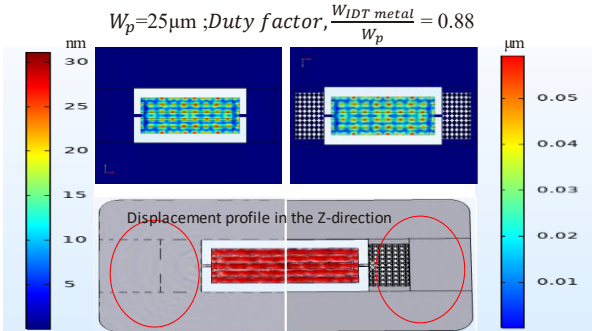
**Fig.3** Transmission model with a) DTP-PnC, b) silicon bar; c) transmission map of DTP-PnC for frequencies in the range of the bandgap, d) transmission map of DTP-PnC for frequencies out of the range of the bandgap, and e) Transmission map of silicon bar as a transmission medium respectively.

**4. Resonator design and analysis**

The resonant frequency of the resonator is given by [5][6]:

$$f_r = \frac{n}{2W_r} \sqrt{\frac{E}{\rho}} = \frac{1}{2W_p} \sqrt{\frac{E}{\rho}} \dots \dots \dots (5)$$

Where  $W_r$  is width of resonant body,  $W_p$  is the Pitch width,  $n$  is harmonic mode of vibration



**Fig.4** Vibration mode shape of 5<sup>th</sup> order MEMS resonator at the resonance frequency ( $f_r$ ) without PnC(left) and with PnC (right).The displacement profile in the Z-direction shows the effect of DTP-PnC in reflecting the elastic waves as it could be noticed from the space marked by red circles.

frequency,  $E$  is the Young’s modulus and  $\rho$  is the density of substrate material used(silicon).The resonant body comprises of 0.5 $\mu$ m thick AlN sandwiched by 1 $\mu$ m thick Al interdigitated (IDT) electrodes and the bottom substrate of 10 $\mu$ m thick silicon(Si).

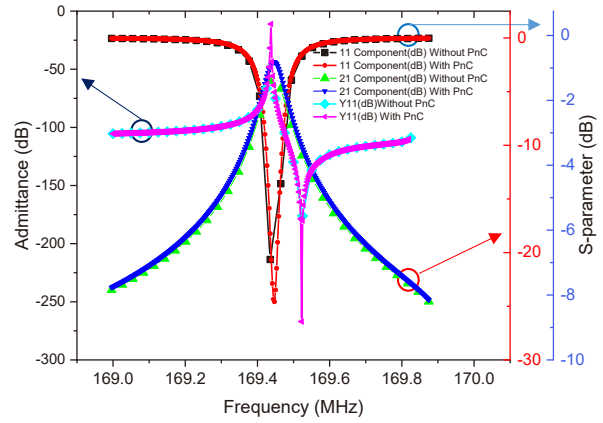
The electromechanical coupling coefficient and Figure of Merit are calculated from the FEM simulation results using the following equations. [5][6]:

$$K_{eff}^2 = \frac{\pi^2}{8} (\frac{f_p^2 - f_s^2}{f_p^2} \dots \dots \dots (6); FoM = K_{eff}^2 * Q_u \dots \dots (7)$$

**5. Results and discussion**

**Fig.5** Illustrates the values obtained from the FEM simulation of the 5<sup>th</sup> order MEMS resonator

with and without DTP-PnC. Compared to the conventional resonator, the results obtained from the resonator with PnC revealed capacity of the DTP-PnC in reflecting acoustic energy as a result of which quality factor (Q) and other performance parameters of the resonator significantly improved.



Without PnC	With PnC
$f_r = 169.45\text{MHz}; R_m = 357\Omega$	$f_r = 169.44\text{MHz}; R_m = 53\Omega$
$K_{eff}^2 = 0.14\%; IL = 1.5\text{dB}$	$K_{eff}^2 = 0.12\%; IL = 0.67\text{dB}$
$Q_{anch} = 85,076$	$Q_{anch} = 3,219,706$
$Q_l = 2,258; Q_u = 14,23$	$Q_l = 3,227; Q_u = 43,469$
$FoM = 1993$	$FoM = 5216$

**Fig.5** Admittance ( $Y_{11}\text{dB}$ ) and S-parameter (dB) FEM simulated results of the MEMS resonator with and without PnCs in the anchoring boundaries.

**6. Conclusion**

This paper demonstrated a novel ultra-wide acoustic bandgap PnC with a BG% of 78% having very high elastic wave isolation capability. With its application in the supporting structures of a MEMS resonator, the results reveal that the DTP-PnC can effectively increase the  $Q_u$  by about 3 folds from 14,237 to 43,469. In line with this,  $R_m$  reduces from 357  $\Omega$  to 53  $\Omega$ .

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