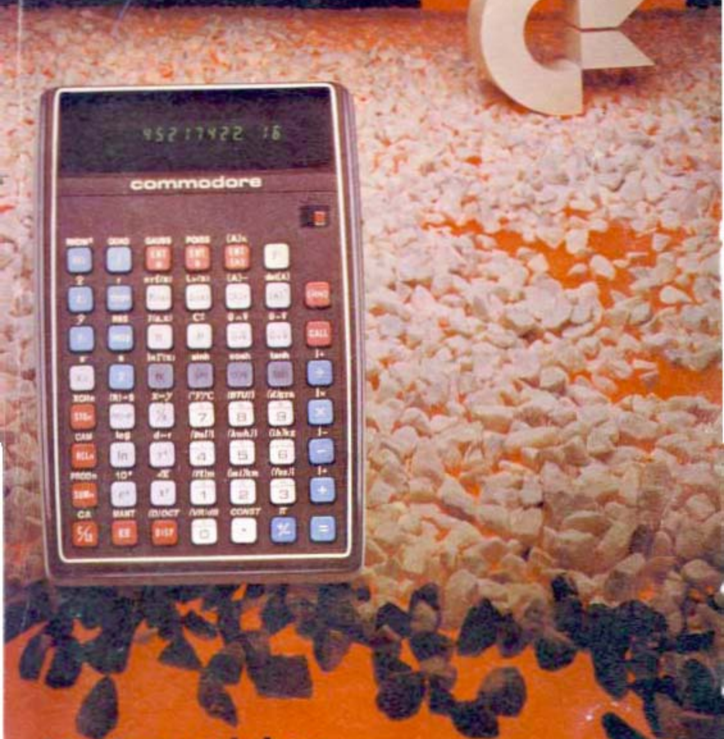
commodore M55



Mathematician Owners Manual

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[.]

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MANT mantissa mode

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(VR)dB	voltage ratio to decibels
(ft)m	feet to meters
(mi)km	miles to kilometers
(foz)1	US fluid oz to liters
(gal)1	US gallons to liters
(kwh)J	kilowatt-hours to joules
(lb)kg	pounds to kilograms
(°F)°C	Fahrenheit to Centigrade
(BTU)J	BTU to joules
(d)gra	degrees to grads
(D)OCT	decimal to octal
d↔r	degrees to radians, changes mode (section 2.13)
(R)→P	rectangular to polar (section 3.4)
(R)→S	rectangular to spherical (section 3.4)

Constants (section 2.12)

h	Planch's constant
k	Boltzman's constant
q	electronic charge
m	electronic rest mass
c	velocity of light
E	permittivity of free space

[μ]	permeability of free space
[G]	Universal Gravitational Constant
[R]	Universal Gas Constant
[N]	Avogadro's number
[π]	Pi

M55 Keyboard Diagram

ON — OFF

RND M ≠	QUAD	GAUSS	POISS	(A) x		
DEL	∫	ENT a	ENT b	ENT (A)	F	
$\hat{x}$	r	erf (x)	L ν (x)	(A) −	det (A)	
x_i	Slope	P ν (x)	J ν (x)	(A) +	$(A)^{-1}$	(inv)
$\hat{y}$	RSS	γ (a,x)	C^n_k	$\bar{u} \wedge \bar{v}$	$\bar{u} - \bar{v}$	
y_i	intcp	((	))	$\bar{u} \cdot \bar{v}$	$\bar{u} + \bar{v}$	CALL
S'	S	lnΓ(x)	sin h	cos h	tan h	j÷
x_n	$\bar{x}$	n!	sin	cos	tan	÷
XCHn	(R) → S	x ←→ y	(°F) °C	(BTU) J	(d) gra	j×
STO n	(R) → P	1/x	G / 7	R / 8	N / 9	×
CAM	log	d ←→ r	(gal) l	(kwh) J	(lb) kg	j−
RCLn	l n	y^x	c / 4	ε / 5	μ / 6	−
$PROD_n$	10^x	$\sqrt{x}$	(ft) m	(mi) km	(foz) l	j+
SUMn	e^x	x^2	k / 1	q / 2	m / 3	+
CA	MANT	(D) OCT	(VR) dB	CONST	π	
C/CE	E E	DISP	h / 0	.	+/−	=

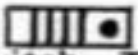
2. Using your MATH 55

- the fundamentals -

2.1 The Display

Power On

The power switch is located in the upper right-hand corner of the keyboard, just under the display. To power on your MATH 55 push this switch to the left; notice a red dot

 appears to the right of the power switch. This red dot signifies that your calculator has been turned on. When your calculator is powered on, your display should contain a zero. For specifics on the power supply refer to Appendix C.

Display Format

The MATH 55 has a 14-digit display.

Sample Display:

·	0.123456789		90
sign of mantissa	mantissa	sign of exponent	exponent

The **mantissa** is a maximum of ten digits with or without a decimal point. The sign of the mantissa is positive if the sign of the mantissa field is blank and negative if the sign of the mantissa field contains a "–" sign.

The **exponent** is a maximum of two digits. The sign of the exponent is positive if the sign of the exponent field is blank and negative if the sign of the exponent field contains a "≟" sign.

Display Indicators

Your calculator has three display indicators: dot indicators to signify radian mode and that [F] has been pressed and a minus "-" indicator for a busy signal.

Sample Displays:

1. To signify your calculator is in **radian mode** (and not degree mode), a special dot is displayed.

   [2.094395 22.]
   dot indicator

2. To indicate that the upper case function key [F] **has been pressed**, a special dot is displayed.

   [- . 2.14675498]
   dot indicator

3. Some functions, such as factorials, on your calculator involve relatively long calculations. During such calculations, the display will be blank except for a **busy signal**, a minus "-" indicator.

   [-]

   This busy signal will disappear when the calculation is finished and the answer is displayed.

The [DISP] key

A special feature on your MATH 55 is the [DISP] key.* This key provides two display modes, significant digits and fixed decimal point.

The **significant digits** display feature will round off the display value to a given number of significant digits. Suppose you want to round 5465.03 to three significant digits, enter [DISP] [3] and the display should read 5470.

The **fixed decimal point** display feature will round off the display to a given number of decimal places. Suppose you want to round 22.5681243 to two decimal places, enter [DISP] [·] [2] and the display should read 22.57.

To return to the normal ten digit mantissa display mode either enter [DISP] [9] or turn your calculator off and on again.

*These displays modes are not valid during matrix arithmetic. (see section 3.14)

Error Conditions

An error condition results when an illegal operation is performed or when the result of an operation overflows or underflows the absolute range of the calculator. (See Appendix C)

When an error condition occurs the word "ERROR" is displayed. Press the clear key [C/CE] to clear the error condition.

2.2 Number Entry

Entering positive and negative numbers

To enter a number use the following keys:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ., +/-

These keys directly enter positive or negative numbers. The [+/-] key will change the sign of the number during or after number entry—but not prior to entry.

For example, to enter a decimal number such as -2.45 press:

[2] [.] [4] [5] [+/-]

Scientific Notation

A number may be entered in decimal or exponential format (i.e. scientific notation), and regardless of the format of the number entered, the resulting number is displayed as decimal unless it is too large to display. In which case, the resulting number is displayed in exponential format. (see Appendix C, Internals).

To convert a decimal number to exponential format simply press [EE] ; this puts your calculator in **exponent mode.*** When in exponent mode, it is possible to alter the exponent value by number entry and alter the sign of the exponent by pressing the change sign key, [+/-].

If your calculator is in exponent mode and you want to modify the mantissa, simply press [F] MANT : this puts your calculator in **mantissa mode**. The exponent is cleared when the calculator is put in mantissa mode. Therefore, after modifying the mantissa, press [EE] and re-enter the exponent.

When the display is in exponent mode and a function key is pressed, the functional results will, if possible, be displayed in decimal format. If not possible, the results will have a decimal point after the most significant mantissa digit and its exponent will be changed accordingly.

*A number can be converted to exponential format only while in number entry; attempting to convert to exponential format after pressing a function, or operation, key will have no effect on the display format.

Example

Display 23.4 x 10^2, change it to 23.41 x 10^{-2}, and add .3 to the displayed number.

KEY ENTRY	DISPLAY	EXPLANATION
23.4	23.4	
[EE]	23.4 00	in exponent mode
2	23.4 02	
[F] MANT	23.4	in mantissa mode
1	23.41	
[EE]	23.41 00	in exponent mode
2	23.41 02	
[+/-]	23.41 -02	
[+]	.2341	
.3	.3	
[=]	.5341	

2.3 Clearing . . .

To clear an erroneous entry while keeping prior numerical entries intact, press [C/CE] once.

For example, 4 [÷] 2 [C/CE] 4 [=] 1

Pressing [C/CE] will also clear an error condition when "ERROR" appears in the display.

To clear a calculation, if needed, and allow for the entering of another calculation, press [C/CE] twice successively.

To clear the user memories, all six of them, and at the same time maintain the display, press [F] CAM .

There are two ways of **clearing your machine.** To clear your machine except for the six user memories, press [F] CA .

And to clear your machine including the six user memories power off and on again.

2.4 Function Control Keys: [F] & [(inv)]

The upper case function key is [F]. An upper case function is any function, or operation, which appears above a key top on the keyboard.*

Example: C^n_k [))] C^n_k is an upper case function.

Whenever an upper case function is needed, press [F] and then press the associated key top for the desired upper case function.

Example: press [F] C^n_k [))] for combinations after parameter entry. If [F] is accidently pressed, immediately press [F] again.

A special dot appears as a display indicator whenever [F] is pressed. This dot will disappear after the associated key top for the upper case function is pressed. (See section 2.1, Display Indicators).

* In this manual, all upper case functions will be underscored with a "⌐¬". Example: the upper case function C^n_k is designated by C^n_k ⌐¬.

The inverse key is [(inv)]. Several functions, or operations, on your MATH 55 require the use of [(inv)]. Those functions are both the trigonometric and hyperbolic functions, and the conversions including rectangular/polar, rectangular/spherical and unit conversions.

For the inverse of any trig function (ie. $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$), press [(inv)] and then press the appropriate trig function key. Example: for $\sin^{-1}$ .5, enter .5 then press [(inv)] [sin] and the display should read 30. (See sections 2.10 and 3.5 for inverse of hyperbolic)

All conversions on your MATH 55 are two-way, which means, for example, feet can be converted to meters and vice versa, meters can be converted to feet. The conversions are represented one-way on the keyboard, i.e. feet to meters.

For converting the other way, ie. meters to feet, the [(inv)] key must be used. Because all conversions except the rectangular/polar conversion are upper case functions, pressing [F] in addition to [(inv)] is required** prior to pressing the appropriate key top. (See Section 2.12)

** When [F] and [(inv)] are both required for any operation, the order in which they are pressed is not important.

2.5 Simple Arithmetic

The simple arithmetic function keys are:

[+] , [-] , [×] , [÷]

Your calculator follows normal algebraic logic which means key entry is done in a straightforward manner. The [=] key terminates any arithmetic key sequence and displays the final results.

Example: Entering 2 [×] 3 [=] will display the result 6.

Remark: Simple arithmetic allows chaining. (See section 2.7)

2.6 Parentheses

Two levels of parentheses are provided on your MATH 55. Parentheses allow for straightforward entry of more complex algebraic expression such as sum of products.

For example, to evaluate $\dfrac{(5 \times 2) + (7 \times 3)}{(4 \times 8) + (9 \times 9)}$

KEY ENTRY	DISPLAY
((	0
5	5
×	5
2	2
))	10
+	10
((	10
7	7
×	7
3	3
))	21
÷	31
((	31
((	31
4	4
×	4
8	8
))	32
+	32

[((]	32
9	9
[×]	9
9	9
[))]	81
[))]	113
[=]	2.743362832 - 01

The trigonometric, logarithmic and exponential functions may be used within parentheses, for example, to evaluate

$$\frac{e^{(\sin 50+\cos 23)} \times 1n\ 8}{2} :$$

KEY ENTRY	DISPLAY
[((]	0
50 [sin]	7.660444431 - 01
[+]	7.660444431 - 01
23 [cos]	9.205048535 - 01
[))]	1.686549297
[e^x]	5.400811913
[×]	5.400811913
8 [1n]	2.079441542
[÷]	11.23067265
2	2
[=]	5.615336326

The contents of user memories may also be recalled within parentheses.

2.7 Chaining Operations

Chaining is the ability to use the result of an initial operation as the first operand of another operation. Four functions allow chaining.

- Simple arithmetic
- Complex arithmetic (see section 3.3)
- Vector arithmetic (see section 3.14)
- Matrix arithmetic (see section 3.15)

An example of simple arithmetic chaining:

KEY ENTRY	DISPLAY
2	2
[×]	2
3	3
[+]	6
4	4
[=]	10

Here, the operands are 2, 3, and 4 and the chain of operations is [×] then [+].

2.8 Arithmetic Functions

1. **Finding square of numbers** [x^2]

   To find the square of a number, enter the number, then press [x^2]

2. **Finding square root of numbers** $\sqrt{x}$

   To obtain the square root of a number, enter the number, then press [F] $\sqrt{x}$

   Note: Valid for $x \geq 0$

3. **Finding reciprocal of numbers** [1/x]

   The reciproval of a number can be obtained by entering in the number and then pressing [1/x]

   Note: Not valid for $x = 0$

2.9 Logarithmic and Exponential Functions

1. **Finding Natural Logarithmn of numbers** [1n]

   To find the natural logarithm of a number, enter the number, then press [1n]

   Note: $x > 0$

2. **Finding e to the power x** [e^x]

   To obtain the e to the power x of a number, enter the number, then key in [e^x]

3. **Finding Common Logarithm of numbers** [F] log

   The Common Logarithm of a number can be obtained by entering the number and then pressing [F] log

   Note: $x > 0$

4. **Finding Common antilog of numbers** 10^x

   To calculate the common antilog of a number, enter the number, then press [F] 10^x

2.10 Trigonometric Functions

Finding Trigonometric Functions [sin] [cos] [tan]

To find the **sine** of a number in degrees, enter the number and then press [sin]. The **cosine** and **tangent** can be obtained similarly. If you want to calculate the **sin** of a number in radians, the calculator has to be set in the radian mode by pressing [F] d↔r and then entering the number, followed by [sin]. The **cosine** and **tangent** can be found similarly.

To find the inverse sin of a number, enter the number then depress [INV] [sin]. The inverse of the cosine and tangent can be obtained similarly.

Note: (1) inverse sine and cosine $\leq$ 1.

(2) Also tan 90° or tan $\pi/2$ is invalid.

4. **Finding Factorials** [n!]

To obtain the factorial of an integer on display, press [n!].

Note: n! is obtained if n < 69. For n > 69, use 1n r(x) (refer to example).

Double Functions

5. **Finding y to the power x** [y^x]

To raise a positive number to any power, enter as follows:

KEY ENTRY	DISPLAY
y	y
[y^x]	y
x	x
[=]	y^x

Note: x can be an integer or a decimal, negative or positive.

6. **Using the Exchange Register Key** [F] $x \leftrightarrow y$

The exchange key, $x \leftrightarrow y$, reverses the order of the operands.

For instance, x ÷ y will become y ÷ x. The exchange register key can be used as follows:

KEY ENTRY	DISPLAY
x	x
[÷]	x
y	y
[F] $x \leftrightarrow y$	x
[=]	y ÷ x

Note: You can use the exchange register key for the following operations: division, subtraction, and power.

2.11 User Memories

There are a maximum of six memories available to the user. The six memories will be referred to as registers from 1 to 6. All six memories may not be available to the user when certain advanced mathematical functions are being evaluated.

Many of the examples in Section 4 show how the memory registers can be used.

1. **Storing the Display in User Memory** - STOn n

   For storing a number on display in a memory, simply depress STOn followed by an arbitary number from 1 to 6 (these are the 6 memory registers available to the user).

   For instance, if we want to store 234 into register 2, simply enter 234, then depress STOn 2

2. **Recalling the Quantity Stored in User Memory** RCLn n

   For recalling a value stored in a memory register, simply depress RCLn followed by the memory register *(number 1 to 6) in which the value is stored. For instance, if we want to recall the value stored in register 2, simply depress RCLn 2, value obtained on the display is 234*.

   * Refer to the example 2 in Section 4.

3. **Exchange User Memory and Display** XCHn n

   A very important operation available in the MATH 55 is the exchange memory key XCHn . The effect of XCHn is to combine the effects of storing a new value and recalling the value stored earlier in one single step. To show how the XCHn key is used, an example is presented below:

KEY ENTRY	DISPLAY	EXPLANATION
5 STOn 1	5	5 in register 1
150 ÷	150	
25	25	

KEY ENTRY	DISPLAY	EXPLANATION
[+]	6	150 ÷ 25
[F] XCHn 1	5	6 in register 1 (new number)
[=]	11	6 + 5
[RCLn] 1	6	

4. **Four Function User Memories and Display**

[SUMn] PROD n

Another important operation available on the MATH 55 is simple arithmetic operations that can be carried out directly to a memory without the need to recall the value. This means that a new value a can be added, subtracted, multiplied or divided directly to a value present in any memory register. A new modified value will then occupy the memory register.

(1) To ADD a to the quantity present in memory register 1, enter a and press [SUM] 1.

(2) To SUBTRACT a from the quantity present in memory register 1, enter a and press [+/-] [SUM] 1.

(3) To MULTIPLY the quantity present in memory register 1 by the value a, enter a and press [F] PROD 1

(4) To DIVIDE the quantity present in memory register 1 by the value a, enter a and press [1/X] [F] PROD 1

To show these operations, the following example is given:

$$P_3^5 = 3!\ C_3^5$$

KEY ENTRY	DISPLAY	EXPLANATION
5 [ENTa]	5	
3 [F] C_k^n	9.999999...	C_3^5
[STOn] 1	9.999999...	Store in memory register 1
3 [n!]	6	
[F] PRODn 1	6	
[RCLn] 1	60	6 x 9.9999....

5. **Clearing the User Memories** CAM

To clear all the user memory registers, depress [F] CAM

If you want to clear only the value in one memory depress 0 [STOn] n (n referring to the memory register that is to be cleared).

6. **User Memory Register Limitations**

All user memory registers are not available under certain conditions. The table below provides the list of the memories not available when using certain functions:

Function	Memory Registers Lost
Rectangular/Polar	6
Rectangular/Spherical	5,6
Quadratic	5,6
Vector Operations	5,6
Mean/Standard deviation	4,5,6 (data base)
Matrix Determinant	5,6
Matrix Inverse	4,5,6
Matrix Arithmetic	1,2,3,4,5,6
Numerical Integration	6
Bessel Function	4,5,6

Function*	Memory Registers Lost
Linear Regression	1,2,3,4,5,6 (data base)
Combinations	6
Legendre	6
Laguerre	6

2.12 Conversions and Constants

1. **Rectangular/Polar and Rectangular/Spherical Coordinate conversions** (section 3.4)

   (R)→P , (R)→S

2. **Degree to radian conversion** d↔r

   (section 2.13)

3. **Unit Conversions**

   The unit conversions available on the MATH 55 are as follows:

Length		Conversion Factor (Unit 1 to Unit 2)	(Unit 2 to Unit 1)
(ft)m	feet to meters	0.3048	3.288839895
(mi)km	miles to kilo-meter	1.609344	0.621371192
Mass			
(lb)kg	pounds to kilo-grams	0.45359237	2.204622622
Volume			
(gal) l	gallons (U.S.) to liters	3.785411784	0.264172052
(foz) l	fluid ounces to liters	0.0295735296	33.81402266
Power, Energy			
(kwh) J	Kilowatt hours to joules	3600000	2.777778×10^{-7}
(BTU) J	BTU to joules	1055.055853	$9.478171203 \times 10^{-4}$
Temperature			
(F) °C	Degrees fahrenheit to degrees centigrade	(°F-32)÷1.8	$(°C \times \frac{5}{9}) + 32$

4. **Miscellaneous Conversions**

(d) gra	degree to grads	1.111111111	0.9
(d)OCT	decimal to octal		
(VR) dB	voltage ratio to decibels		

To convert the display in UNIT 1 to UNIT 2, enter [F] (unit 1) unit 2

To convert the display in UNIT 2 to UNIT 1, enter [F] [INV] (unit 1) unit 2

5. **Physical Constants**

The physical constants available on the MATH 55 are as follows:

KEY	NAME	QUANTITY	UNITS
n	Planck's constant	6.634×10^{-34}	Joules-sec
k	Boltzman's constant	1.381×10^{-23}	Joules/K
q	electronic charge	1.602×10^{-19}	Coulomb
m	electron rest mass	9.109×10^{-31}	kg
c	velocity of light	2.998×10^{8}	metres/sec
E	permittivity of free space	8.85×10^{-12}	F/cm
μ	permeability of free space	1.257×10^{-6}	H/m
G	Universal Gravitational Constant	6.6732×10^{-11}	$N.M^2/kg^2$
R	Universal Gas Constant	8.314	$J.K^{-1}mol^{-1}$
N	Avogadro's number	6.023×10^{26}	atoms/molecules per gm-mole
π	Pi	3.141592654	

To obtain one of the above physical constants (except for Pi) simply depress: [F] CONST and then the key top for the desired constant.

To obtain the constant Pi, enter [F] π

The constants listed above are stored in the calculator to 10 significant figures.

2.13 Degree/Radian Conversions & Modes

When you require either a degree/radian conversion or a change of degree/radian mode, press:

[F] d↔r

Pressing the above will both do the conversion and reset the mode. In other words, if your calculation is in degree mode and [F] d↔r is pressed, a degree to radian conversion is done and your calculator is put in radian mode. Likewise, if your calculator is in radian mode and [F] d↔r is pressed, a radian to degree conversion is done and your calculator is put in degree mode.

Rule for determining your calculator's mode are:

1) When turned on, your calculator is **initially in degree mode.**

2) If there is a decimal point in the exponent field of the display, your calculator is in radian mode. If not, your calculator is in degree mode. (See section 2.1, Display Indicators)

3. Using your MATH 55

the specialist functions

3.1 Special Parameter Entry Keys - [ENT a], [ENT b], [ENT [A]]

[ENT a], [ENT b], and [ENT [A]] keys are used with functions requiring more than one parameter.

A. The following functions require the key [ENT a] :*

(1) Rectangular/Polar coordinate conversions

(2) Numerical Integration

(3) Combinations

(4) Poisson

(5) Incomplete Gamma

(6) Bessel

(7) Legendre polynomial

(8) Laguerre polynomial

B. The following functions require both [ENT a] and [ENT b] :*

(1) Rectangular/Spherical coordinate conversions

(2) Quadratic Solution

(3) Vector Operations

C. The entry key [ENT [A]] is required for matrix operations.*

*Refer to the appropriate section for the specific key sequence.

3.2 The [CALL] key

The [CALL] key is used for all functions that require more than one answer for a solution. Generally speaking, five functions on your MATH 55 use the [CALL] key.

- **matrix operations** call for the complex elements of the resultant matrix. (See Section 3.15)
- **vector operations** call for the components of the resultant vector and the angle between the given vectors. (See Section .14)
- **complex arithmetic** calls for the real and imaginary parts (See Section 3.3.)
- **rectangular/polar and rectangular/spherical conversions** call for the appropriate coordinate (See Section 3.4).
- **quadratic solution** calls for the real and imaginary part of the roots (See Section 3.10)

Remark: The [CALL] key is only a result key and **must not** be used during function entry.

3.3 Complex Arithmetic

Suppose $(x_1 + jy_1)$ $(x_2 + jy_2)$ $(x_3 + jy_3)$ are complex numbers. To perform complex arithmetic, enter the following key sequence.

KEY ENTRY	DISPLAY	EXPLANATION
x_1	x_1	
ENT a	x_1	
y_1	y_1	
F j+	y_1	or any complex operation (F j-, F jx, F j÷)
x_2	x_2	
ENT a	x_2	
y_2	y_2	

For the results, enter the following:

=	x_a	real part of result A
CALL 0	y_a	imaginary part of result A
CALL 1	x_a	real part of result A, again

To perform complex arithmetic on the results (chaining), enter the following:

F j+	x_a	or any complex operation (F j-, F jx, F j÷)
x_3	x_3	
ENT a	x_3	
y_3	y_3	
=	x_b	real part of result B
CALL 0	y_b	imaginary part of result B
CALL 1	x_b	real part of result B, again

Remarks: No parentheses may be used while doing complex arithmetic but chaining capability is provided.

3.4 Rectangular/Polar and Rectangular/Spherical Conversions

Rectangular/Polar

When converting (x,y) to (r,θ), use the following key sequence:

KEY ENTRY	DISPLAY
x	x
[ENT a]	x
y	y
[(R)→P]	r
[CALL] 2	θ
[CALL] 1	r

To convert the inverse, or rather (r,θ) to (x,y), use the following:

KEY ENTRY	DISPLAY
r	r
[ENT a]	r
θ	θ
[(inv)] [(R)→P]	x
[CALL] 2	y
[CALL] 1	x

Note: User memory 6 is not available during rectangular/polar conversion.

Rectangular/Spherical

When converting (x,y,z) to (r,θ,φ), use the following key sequence:

KEY ENTRY	DISPLAY
x	x
ENT a	x
y	y
ENT b	y
z	z
F (R)→S	r
CALL 2	θ
CALL 3	φ
CALL 1	r

To convert the inverse, or rather (r,θ,φ) to (x,y,z), use the following:

KEY ENTRY	DISPLAY
r	r
ENT a	r
θ	θ
ENT b	θ
φ	φ
F (inv) (R)→S	x
CALL 2	y
CALL 3	z
CALL 1	x

Note: User memories 5 and 6 are not available during rectangular/spherical conversion.

3.5 The Hyperbolic Functions

The hyperbolic functions are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

To obtain the hyperbolic sine of x, enter x and press [F] sinh. The hyperbolic cosine and hyperbolic tangent can be obtained similarly. To calculate the inverse of the hyperbolic functions, enter the number followed by [F] [INV] Sinh. The inverse of tanh and cosh can be obtained similarly.

Remark: The inverse hyperbolic functions are designed as follows:

$$\sinh^{-1} x = \ln[x + (x^2 + 1)^{½}]$$

$$\cosh^{-1} x = \ln[x + (x^2 - 1)^{½}] \quad x \geq 1$$

$$\tanh^{-1} x = ½ \ln(\frac{1 + x}{1 - x}) \quad 0 \leq x^2 < 1$$

3.6 Linear Regression

Before data is entered for a linear regression, you have to clear the six memory registers by depressing [F] CAM. To enter the data, the x value is entered first followed by [x_i]. The y value is entered next followed by [y_i]. The display will show the number of pairs of points entered at this time.

A powerful feature of your machine is its ability to **preserve the data base.** This allows the user to do a linear regression and calculate the relevant parameters, then remove certain points from the data base by using [DEL] if desired or continue to add more points. Performing linear regression calculations does not destroy the data base.

Suppose we have a set of points (x_i, y_i) with which we want to fit a straight line

$$y = \alpha + \beta x$$

x	3	4	6	8
y	5	7	9	13

We want to calculate,

(a) the slope b(the best estimate of β) [slope]

(b) the intercept, a(the best estimate of α) [intcp]

(c) the residual sum of squares, RSS [F] RSS

where $$RSS = \sum_{i=1}^{N} [y_i - (\alpha + \beta x_i)]^2$$

(d) the coefficient of correlation [F] r

$$\text{where } r = \frac{N\sum_{i=1}^{N} x_i y_i - (\sum_{i=1}^{N} x_i)(\sum_{i=1}^{N} y_i)}{\sqrt{[N\sum_{i=1}^{N} x_i^2 - (\sum_{i=1}^{N} x_i)^2][N\sum_{i=1}^{N} y_i^2 - (\sum_{i=1}^{N} y_i)^2]}}$$

$0 \leq r^2 \leq 1$

$r^2 = 1$ is a perfect fit.

(e) fitted value of y for a corresponding x,

where $\hat{y} = \alpha + \beta x$ let $x = 9$

(f) fitted value of x for a corresponding y,

where $\hat{x} = \frac{y - \alpha}{\beta}$ let y = 15

Then the data may be entered as follows:

KEY ENTRY	DISPLAY
[F] CAM	0
3	3
[x_i]	3
5	5
[y_i]	1
4	4
[x_i]	4
7	7
[y_i]	2
6	6
[x_i]	6
9	9
[y_i]	3
8	8
[x_i]	8
13	13
[y_i]	4
[SLOPE]	1.525423723
[INTCP]	0.491525423
[F] r	0.990267408
[F] RSS	0.677966102
9 [F] $\hat{y}$	14.22033898
15 [F] $\hat{x}$	9.511111111

The contents of the data base can be obtained by entering as follows:

KEY ENTRY	DISPLAY	EXPLANATION
RCLn 6	4	i
RCLn 5	21	Σx_i
RCLn 4	125	Σx^2_i
RCLn 3	34	Σy_i
RCLn 2	324	Σy^2_i
RCLn 1	201	$\Sigma x_i y_i$

Remarks: (1) Clear all memory registers prior to initial point entry.

(2) All memory registers are used in the calculations.

(3) The number of points entered is unrestricted.

(4) The data must be entered in pairs with the x value entered first.

3.7 Mean and Standard Deviation

Before entering data for mean and standard deviation, memory registers 4,5 and 6 have to be cleared by entering O [STOn] 4 followed by O [STOn] 5 and O [STOn] 6.

Just as in linear regression, your data base is preserved, and therefore depressing [$\overline{X}$] or [F] s' or [F] s does not destroy the data base. Also deleting values and adding additional values is possible. Values can be deleted by entering the number and then depressing [DEL].

The following quantities are stored in the memory registers specified:

Quantity	Memory Register
Σ_i	6
$\sum_{i=1}^{N} x_i$	5
$\sum_{i=1}^{N} x_i^2$	4

The quantities can be obtained by depressing [RCLn] followed by the required memory register.

Supposing we are given a set of number (5.1, 5.8, 4.5, 5.5) and we want to evaluate

(a) Mean, [$\overline{X}$] where

$$\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

(b) Standard deviation of the sample (unbiased), [F] s

$$S = \sqrt{\frac{\sum_{i=1}^{N} x_i^2 - N\overline{x}^2}{(N-1)}}$$

(c) Standard deviation of the population (biased),

[F] s'

$$s' = \sqrt{\frac{\sum_{i=1}^{N} x_i^2 - N\bar{x}^2}{N}}$$

(d) Standard error of sample,

where $s_{\bar{x}} = \frac{s}{\sqrt{N}}$

Then we can enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
5.1	5.1	
[x_n]	5.1	
5.8	5.8	
[x_n]	5.8	
4.5	4.5	
[x_n]	4.5	
5.5	5.5	
[x_n]	5.5	
[$\bar{x}$]	5.225	
[F] s	0.56199051	standard deviation of sample
[F] s'	0.486698058	standard deviation of population

and for the standard error of sample,

KEY ENTRY	DISPLAY	EXPLANATION
[RCLn] 6	4	N
[$\sqrt{x}$]	2	
[1/x]	0.5	
[×]	×	
[F] s	0.56199051	
[=]	0.28095255	$s_{\bar{x}}$ (standard error)

NOTE: (1) Clear memory registers 4, 5 and 6 prior to entering data.

(2) Memory registers 4, 5 and 6 are not available for user.

(3) The number of sample values is unrestricted.

(4) N is a positive integer, $>$ 1.

3.8 Random Number Generator

To generate a pseudo random number enter any number of up to five digits and press:

[F] RNDM#

Successively pressing the above will give you a sequence of pseudo random numbers.

3.9 Combinations, Poisson & Gaussian

Combinations

The number of combinations of n objects taken k at a time is denoted by C_k^n and is given by:

$$C_k^n = \frac{n(n-1)\ldots(n-r+1)}{r!} = \frac{n!}{r!\,(n-r)!}$$

KEY ENTRY	DISPLAY
n	n
[ENT a]	n
k	k
[F] C_k^n	" C_k^n "

NOTE: 1. Do not use parentheses.
2. User memory 6 is not available.

Poisson Distribution

The Poisson probability with k successes out of an infinite number of trials assuming a frequency λ, is given by:

$$\text{Poiss}\ (k) = \frac{\lambda^k e^{-\lambda}}{k}$$

where $x > 0$ and $k = 0, 1, 2, \ldots$

To obtain the Poisson probability mass function:

KEY ENTRY	DISPLAY
k	k
[ENT a]	k
λ	λ
[F] POISS	"POISS (k)"

NOTE: Parentheses not available for user.

Gaussian (Normal) Distribution

The Gaussian probability distribution (Φ) is evaluated using:

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy$$

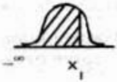
where

z is given by $z = \frac{x_i - \mu}{\sigma}$

and x_i = any value from a normal population

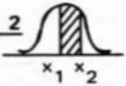
μ = mean

σ = standard deviation

Case 1 

To find the area under the curve between -∞ and x_1, where μ and σ are known, find z_1 by simple arithmetic, then enter the following:

KEY ENTRY	DISPLAY	EXPLANATION
z_1	z_1	
[F] GAUSS	$\Phi(z_1)$	area under the curve

Case 2 

To find the area between x_1 and x_2, find z_2 by simple arithmetic and then enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
z_2	z_2	
[F] GAUSS	$\Phi(z_2)$	
[STO$_n$] [1]	$\Phi(z_2)$	
x_1	z_1	
[-]	x_1	
μ	μ	
[÷]	$x_1 - \mu$	
σ	σ	
[=]	z_1	
[F] GAUSS	$\Phi(z_1)$	
[+/-]	$-\Phi(z_1)$	
[+]	$-\Phi(z_1)$	

$\Phi(z_2)$	$\Phi(z_2) - \Phi(z_2)$	area under the normal curve between x_1 and x_2

Remark: Parentheses not available for user.

3.10 Numerical Integration & Quadratic Solution

Numerical Integration

Numerical Integration in this calculator is done by using the trapezoidal rule, which is given by:

$$\int_{x}^{xn} y(x)dx \sim \tfrac{1}{2}h[y_o + 2y_1 + \ldots + 2y_{n-1} + y_n]$$

To numerically integrate between two points, say (x_1, y_1) and (x_2, y_2), enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[F] CA	0	clear calculator excluding user memories
x_1	x_1	
[ENT a]	x_1	
y_1	y_1	here, simple arithmetic may be used to evaluate $y = f(x)$
[∫]	0	
x_2	x_2	
[ENT a]	x_2	
y_2	y_2	here, simple arithmetic may be used to evaluate $y = f(x)$
[∫]	" $\int_{x_1}^{x_2}$ "	

Additional points may be entered to further define the function f(x).

Do this by entering: (Assume (x_3, y_3) is an additional point.)

x_3	x_3	
[ENT a]	x_3	

y_3	y_3	here, simple arithmetic may be used to evaluate y = f(x)
[∫]	" $\int_{x_1}^{x_3}$ "	

NOTE:

1. User memory 6 is not available.
2. Do not use parentheses.
3. Clear your calculator (excluding user memories) prior to initial key entry and after the computation.
4. Remember the function f(x) is defined by the way in which points are entered.
5. Trunction error is approximately

$$\frac{-(Xn - Xo)\, h^2\, y^2}{12}$$

Quadratic Solution

To find the solution (x_1 and x_2) to the equation $ax^2 + bx + c = 0$, enter the following:

KEY ENTRY	DISPLAY	EXPLANATION
a	a	
[ENT a]	a	
b	b	
[ENT b]	b	
c	c	
[F] QUAD	x_1^R	real part of root 1
[CALL] 2	x_1^I	imaginary part of root 1
[CALL] 3	x_2^R	real part of root 2
[CALL] 4	x_2^I	imaginary part of root 2
[CALL] 1	x_1^R	real part of root 1, again

NOTE:

1. User memories 5 and 6 are not available.
2. The entering of all parameters is required, even if they are zero.
3. The formula used for obtaining the quadratic solution is given by:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4\,ac}}{2a}$$

3.11 Natural Log of Gamma, Incomplete Gamma & Error Functions

Natural Log of Gamma

The Gamma function is given by the formula:

$$\Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\, dt$$

The natural log of Gamma as opposed to Gamma is given in order to extend the range of x value for which Gamma or factorial can be evaluated.

To obtain the $1n\Gamma(x)$, enter the following:

KEY ENTRY	DISPLAY
x	x
[F] 1nΓ(x)	"1nΓ(x)"

Remarks: 1. Do not use parentheses.

2. Applications of the Gamma function are found in mathematical physics and engineering.

Incomplete Gamma

The Incomplete Gamma function is given by the formula:

$$\gamma(a,x) = \int_0^x e^{-t}\, t^{a-1}\, dt$$

$$= x^a e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{a(a+1)\ldots(a+n)}$$

To obtain Y (a,x), enter the following:

KEY ENTRY	DISPLAY
a	a
[ENT a]	a
x	x
[F] γ (a,x)	"γ (a,x)"

Remarks: 1. Do not use parentheses.

2. Applications of the incomplete gamma are in mathematics, engineering, and statistics; χ^2 can be obtained using Incomplete Gamma.

Error Function

The Error Function is given by the formula:

$$\mathrm{erf}(x) = \frac{2}{\pi} \int_0^x e^{-t^2} \, dt$$

$$= \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1,3 \ldots (2n+1)} x^{2n+1}$$

To obtain erf(x), enter the following:

KEY ENTRY	DISPLAY
x	x
[F] erf(x)	"erf(x)"

Remarks: 1. Do not use parentheses.

2. Applications of the error function are in data communication and diffusion processes.

3.12 The Orthogonal Polynomials (Legendre and Laguerre polynomials)

Legendre Polynomial

The Legendre Polynomial of order n of the first kind is given by

$P\nu(x)$.

$$L\nu(x) = \frac{e^x}{\nu!} \frac{d^\nu}{dx^\nu} (e^{-x}x^\nu)$$

where $\nu = 0, 1, 2, 3, ...$

To obtain the Legendre polynomial, enter as follows:

KEY ENTRY	DISPLAY
for $P\nu(x)$:	
ν	ν
[ENT a]	ν
x	x
[Pν(x)]	"Pν(x)"

Remarks: 1. Do not use parentheses.

2. Applications are found in mathematical physics, especially spherical harmonics problems.

3. User memory 6 is not available.

Laguerre Polynomial

The Laguerre polynomial is given by:

$$P\nu(x) = \sum_{k=0}^{k=\nu/2} \frac{(-1)^k (2\nu - 2k)!\, x^{\nu-2k}}{2^\nu k!\, (\nu-k)!\, (\nu-2k)!}$$

To obtain the Laguerre polynomial, enter the following:

KEY ENTRY	DISPLAY
ν	ν
ENT a	ν
x	x
F $L_\nu(x)$	"$L_\nu(x)$"

Remarks: 1. Do not use parentheses.

2. Applications are found in mathematical physics.

3. User memory 6 is not available.

3.13 Bessel Function

The Bessel function of order ν is given by:

$$J\nu(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (\frac{1}{2} x)^{\nu+2r}}{r! \Gamma(\nu+r+1)}$$

where ν = positive or negative integar

To obtain the Bessel function, we enter the following: (Note: user memories 4, 5 and 6 are destroyed.)

KEY ENTRY	DISPLAY
ν	ν
[ENT a]	ν
x	x
[Jν(x)]	"Jν(x)"

NOTE:

1. User memories 4, 5 and 6 are not available.
2. Do not use parentheses.
3. Applications for the Bessel function are found in cylindrical problems and used widely in communications and electro-magnetic theory.

3.14 Vector Operations

Consider vectors $\overline{V}_1 = (x_1, y_1, z_1)$, $\overline{V}_2 = (x_2, y_2, z_2)$ and $\overline{V}_3 = (x_3, y_3, z_3)$.

The initial key entry for vector addition, vector subtraction, and both dot and cross products is as follows:

KEY ENTRY	DISPLAY	EXPLANATION
x_1	x_1	
[ENT a]	x_1	
y_1	y_1	
[ENT b]	y_1	
z_1	z_1	
[$\overline{u} + \overline{v}$]	z_1	for vector addition*
x_2	x_2	
[ENT a]	x_2	
y_2	y_2	
[ENT . b]	y_2	
z_2	z_2	

To find the **result of the dot product**, enter:

KEY ENTRY	DISPLAY	EXPLANATION
=	"dot product"	
[CALL] 0	"angle"	angle between vectors $\overline{V}_1$ and $\overline{V}_2$
[CALL] 1	"dot product"	dot product, again

To find the **results of all vector operations except dot product**, enter:

(Assume (x_a, y_a, z_a) is the resultant vector.)

=	x_a	first component of resultant vector, $\overline{V}_A$
CALL 2	y_a	second component of resultant vector, $\overline{V}_A$
CALL 3	z_a	third component of resultant vector, $\overline{V}_A$
CALL 1	x_a	first component of resultant vector, $\overline{V}_A$, again
CALL 0 (only for cross product)	"angle"	angle between vectors $\overline{V}_1$ & $\overline{V}_2$

For chaining on the resultant vector ($\overline{V}_A$), the key entry should be as follows:

$\overline{u}+\overline{v}$		for vector addition*
x_3	x_3	
ENT a	x_3	
y_3	y_3	
ENT b	y_3	
z_3	z_3	

*for vector subtractions, press F $\overline{u}$-$\overline{v}$

for dot product, press: $\overline{u}\cdot\overline{v}$

for cross product, press: F $\overline{u}\wedge\overline{v}$

Depending on the vector operation, follow the appropriate given key entry for the chaining results.

NOTE:

1. User memories 5 and 6 are not available.
2. The angle between vectors is calculated for dot product and cross product.
3. A value must be entered for each vector component. In the two-dimensional vector case, enter zeros for the third components.
4. A chaining capability is provided.
5. Do not use parentheses.

3.15 Matrix Operations

The matrix operations available on your MATH 55 are the inverse, the determinant, and matrix arithmetic. Each operation will handle 2 x 2 complex (on non-complex) matrices. The elements of these matrices are limited to five significant digits, and if the matrix is complex both the real and imaginary parts are limited to five significant digits. Truncation occurs if this limitation is exceeded.

Let A, B, and C be complex matrices:

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \quad B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

Denote the real part of any element as:

a_i^R for matrix A

b_i^R for matrix B

c_i^R for matrix C

And the imaginary part of any element as:

a_i^I for matrix A

b_i^I for matrix B

c_i^I for matrix C

3.15 The Inverse and the Determinant

The initial key entry for both the inverse and determinant of a matrix follows: (Note: user memories 5 and 6 are destroyed during determinant evaluation and 4, 5 and 6 are destroyed during inverse evaluation.)

KEY ENTRY	DISPLAY	EXPLANATION
a_1^R	a_1^R	real part of element a_1
ENT a	a_1	
a_1^I	a_1^I	imaginary part of element a_1
ENT [A]	a_1^I	enter matrix element a_1
a_2^R	a_2^R	real part of element a_2
ENT a	a_2^R	
a_2^I	a_2^I	imaginary part of element a_2
ENT [A]	a_2^I	enter matrix element a_2
a_3^R	a_3^R	real part of element a_3
ENT a	a_3^R	
a_3^I	a_3^I	imaginary part of element a_3
ENT [A]	a_3^I	enter matrix element a_3
a_4^R	a_4^R	real part of element a_4
ENT a	a_4^R	

a_4^I	a_4^I	imaginary part of element a_4
[ENT [A]]	a_4^I	enter matrix element a_4

<u>To evaluate the determinant</u>, enter the following:

[F] det [A]	"real part"	real part of the determinant
[CALL] 0	"imaginary part"	imaginary part of the determinant
[CALL] 1	"real part"	real part, again

<u>To evaluate the inverse</u>, enter the following:

(Let X be the resultant matrix, ie. the inverse.)

$[A]^{-1}$	x_1^R	real part of element x_1
[CALL] 0	x_1^I	imaginary part of element z_1
[CALL] 2	x_2^R	real part of element x_2
[CALL] 0	x_2^I	imaginary part of element x_2
[CALL] 3	x_3^R	real part of element x_3
[CALL] 0	x_3^I	imaginary part of element x_3
[CALL] 4	x_4^R	real part of element x_4
[CALL] 0	x_4^I	imaginary part of element x_4
[CALL] 1	x_1^R	real part of element x_1, again.
[CALL] 0	x_1^I	imaginary part of element x_1

At this point, chaining may be done using the inverse matrix. Refer to the chaining procedure in <u>Matrix Arithmetic</u>.

Matrix Arithmetic

The initial key entry for matrix addition, subtraction and multiplication is as follows: (Note: all user memories are not available.)

KEY ENTRY	DISPLAY	EXPLANATION
a_1^R	a_1^R	real part of element a_1
ENT a	a_1^R	
a_1^I	a_1^I	imaginary part of element a_1
ENT [A]	a_1^I	enter element a_1
a_2^R	a_2^R	real part of element a_2
ENT a	a_2^R	
a_2^I	a_2^I	imaginary part of element a_2
ENT [A]	a_2^I	enter element a_2
a_3^R	a_3^R	real part of element a_3
ENT a	a_3^R	
a_3^I	a_3^I	imaginary part of element a_3
ENT [A]	a_3^I	enter element a_3
a_4^R	a_4^R	real part of element a_4
ENT a	a_4^R	
a_4^I	a_4^I	imaginary part of a_4

ENT [A]	a_4^I	enter element a_4
[A]+	a_4^I	for matrix addition*
b_1^R	b_1^R	real part of element b_1
ENT a	b_1^R	
b_1^I	b_1^I	imaginary part of element b_1
ENT [A]	b_1^I	enter element b_1
b_2^R	b_2^R	real part of element b_2
ENT a	b_2^R	
b_2^I	b_2^I	imaginary part of element b_2
ENT [A]	b_2^I	enter element b_2
b_3^R	b_3^R	real part of element b_3
ENT a	b_3^R	
b_3^I	b_3^I	imaginary part of element b_3
ENT [A]	b_3^I	enter element b_3
b_4^R	b_4^R	real part of element b_4
ENT a	b_4^R	

b_4^I	b_4^I	imaginary part of element b_4
[ENT [A]]	b_4^I	enter element b_4

* for matrix subtraction, press [F] [A] -
  for matrix multiplication, press: [F] [A] x

For the results, enter the following:
(Let X be the resultant matrix.)

=	x_1^R	real part of element x_1
[CALL] 0	x_1^I	imaginary part of element x_1
[CALL] 2	x_2^R	real part of element x_2
[CALL] 0	x_2^I	imaginary part of element x_2
[CALL] 3	x_3^R	real part of element x_3
[CALL] 0	x_3^I	imaginary part of element x_3
[CALL] 4	x_4^R	real part of element x_4
[CALL] 0	x_4^I	imaginary part of element
[CALL] 1	x_1^R	real part of element x_1, again
[CALL] 0	x_1^I	imaginary part of element x_1

For chaining on the resultant matrix, enter the following:

[[A] +]		for matrix addition**
c_1^R	c_1^R	real part of element c_1

[ENT a]	c_1^R	
c_1^I	c_1^I	imaginary part of element c_1
[ENT [A]]	c_1^I	enter element c_1
c_2^R	c_2^R	real part of element c_2
[ENT a]	c_2^R	
c_2^I	c_2^I	imaginary part of element c_2
[ENT [A]]	c_2^I	enter element c_2
c_3^R	c_3^R	real part of element c_3
[ENT a]	c_3^R	
c_3^I	c_3^I	imaginary part of element c_3
[ENT [A]]	c_3^I	enter element c_3
c_4^R	c_4^R	real part of element c_4
[ENT a]	c_4^R	
c_4^I	c_4^I	imaginary part of element c_4
[ENT [A]]	c_4^I	enter element c_4

** for the determinant, press [F] det [A]

for the inverse, press: [[A]$^{-1}$]

for the matrix subtraction, press: [F] [A] -

for matrix multiplication, press: [F] [A] x

Depending on the matrix operation, follow the appropriate given key entry for the results of chaining.

Remarks: 1. The fixed point and significant digits display features will not work during matrix arithmetic- addition, subtraction and multiplication. (See section 2.1, The [DISP] key)

2. Do not use parentheses.

3. Entering the real part of an element is always required even if it is zero. Where as, entering the imaginary part of an element is not. In other words, if an element has no imaginary part enter element directly using just the [ENT [A]] key, and if the element is zero, a zero must be entered using the [ENT [A]] key.

4. User memories 4, 5 and 6 are not available during inverse evaluation. User memories 5 and 6 are not available during determinant evaluation. All user memories are not available in matrix arithmetic.

5. Both the real and imaginary part of an element are limited to five significant digits. If the five digits limit is exceeded, the right most digits will be truncated.

6. A chaining capability is provided.

4. Applications of the Special Functions Present in Your Calculator

Your Math 55 has a whole range of unique functions such as the Bessel, Laguerre and Legendre functions, functions which have wide applications in the field of engineering (especially chemical, mechanical and electrical); in mathematical physics (electromagnetic theory & quantum mechanics); in the solution of differential equations; series solution of the wave equation and other boundary value problems. Your calculator has complex arithmetic and matrix operations, features which are important in the field of electrical engineering. It has the quadratic solution and the numerical integration, besides having several unique and special functions. It has statistical functions such as mean and standard deviation, linear regression, Poisson density function, and the Gaussion distribution. In this section, we present a few examples to illustrate some of the applications of the various functions present in your calculator. We hope that by going through the examples, you would be able to think of other examples that you can apply towards your work. In the paragraph below, we present some of the topics in which the unique functions are applied.

Bessel function - diverse applications in physics, engineering and mathematical analysis, ranging from abstract number theory and theoretical astronomy to concrete cylindrical problems of physics and engineering. Some fields in which it is applied are the electromagnetic theory, conduction of heat and in communications.

Error function - applications in probability theory, theory of errors, theory of vibrations, theory of heat conduction; diffusion and transport phenomena.

Gamma function - prerequisite for the study of specialized functions and in complex variable theory. Solution to differential equations.

Incomplete Gamma function - applied in physics, engineering and statistics.

Laguerre polynomials - mathematical physics covering topics such as quantum mechanics and filters; approximation theory; and theory of mechanical quadratures.

Legendre first order polynomial - mathematical physics and engineering - approximation theory, solution of helmoltz equation, potential theory and examples in spherical harmonies.

Example

I **Statistical Functions**

1. Combination
2. Permutation
3. Use of Combination and y^x to obtain in Binomial Distribution
4. Gaussian Distribution
5. Mean and Standard Deviation
6. Linear Regression
7. General Curve Fitting
8. Poisson Distribution
9. Quality Control Applications
10. Using Incomplete Gamma Function to Obtain Chi-square Distribution

II **Other Mathematical Functions**

1. Rectangular/Spherical Conversion
2. Vector Cross Product
3. Vector Addition/Subtraction
4. Determinant of a 3 x 3 Non Complex Matrix
5. Quadratic Equation
6. Electrical Engineering Examples
7. Using $\ln\Gamma(x)$ to find 120!
8. Solving Definite Integral Using Gamma Function
9. Probability Example Using Error Function
10. Diffusion Example Using Error Function
11. Error Function on Heat Conduction
12. Laguerre Polynomial Example on the theory of propogation of Electromagnetic Waves
13. Legendre Polynomial Example
14. Bessel Function Examples (on Heat Loss and FM)

15. **Hyperbolic Functions on Resonant Circuits**

16. **Using the Exchange Key (x↔y)
to Solve: 3 ln2 + sin 30**

1. Combination

In how many ways can a committee of 5 people be chosen out of 9 people?

Solution: C_k^n where n = 9 k = 5

KEY ENTRY	DISPLAY
9	9
[ENT a]	9
5	5
[F] C_k^n	126

A committee of 5 people can be chosen 126 different ways.

2. **Permutation**

In how many ways can 10 people be seated on a bench if only 5 seats are available?

Solution: n = 10 k = 5

$$P_k^n = \frac{n!}{(n-k)!} = C_k^n \cdot k! = \frac{10!\ 5!}{(10-5)!\ 5!}$$

Enter as follows:

KEY ENTRY	DISPLAY
10	10
[ENTa]	10
5	5
[F] C_k^n	252
[STOn] 1	252
5 [n!]	120
[x] [RCLn] 1	252
[=]	30240

Ten people can be seated in 30,240 different ways if only 5 seats are available.

3. **Use of Combination and y^x to obtain Binomial Distribution**

1. Find the probability of getting exactly 2 heads in 6 tosses of a fair coin.

Solution:

We use the binomial distribution whereby n = 6
k = 2

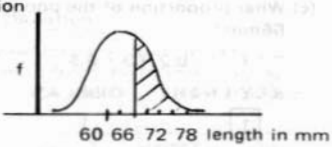
$$P(k) = \underline{C^n_k \cdot p^k \cdot q^{n-k}} \qquad p = q = 1/2$$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
6	6	
[ENT a]	6	
2	2	
[F] [C^n_k]	15	
[STOn] 1	15	
0.5	0.5	
[y^x]	5 -01	
2	2	
[=]	2.5 -01	
[F] PRODn 1	2.5 -0.1	Have C^n_k in memory register 1
0.5	0.5	
[y^x]	5 -01	
4	4	
[=]	6.25 -02	
[F] PRODn 1	6.25 -02	
[RCLn] 1	2.34375 -01	$C^n_k \cdot p^k \cdot q^{n-k}$

∴ the probability of getting exactly 2 heads in 6 coins is **0.234**

4. Gaussian Distribution

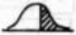


Calculate proportions of a normal distribution of bone lengths,

where μ = 60mm, σ = 10mm and N = 2,000.

(a) What proportion of the population of bone lengths is larger than 66 mm?

$$z = \frac{x_i - \mu}{\sigma} = \frac{66mm - 60mm}{10mm}$$

KEY ENTRY	DISPLAY	EXPLANATION
66	66	
[-] 60	60	
[÷]	6	
10 [=]	6 -01	
[F] Gauss	7.257468823 - 01	
[+/-] [+] 1 [=]	2.742531178 -01	

0.274 of the proportion of bone lengths is larger than 66 mm.

(b) How many bone lengths in this population are greater than 66mm?

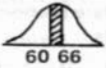
Solution: p(z) × N =

KEY ENTRY	DISPLAY	EXPLANATION
$p(z > 0.6)$	2.74253... - 01	
[×]	2.74253... - 01	
2000	2000	
[=]	548.506...	[≈549] bone lengths

549 bone lengths in this population are greater than 66m

(c) What proportion of the population lies between 60 and 66mm?

1 - 0.2763 + 0.5

KEY ENTRY	DISPLAY	EXPLANATION
[1] -	1	
p(z>0.6) [ie., 0.2763...]	0.274253...	
[+] .5 [=]	0.7743	

0.774 of the proportion lies between 60 and 66 mm.

(d) How many bone lengths in the population are larger than 77.5mm?

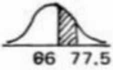
Solution First we have to find the probability of finding bone lengths greater than 77.5mm.

KEY ENTRY	DISPLAY	EXPLANATION
77.5	77.5	
[-] 60	60	
[÷]	17.5	
10 [=]	1.75	
[F] GAUSS	9.599408431 -01	
[+/-] [+] 1 [=]	4.0059 ... -02 P(x)>77.5mm	
[x]	4.0059 ... -02	
2000 [=]	80.118 ... [~80] bone lengths	

80 bone lengths in the population are larger than 77.5mm.

(e) What is the probability of selecting at random from this population a bone measuring between 66 and 77.5 mm in length?

$p(x_i > 66mm) - p(x_i > 77.5mm) =$

KEY ENTRY	DISPLAY	EXPLANATION
0.2743	0.2743	$p(x_1) > 66$
[-]	2.743 - 01	$p(x_1) > 77.5$
0.0401	0.0401	
[=]	2.342 -01	

Probability = **0.2342**

5. Mean and Standard Deviation

a) Find the average height of ten sixteen year old boys in a high school from the following data. Also find the precision, or the unbiased standard deviation.

Subject	Height in inches
1	66
2	62
3	60
4	58
5	65
6	65
7	57
8	61
9	60
10	62

Enter data as follows:

KEY ENTRY	DISPLAY	EXPLANATION
66	66	
[X_n]	66	
62	62	
[X_n]	62	
60	60	
[X_n]	60	
,	,	
,	,	continue entering data
,	,	
62	62	
[X_n]	62	
[$\bar{X}$]	61.6	
[F] s	3.025814858	

Therefore, the mean height for a sixteen year old boy is 61.6 inches with a precision of 3.03 inches.

b) Find the probability of finding the mean height of 65 inches when 10 measurements are taken.

Solution: First we find

$$z = \frac{(65 - 61.6)\sqrt{10}}{3.026}$$

and then find the Gaussian distribution.

Enter as follows:

KEY ENTRY	DISPLAY
65	65
[-]	65
[$\bar{x}$]	61.6
[×]	3.4
10	10
[F] $\sqrt{x}$	3.1627766
[÷]	10.75174404
[F] s	3.025814858
[=]	3.553338373
[F] GAUSS	9.998098125 -01
[+/-]	-9.998098125 -01
[+]	-9.998098125 -01
1	1
[=]	1.9018748 -04

Therefore, the probability of finding sixteen year old boys with mean height of 65 inches is only 0.00019 when 10 measurements are taken.

6. Linear Regression

In an experiment to measure the stiffness of a spring, the length of the spring under different loads was measured as follows:

x = Load (lb)	0	1	2	3	4	5	6
y = Length (inches)	4.2	5	6.7	8.0	9.0	10	11

a) Find the regression equation to fit the data.

b) Predict the length of the spring if the load is 11 lbs.

c) Find the correlation coefficient for the regression equation.

a) To find the regression equation, enter the data as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[F] CAM	0	Clear all memory registers
0	0	
[x_i]	0	
4.2	4.2	
[y_i]	1	1 pair entered so far
.	.	
.	.	Continue entering data
.	.	
.	.	
6	6	
[x_i]	6	
11	11	
[y_i]	7	7 pairs entered
[slope]	1.167857143	
[intcp]	4.196428571	
[DISP] 3	4.2	to set in 3 significant digits

slope	1.17

Therefore the regression equation is

$$y = 4.20 + 1.17 x$$

b) To find the length if the load is given, enter:

KEY ENTRY	DISPLAY
11	11
F $\hat{y}$	17

The length is 17 inches if the spring load is 11 lbs.

c) To find the correlation coefficient, enter:

KEY ENTRY	DISPLAY
F r	9.96 -01

Therefore the correlation coefficient is 0.996.

ENTRY	DISPLAY	COMMENTS
CLR F CAM	0	
11	11	
x_i	11	
12	12	
1n	2.48	
y_i	1	continue entering data
.	.	
.	.	
.	.	
12	12	
x_i	12	
15	15	
In	2.71	
y_i	15	Number of pairs entered
F RSS	9.476935891	
SLOPE	6.101639534 -02	
INCPT	1.905662729	

Since RSS for $y' = \frac{y^{0.5}-1}{0.5}$ is smaller than the other transformed y,

the best fit is:

$$y' = -1.37 + 0.618x$$

$$y^{0.5}-1 = 0.5(-1.37 + 0.618x)$$

$$y^{0.5}-1 = -0.685 + 0.309x$$

$$y^{0.5} = 0.315 + 0.309x$$

$$y = (0.315 + 0.309x)^2$$

8. **Poisson Distribution**

If 5% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 120 bulbs (a) 0 (b) 1 (c) 2 bulbs will be defective.

Solution:

We can use the poisson distribution where P = 0.05 and n = 120

k = 0, 1, 2 for (a), (b) and (c) respectively

To solve the problem, enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
120.	120	
[×]	×	
.05	.05	
[=]	6	11p
[STOn] 1	6	
0	0	
[ENT a]	0	
[RCLn] 1	6	
[F] Poiss	2.478752176 -03	
1	1	
[ENT a]	1	
[RCLn] 1	6	
[F] Poiss	1.487251306 -02	
2	2	
[ENT a]	2	
[RCLn] 1	6	
[F] Poiss	4.461753918 -02	

The probability that in a sample of 120,
(a) 0 bulb will be defective is 0.0025
(b) 1 bulb will be defective is 0.0148
(c) 2 bulbs will be defective is 0.0446

9. Quality Control Applications

(a) Variable sampling plan for accepting or rejecting lots by vendor or buyer. Deriving an O.C. curve for the plan.

The lot of apples will be accepted if there is 95% assurance that they contain a minimum of 15% sucrose. Take precision (e) to be 0.2% and standard deviation (s) to be 1. Also let $\overline{X}$ = 15.2%.

Determine the number to be sampled and derive an O.C. curve.

Solution:

(a) $n = (\frac{ks}{e})$ k for 95% assurance ~ 1.65

∴ n can be found by entering as follows:

KEY ENTRY	DISPLAY
1.65 [÷]	1.65
0.2	0.2
[=]	68

(b) To find O.C. curve we use the normal distribution to find Pa. Pa is found by finding k which can be obtained by

$k = (e)(\sqrt{n}/s)$,

where e is variable.

$(\sqrt{n}/s) = \sqrt{68}/1 = 8.246 =$ 8.25 to 2 decimal places

∴ Pa can be found by entering as follows:

ENTRY	DISPLAY	EXPLANATION
8.25	8.25	
[STOn] 1	8.25	$(\sqrt{n}/s)$ in memory register 1
[×]	8.25	
0.01	0.01	e_1
[=]	8.25 -02	
[F] Gauss	5.328754408 -01	
[+/-]	-5.328754408 -01	

[+] 4.671245592 -01

1 1

[=] -5.328754408 -01 Pa for e^2

. .

. .

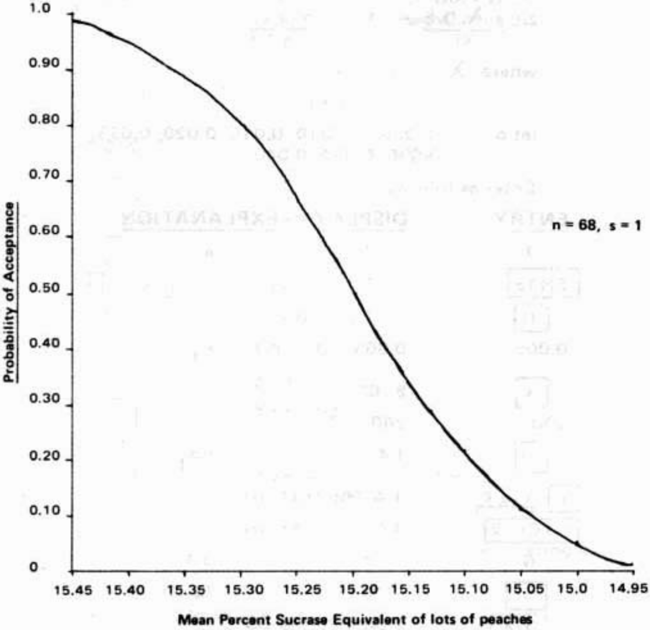
. . Continue entering data (ie. e to obtain pa)

and so on for e = 0.02, 0.04, 0.06, 0.08, 0.10, 0.15, 0.20, ... up to 0.40

The O.C. curve is constructed by beginning with the mean value of 15.2 at 0.5 probability, then adding the e value and also subtracting e from 15.2. We obtain:

p	Pa	p	Pa
15.20	0.50	15.20	0.50
15.21	0.53	15.19	0.47
15.22	0.56	15.18	0.44
15.24	0.63	15.16	0.37
15.26	0.69	15.14	0.31
15.28	0.75	15.12	0.25
15.30	0.80	15.10	0.20
15.35	0.89	15.05	0.11
15.40	0.95	15.00	0.05
15.45	0.98	14.95	0.02
15.50	0.99	14.90	0.01

Fig : O.C. curve for the Variable sampling problem with n = 68, st. d = 1



(b) Attribute single sampling plan for accepting or rejecting lots by vendor and buyer. Deriving an O.C. curve for the plan. Find an O.C. curve for a plan in which n = 280, c = 6, and the lot is 5,000.

Solution:

We use the cumulative poisson binomial limit which can be obtained from the calculator using the incomplete gamma function.

$$\Sigma \frac{e^{-\lambda} . \lambda^k}{k!} = 1 - \frac{\Upsilon(a,x)}{(a-1)!}$$

where $\lambda = np = x$

$\hat{a} = C+1$

let p = 0, 0.005, 0.010, 0.015, 0.020, 0.025, 0.030, 0.035, 0.040

Enter as follows:

ENTRY	DISPLAY	EXPLANATION
7	7	a
[ENTa]	7	
[((]	7	
0.005	0.005	P_1
[x]	5 -03	
280	280	n
[))]	1.4	np_1
[F] Y(a,x)	4.480667151 -01	
[STOn 2]	4.480667151 -01	
6	6	a-1
[n!]	720	(a-1)!
[1/x]	1.388888889-03	
[+/-]	-1.388888889-03	
[STOn 1]	-1.388888889-03	-1/(a-1)! stored in memory register 1 for later use

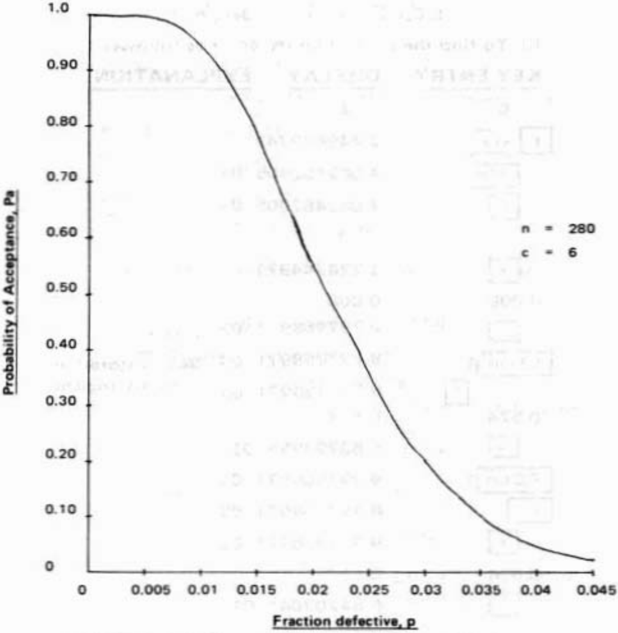
[×]	-1.388888889 -03	
[RCLn] 2	4.480667151 -01	
[=]	-6.223148821 -04	
[+]	-6.223148821 -04	
1	1	
[=]	9.993776851 -01	
[DISP] 3	9.99 -01	to work with 3 significant digits
7	7	
[ENTa]	7	
[((]	7	
0.010	0.010	p_2
[×]	1. -02	
280	280	
[))]	2.8	
[F] Y(a,x)	17.6	
[×]	17.6	
[RCLn] 1	-1.39 -03	
[=]	-2.44 -02	
[+]	-2.44 -02	
1	1	
[=]	9.76 -01	p_a for p_2
.	.	
.	.	continue entering data
.	.	
.	.	

and so on, to obtain:

p(fraction defective)	Pa(prob. of acceptance)
0	1
0.005	0.999
0.010	0.976
0.015	0.867
0.020	0.670
0.025	0.450
0.030	0.267
0.035	0.143
0.040	0.071

Figure : An O.C. curve for the above problem with $n = 280$ and $c = 6$.

Fig : An O.C. curve for the attribute sampling problem with n = 280 and c = 6



(c) Control Charts

A machine is constructed to produce ball bearings having a mean diameter of 0.574 inches and a standard deviation of 0.008 inches. Determine the Upper and Lower Control limits giving better than 99% assurance. Also find the modified control limits for an individual ball bearing. Let n = 6 for testing.

Solution:

$$U.C.L._{\bar{x}} = \bar{X} + 3s/\sqrt{n}$$

$$L.C.L._{\bar{x}} = \bar{X} - 3s/\sqrt{n}$$

(i) To find the control limits, enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
6	6	
[F] [$\sqrt{x}$]	2.449489743	
[1/x]	4.082482905 -01	
[×]	4.082482905 -01	
3	3	
[×]	1.224744871	
0.008	0.008	
[=]	9.797958971 -03	$3s/\sqrt{n}$
[STOn] 1	9.797958971 -03	$3s/\sqrt{n}$ stored in
[+]	9.797958971 -03	memory register 1
0.574	0.574	
[=]	5.83797959 -01	
[RCLn] 1	9.797958971 -03	
[+/-]	-9.797958971 -03	
[+]	-9.797958971 -03	
0.574	0.574	
[=]	5.64202041 -01	

∴ control limits are: $UCL_{\bar{x}}$ = 0.584 to 3 decimal
$LCL_{\bar{x}}$ = 0.564 places

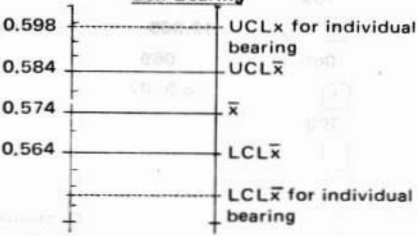
(ii) Modified control limits for individual bearings $= \overline{X} + 3s$

KEY ENTRY	DISPLAY	EXPLANATION
0.008	0.008	
[x]	8 -03	
3	3	
[=]	2.4 -02	
[STOn] 1	2.4 -02	3s stored in memory register
[+]	2.4 -02	
0.574	0.574	
[=]	5.98 -01	Modified upper limit
0.574	0.574	
[-]	5.74 -01	
[RCLn] 1	2.4 -02	
[=]	5.5 -01	Modified lower limit

∴ the modified control limits for individual bearings are:

upper = 0.598   lower = 0.550

Figure : Control Limits Chart for Ball Bearing



10. Using Incomplete Gamma Function to find Chi-Square

Consider the occurrence, past and present, of peptic ulcers in 4,871 men, selected at random. The men were interviewed by age groups and the following data was the result.*

Age group	14-	20-	25-	35-	45-	55-	65-	Total
Number of men	199	300	1128	1375	1089	625	155	4871
Number of ulcers	1	8	38	96	105	56	12	316

**Based on studies done by Coll and Jones of Central Middlesex Hospital*

Can it be concluded that the frequency of ulcers is constant between age groups?
In total 316 in 4,871 men have, or have had in the past, peptic ulcers, so the constant frequency of cases should be 316/4,871, or 6.5%. To find the expected cases for each age group do the following key sequence.

KEY ENTRY	DISPLAY	EXPLANATION
.065	.065	
[×]	6.5 -02	
199	199	
[=]	12.935	approx. 13
.065	.065	
[×]	6.5 -02	
300	300	
[=]	19.5	
.	.	
.	.	Continue for all age groups
.	.	
.	.	

Therefore, the total expected cases for all age groups are as follows:

Age Group	14-	20-	25-	35-	45-	55-	65-
Number expected	13	19.5	73	89	71	40.6	10

To reach a conclusion, we must find X^2, where

$$X^2 = \Sigma \frac{(\text{actual cases} - \text{expected cases})^2}{\text{expected cases}}$$

The key sequence is:

KEY ENTRY	DISPLAY	EXPLANATION
1	1	
[-]	1	
13	13	
[=]	-12	
[x^2]	144	
[÷]	144	
13	13	
[=]	11.07692308	
[STOn] 1	11.07692308	
8	8	
[-]	8	
19.5	19.5	
[=]	-11.5	
[x^2]	132.25	
[÷]	132.25	
19.5	19.5	
[=]	6.782051282	
[SUMn] 1	6.782051282	
38	38	
[-]	38	
73	73	
[=]	-35	
[x^2]	1225	
[÷]	1225	
73	73	
[=]	16.78082192	
[SUMn] 1	16.78082192	
.	.	
.	.	Continue for all age groups
.	.	

The Results, X^2, is stored in memory register 1 at the end of calculation and is approximately 57.6. Since there are seven age groups, there are 6 degrees of freedom.

Realizing $\frac{Y(a,x)}{\Gamma(a)} = P(x^2/v)$

where $a = \frac{v}{2}$ and $x = \frac{X^2}{2}$

we have

$$P(57.6,6) = \frac{Y(3,28.8)}{\Gamma(3)}$$

For the solution, use the following key sequence

KEY ENTRY	DISPLAY	EXPLANATION
3	3	
F lnΓ(x)	.69314718	
e^x	2	
STOn 1	2	
3	3	
ENT a	3	
28.8	28.8	
F Y(a,x)	1.999999999	
÷	1.999999999	
RCLn 1	2	
=	9.999 ... -01	

To determine any significance in the hypothesis, enter the following:

KEY ENTRY	DISPLAY	EXPLANATION
+/-	-9.999 .... -01	
+	-9.999 .... -01	
1	1	
=	3.6 -10	A small number implies little or no significance

Therefore, one cannot hypothesize a constant frequency of ulcers within age groups.

B. Other Mathematical Functions

1. Rectangular/Spherical

Convert Rectangular to Spherical coordinates

(x, y, z) = (1, -5, 3)

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
1	1	x entry
[ENT a]	1	
5 [+/-]	-5	y entry
[ENT b]	-5	
3	3	z entry
[F] (R)→S	5.916079783	r computed
CALL 2	-78.69006753	θ computed
CALL 3	59.52964053	ϕ computed
CALL 1	5.916079783	

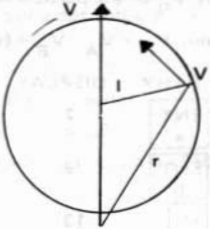
Therefore r = 5.916, θ = -78.69, and ϕ = 59.529

2. Cross Product (vector)

Find the linear velocity of a particle rotating with an angular velocity V = 7i + 5j - 13k about a fixed axis if the displacement vector on the axis of rotation is given by,
r = 3i - 3j + 8k. Also find the angle between V and r.

Solution: V = Vxr

$v = (v_1, v_2, v_3)$



KEY ENTRY	DISPLAY	
7 [ENT a]	7	
5 [ENT b]	5	
13 [+/-]	-13	
[F] u ∧ v	-13	
3 [ENT a]	3	
3 [+/-] [ENT b]	-3	
8 =	1	v_1
CALL 2	-95	v_2
CALL 3	-36	v_3
CALL 1	1	v_1
CALL 0	46.03250219	∡ r

Linear velocity, v = (1, -95, -36). The angle between r and v = 46°.

3. Vector addition/subtraction

The velocity of particle A is described by the equation $V_A = 2i + 9j - 13k$ while that of particle B is given by $V_B = 5i + 7j - 10k$. Calculate the velocity V_R of the particle A relative to particle B.

Solution: $V_R = V_A - V_B = (v_1, v_2, v_3)$

KEY ENTRY	DISPLAY	EXPLANATION
2 [ENT a]	2	
9 [ENT b]	9	
13 [+/-]	-13	
[F] [$\overline{u} - \overline{v}$]	-13	
5 [ENT a]	5	
7 [ENT b]	7	
10 [+/-]	-10	
[=]	-3	v_1
CALL 2	2	v_2
CALL 3	-3	v_3

The velocity, Vr, of particle A relative to B is (-3, 2, -3).

4. Determinant of a 3 x 3 non complex matrix

Find the determinant of the matrix below:

2	7	-3
-1	0	6
2	3	4

The solution requires both a dot and a cross product calculation:

[(-1, 0, 6) x (2, 3, 4)] · (2, 7, -3)

= a · (b x c) scalar triple product

The data may be entered as follows:

KEY ENTRY	DISPLAY	EXPLANATION
1	1	
[+/-]	-1	
[ENT a]	-1	
0	0	
[ENT b]	0	
6	6	
[F] [$\overline{u} \wedge \overline{v}$]	6	
2	2	
[ENT a]	2	
3	3	
[ENT b]	3	
4	4	
[=]	-18	
[u·v]	-18	
2	2	
[ENT a]	2	
7	7	
[ENT b]	7	

3	3
[+/-]	-3
[=]	85

Therefore, the determinant of the 3 x 3 matrix is 85.

5. Quadratic Equation

Solve the following quadratic equation : $2x^2 + 2x + 3$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
2	2	
[ENT a]	2	
2	2	
[ENT b]	2	
3	3	
[F] QUAD	-5.0 -01	real part of root 1
CALL 2	1.118033989	imaginary part of root 1
CALL 3	-5.0 -01	real part of root 2
CALL 4	-1.118033989	imaginary part of root 2

∴ the solutions are:

$$x_1 = -0.5 + 1.118033989j$$

$$x_2 = -0.5 - 1.118033989j$$

6. Electrical Engineering Examples using Matrices and Complex Arithmetic

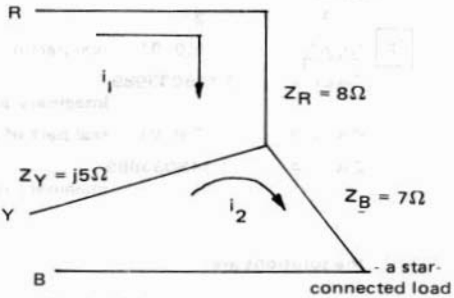
A. A 400-v 3-phase supply feeds a star-connected load (WYE-load) with:

$$Z_R = (8 + j0)\Omega$$

$$Z_Y = (0 + j5)\Omega$$

$$Z_B = (7 + j0)\Omega$$

If the phase sequence is RYB (or ABC), what are the line currents?



Solution:

$V_{RY} = 400(\cos 0 + j \sin 0)$

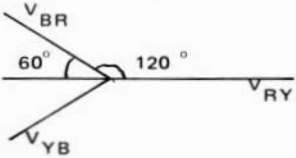
$= 400$

$V_{YB} = 400(\cos 240^\circ + j \sin 240^\circ)$

$= 400(-1/2 - j \frac{\sqrt{3}}{2})$

$= -200 - j\, 200\sqrt{3}$

$= -200 - j(346.41)$



Remembering that V = [Z] 1, we have:

$$\begin{bmatrix} 400 \\ -200 - j(346.41) \end{bmatrix} = \begin{bmatrix} 8 + j5 & -j5 & i_1 \\ -j5 & 7 + j5 & i_2 \end{bmatrix}$$

or rather,

$$\begin{bmatrix} 8 + j5 & -j5 \\ -j5 & 7+j5 \end{bmatrix}^{-1} \begin{bmatrix} 400 \\ -200 - j(346.41) \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

The key sequence to solve the above for i_1 and i_2 is:

KEY ENTRY	DISPLAY	EXPLANATION
8	8	
[ENT a]	8	
5	5	
[[A] ENT]	5	
0	0	
[ENT a]	0	
5	5	
[+/-]	-5	
[[A] ENT]	-5	
0	0	
[ENT a]	0	
5	5	
[+/-]	-5	
[[A] ENT]	-5	
7	7	
[ENT a]	7	
5	5	
[[A] ENT]	5	
[$[A]^{-1}$]	8.7547 -02	
[F] [[A] ×]	8.7547 -02	
400	400	
[[A] ENT]	400	
0	0	
[[A] ENT]	0	
200	200	
	100.	

[+/-]	-200	
[ENT a]	-200	
346.41	346.41	
[+/-]	-346.41	
[[A] ENT]	-346.41	
0	0	
[[A] ENT]	0	
[=]	37.528	
CALL 1	37.528	
[STOn] 1	37.528	real of i_1 stored in memory register 1
CALL 0	-32.404	
[STOn] 2	-32.404	imaginary of i_1 stored in memory register 1
CALL 3	-14.319	
[STOn] 3	-14.319	real of i_2 stored in memory register 3
CALL 0	-12.453	
[STOn] 4	-12.453	imaginary of i_2 stored in memory register 4

To find the line currents:

1. Recall memories 1 and 2 to find I_R

   $I_R = i_1 = 37.528 - j\,32.404$ with magnitude, $[I_R]$ given by the key sequence:

KEY ENTRY	DISPLAY	EXPLANATION
[RCLn] 1	37.528	
[ENTa]	37.528	
[RCLn] 2	-32.404	
[(R)→P]	49.58195236	$[I_R]$

Therefore, $I_R = 37.528 - j\,32.404$ amps $[I_R] = 49.58$

2. $I_B = -i_2$ To find (I_B), enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
RCLn 3	-14.319	
+/-	14.319	
ENT a	14.319	
RCLn 4	-12.453	
+/-	12.453	
(R)→P	18.97659005	(I_B)

Therefore, I_B = 14.319 + j12.453 amps and (I_B) = 18.98

3. Since $I_Y = i_2 - i_1$, I_Y can be found by entering as follows:

KEY ENTRY	DISPLAY	EXPLANATION
RCLn 1	37.528	
+/- SUM 3	-37.528	Real part of I_Y stored in memory register 3
RCLn 2	-32.404	
+/- SUM 4	32.404	Imaginary part of I_Y stored in memory register 4
RCLn 3	51.847	
RCLn 4	19.951	

The magnitude of I_Y is obtained by entering:

KEY ENTRY	DISPLAY	EXPLANATION
RCLn 3	-51.847	
ENT a	-51.847	
RCLn 4	19.951	
(R)→P	55.55316202	(I_Y)

Therefore, $I_Y = -51.847 + j19.951$ amps and $(I_Y) = 55.55$

B. The current in a circuit is given by (4.5 + j12)A when applied voltage is (100 + j150) volts. Determine a) the complex expression for the impedance, stating whether it is inductive or capacitive, b) the power, c) the phase angle between voltage and current.

Solution:

impedance = V/Amps = a + bj/c + dj

Power = ac + bd

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
100	100	
[ENT a]	100	
150	150	
[F] j÷	150	
4.5	4.5	
[ENT a]	4.5	
12	12	
=	13.6986 . . .	real part of impedance
[STOn] 1	13.6986 . . .	
CALL 0	-3.19634 . . .	imaginary part of
[STOn] 2	-3.19634 . . .	impedance
[RCLn] 1	13.6986	
100	100	
[×] 4.5	4.5	
[+]	450	
((150	150	
[×] 12	12	
[))]	1800	
=	2250	Power in watts
[RCLn] 2	-3.19634	
[÷] [RCLn] 1	13.6986	

[=]	-2.3333 .... -01	
[(INV)] [tan]	-13.1340	Ø ∡ between voltage & current

(a) impedance is capacitive since the imaginary part is negative;
= 13.6986 - 3.19634j

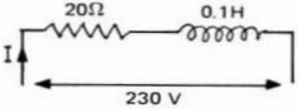
(b) Power = 2250 watts

C. State the impedances of each of the following circuits at a frequency of 50 c/s:

a) a resistance of 20Ω in series with an inductance of 0.1 H.

b) a resistance of 50Ω in series with a capacitance of 40μF. If the terminal voltage is 230 volts, find the value of the current in each case and the phase of each current relative to the applied voltage.

Solution: a)



a) $\omega = 2\pi f$

$z_1 = R_1 + j\omega \times H$

$I = v / |z_1|$

$\emptyset$ = phase difference between the appplied voltage and the current

$$\emptyset = \tan^{-1} \frac{\omega \times H}{R_1}$$

To solve the problem, enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[F] π	3.1415926 ...	
[×] 2	2	
[×]	6.283185	
50	50	f
[=]	314.1592654	$\omega = 2\pi f$
[STOn] 3	314.1592654	
[×] 0.1	0.1	
[=]	31.41592654	imaginary part of Z_1
[STOn] 1	31.41592654	
[ENT a]	31.41592654	
20	20	
[STOn] 2	20	

KEY ENTRY	DISPLAY	EXPLANATION
[(R)→P]	37.2419 ...	
[1/x]	2.685 .... -02	
[×] 230	230	voltage I = v/[z]
[=]	6.1758 ...	Current in amps
[RCLn] 1	31.4959 ...	
÷ [RCLn] 2	20	
[=]	1.57079 ...	
[inv] [tan]	57,518	Ø Current lagging

b) $z_2 = R_2 - j \frac{1}{wH}$ z = 20 + j31.42Ω
Ø = 57.518 Current lagging

$\therefore$ $I = v/[Z_2]$ and $Ø = \tan^{-1} \frac{\omega H}{R_2}$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[RCLn] 3	314.1592654	$\omega = 2\pi f$
[1/x]	3.183098861	-03
[÷] 40	7.957747155	-05
[×] 1 [EE] 6		
[=]	79.57747155	imaginary part z_2
[ENT a]	79.57747154	
[STOn] 1	79.57747154	
50	50	
[STOn] 2	50	
[(R)→P]	93.98177471	
[1/x]	1.064036089	-02
[×] 230	230	voltage I = v/1
[=]	2.447283005	I = v/[z]
[RCLn] 1	79.577 ...	
[+/-]	-79.57747 ...	
[÷] [RCLn] 2	50	

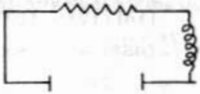
=		-1.591549431	
inv	tan	-57.858 ...	Ø, Current leading

$\therefore\ z_2 = 50 + j79.58\Omega$

Ø = -57.858 current leading

D. If the potential difference across a circuit is represented by 40 + j 25 v, and the circuit consists of a resistance of 20Ω in series with an inductance of 0.06 H and the frequency is 79.5c/sec, find the complex number representing the current in amperes.

Solution:



voltage = 40 + j25
frequency = 79 c/s

for 79c/s, $\omega = 2\pi \times 79$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[F] π	3.14 ...	
[x] 2	2	
[x] 79.5	79.5	
[x] 0.06	0.06	
[=]	29.97079 ...	imaginary part of z
[STOn] 1	29.97079 ...	
20 [ENT a]	20	
[RCLn] 1	29.97079 ...	
[(R)→P]	36.031215 ...	
[1/x]	2.77537 ...-02	
[ENT a]	2.77537 ...-02	
0 [F] jx	0	
40 [ENT a]	40	
25	25	
[=]	1.11014 ...	real part of I
CALL 0	6.9384 ... -01	imaginary part of I

I = 1.115 + j 0.694 amps

7. Using 1nΓ(x) [natural log of gamma function) to find 120!

Solution: By using the relationship n! = Γ(n+1), we can find Γ(121) to give 120!
[1nΓ(121) - 10^{99}1n - 10^{50}1n]

KEY ENTRY	DISPLAY	EXPLANATION
121	121	
F lnΓ(x)	457.812388	
-	457.812388	
1 EE 99	1. 99	
1n	227.9559242	
=	229.8564642	By trial & error find that it is overloaded
-	229.856464	
1 EE 50	1. 50	
1n	115.1292547	
=	114.7272091	
e^x	6.68950291 49	

∴ 120! = $6.6895 \times 10^{49} \times 10^{99} \times 10^{50}$

= 6.69×10^{198}

8. Solving definite integral of $\sin^4 u$ using Gamma function

Solve : $\int_0^{\pi/2} \sin^4 u \, du$

We can solve the problem by the use of the following relationship:

$$\int_0^{\pi/2} \sin^n u du = \frac{\sqrt{\pi}\,\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)} \qquad n > -1$$

in this case n = u

$$\int_0^{\pi/2} \sin^4 u du = \frac{\sqrt{\pi}}{2} \; \frac{\Gamma(1.5)}{\Gamma\ (3)}$$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
3	3	
[F] $1n\Gamma(x)$	6.9314 ... -01	
[e^x]	2	
[1/x]	5. 0-1	
[×]	5. 0-1	
[F] π	3.14159 ...	
[F] $\sqrt{x}$	1.77245 ...	
[÷]	8.86226 .... -01	
2	2	
[=]	4.43113462 -01	
[STOn] 1	4.43113462 -01	
1.5	1.5	
[F] $1n\Gamma(x)$	-1.2078 ... -01	
[e^x]	8.8622 ... -01	
[×]	8.8622 ... -01	
[RCLn] 1	4.43113 ... -01	
[=]	3.926990817 -01	

$$\therefore \int_0^{\pi/2} \sin^4 u du = 0.393$$

Probability example using Error functions:

9. A structure is tested for metal fatigue. The logarithmn (base 10) of the time until failure, in hours, is normally distributed with average value 3 and standard deviation 1. Determine the probability of a failure as a function of the test duration T = 15,000 hrs.

   (≃ 1 3/4 years)

   Solution: Denoting logT by ν, the probability distribution function is

$$p(v) = e^{-(\nu - \overline{v})^2/2\sigma^2}$$

since $\sigma = 1$ and $\overline{v} = 3$. The probability of failure in a time T is

$$P(T) = \int_{\log o}^{\log t} P(v)\,dv = \int_{-\infty}^{\log t} P(v)\,dv = \int_{-\infty}^{\log t} e^{-(v-3)^2/2}\,dv$$

substituting $w = \frac{v - 3}{2}$ and, hence $dw = \frac{1}{2}\,dv$,

$$P(T) = \int_{-\infty}^{\frac{1}{\sqrt{2}}(\log T - 3)} \frac{1}{\sqrt{\pi}} e^{-w^2}\,dw$$

$$= \int_{-\infty}^{o} \frac{1}{\sqrt{\pi}} e^{-w^2}\,dw + \int_{o}^{\frac{1}{\sqrt{2}}(\log T - 3)} \frac{1}{\sqrt{\pi}} e^{-w^2}\,dw$$

$P(W) = (½ + ½ \text{ erf}(w))$

Thus, $P(T) = \frac{1}{2}\left\{1 + \text{erf}\left[\frac{1}{\sqrt{2}}(\log(T - 3))\right]\right\}$

∴ to solve the problem, enter as follows:

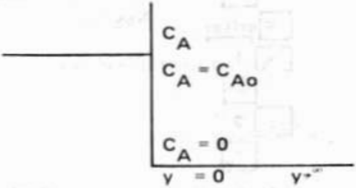
KEY ENTRY	DISPLAY
15 000	15000
[F] log	4.17609 ....
[-] 3	3
[x]	1.17609 ....
2 [F] √x	1.414213 ....
[1/x]	7.07106 -01
[=]	8.316 -01
[F] erf(x)	7.6044 -01
[+] 1	1
[=]	1.76044 ...
[÷] 2	2
[=]	8.8022 -01

Probability of Failure = 0.88

10. Diffusion Example Using Error Function

A plane membrane, impervious to the transfer of mass, separates an infinite solid into two equal parts. One-half of the solid concentration is initially at $C_{Ao} = 7.5$ moles/ft^3 while the other half is at zero. At time $t = 0$, the membrane is removed and the solids brought to direct contact with each other. Calculate the concentration in the solid after time 1 hour and a distance of 5 ft. Obtain the mass flux at the interface. Let diffusion coefficient, $D_{AB,} = 3.5$ ft^2/sec.

Solution:



The initial concentration profile is given above. The equation describing this is given by

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial C_A}{\partial y^2}$$

$$\text{where } C_A = C_A(y,t)$$

The Boundary conditions (BC) and Intitial Conditions (IC) are:

BC(1)	$C_A =$	finite at $y = \infty$
BC(2)	$C_A =$	finite at $y = -\infty$
IC(1)	$C_A =$	C_{Ao} at $t = 0$, for $-\infty \leq y \leq 0$
IC(2)	$C_A =$	0 at $t = 0$, for $0 \leq y \leq +\infty$

The solution then takes the following form:

$$C_A(y,t) = \int_{y' = -\infty}^{y'} \frac{C_A(y')}{\sqrt{4\pi D_{AB} t}} e^{-[(y-y')^2/4D_{AB}t]} dy'$$

where $C_A(y')$ = initial concentration profile

Inserting the IC into (2) gives

$$C_A = \frac{1}{\sqrt{4\pi D_{AB}t}} \left[\int_{-\infty}^{0} C_{A_0} e^{-[(y-y^1)^2/4D_{AB}t]} dy^1 \right.$$

$$\left. + \int_{0}^{\infty} (0) e^{-[(y-y^1)^2/4D_{AB}t]} dy^1 \right] \quad (3)$$

$$= \frac{C_{A_0}}{\sqrt{4\pi D_{AB}t}} \int_{\infty}^{0} e^{-[(y-y^1)^2/4D_{AB}t]} dy^1$$

If we introduce the variable

$$\xi = \frac{y-y^1}{4D_{AB}t}$$

equation (3) becomes

$$C_A = - \frac{C_{A_0}}{\sqrt{4\pi D_{AB}t}} \int_{\xi=\infty}^{\xi = \frac{y}{\sqrt{4D_{AB}t}}} e^{-\xi^2} \sqrt{4D_{AB}t}\, d\xi$$

$$= - \frac{C_{A_0}}{\sqrt{\pi}} \int_{\infty}^{y/\sqrt{4D_{AB}t}} e^{-\xi} d\xi$$

$$= \frac{C_{A_0}}{\sqrt{\pi}} \left[\int_{0}^{\infty} e^{-\xi^2} d\xi - \int_{0}^{y/\sqrt{4D_{AB}t}} e^{-\xi^2} d\xi \right]$$

$$= \frac{C_{A_0}}{\sqrt{\pi}} \left[\frac{\pi}{2} - \frac{\pi}{2} \operatorname{erf}\left(\frac{y}{\sqrt{4D_{AB}t}} \right) \right]$$

$$= \frac{C_{A_0}}{2} \left[1 - \operatorname{erf}\left(\frac{y}{\sqrt{4D_{AB}t}} \right) \right]$$ concentration of solid.

Concentration of solid

(a) To obtain C_A, enter as follows:

KEY ENTRY	DISPLAY
4	4
[×]	4
3.5	3.5
[×]	14
60	60
[x^2]	3600
[=]	50400
[F] $\sqrt{x}$	224.4994432
[1/x]	4.45435032 -03
[×]	4.45435032 -03
5	5
[=]	2.227177016-02
[F] erf(x)	2.512684682-02
[+/-]	-2.512684682-02
[+]	-2.512684682-02
1	1
[÷]	9.748731532 -01
2	2
[×]	4.874365766 -01
7.5	7.5
[=]	3.655774324
[DISP] 3	3.66 to 3 significant digits

Therefore, concentration of solid is 3.66 moles/ft^3

(b) To obtain the mass flux at the interface,

$$\text{Mass/unit area transferred across } y = 0 \text{ from time } 0 \text{ to time } \theta = \int_0^\theta \frac{C_{A_0} D}{\sqrt{4\pi t}} dt$$

$$= C_{A_0} \sqrt{\frac{D\theta}{\pi}}$$

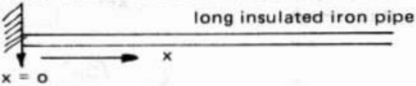
enter as follows:

KEY ENTRY	DISPLAY
3.5	3.5
[×]	3.5
60	60
[x^2]	3600
[÷]	12600
[F] π	3.141592654
[=]	4010.704566
[F] √x	63.33012368
[×]	63.33012368
7.5	7.5
=	474.9759275

Therefore, the transfer flux is given by 475 lb /ft^2 to 3 significant digits.

II. Error Function on Heat Conduction

A very long insulated iron pipe at -40°C is heated at one end so that a constant temperature is maintained at that end (with boiling water). Find the temperature 3 meters from the heated end after 15 hours.



The unknown temperature is a function 0 of distance "x" and time "t".

Note:

1) at time t = 0, $\theta(x,0) = 40^\circ C$
2) at distance x = 0, $\theta(0,t) = 100^\circ C$ for $t > 0$
3) in general, $\theta(x,t) =$
   $(100\text{-}Ti)\ [1\text{-}erf\,(\frac{x}{2a\sqrt{t}})] + Ti$ where Ti is the initial temperature, knowing the conductivity constant, a^2, of iron,

$a = .471 \times 10^{-2}\ m/\sqrt{sec}$

and from the stated problem

$T_i = -40^\circ C$

$x = 3$ meters

$t = 15$ hours $= 15 \times 60 \times 60 = 5.4 \times 10^4$ seconds

Therefore,

$$\theta(x,t) = (100 + 40)\ (1\text{-}erf\ \frac{3}{2(0.471 \times 10^{-2})\ (\sqrt{5.4 \times 10^4})}) - 40$$

Key sequence:

KEY ENTRY	DISPLAY	EXPLANATION
3	3	
[÷]	3	
[((]	3	

((	3
2	2
×	2
.471	0.471
EE	.471 00
2	.471 02
+/-	.471 -02
))	9.42 -03
×	9.42 -03
5.4	5.4
EE	5.4 00
4	5.4 04
F √x	232.3790008
))	2.189010187
=	1.37048243
F erf(x)	9.473956653 -01
—	9.473956653 -01
1	1
=	-5.260433468 -02
+/-	5.260433468 -02
×	5.260433468 -02
((	5.260433468 -02
100	100
+	100
40	40
))	140
=	7.364606855
—	7.364606855
40	40
=	-32.63539314 The temperature in °C

Temperature is -32.63539314°C.

12. Application of the Laquerre Polynomials to the theory of propagation of electromagnetic waves

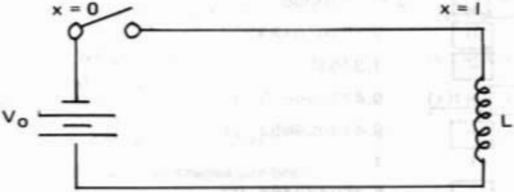
Reflection from the end of a long transmission line terminated by a lumped inductance.

Consider the problem of propagation of electromagnetic waves along a transmission line of length l. Suppose the line terminates at one end in a coil of inductance L_o, while at the other end, a source of constant d-c voltage V_o is suddenly switched on at time t = 0. Find the voltage, V in terms of V_o if $T = \frac{\pi}{4}$, $\alpha = 1.3$

N = 3 and t = 2 seconds

(See figure below) Note that $\alpha = \frac{2}{L_o}$, and

$T = \frac{l}{V}$ where $V = \frac{1}{\sqrt{L}}$



Let the instantaneous values of the voltage and current be denoted by V = V(x,t) and I(x,t), and let the inductance and capacitance per unit length of the line be denoted by L and C. Then the problem reduces to the integration of the following system of linear differential equations.

$$\frac{-\partial V}{\partial x} = \frac{L\partial I}{\partial t} \quad \frac{-\partial I}{\partial x} = \frac{C\partial V}{\partial t}$$

$$V\big|_{t=0} = I\big|_{t=0} = 0$$

and boundary conditions,

$$V\big|_{x=0} = V_o$$

$$V\big|_{x=L} = L_o \frac{dI}{dt}\bigg|_{x=L}$$

To solve these equations, we use the method of the Laplace transform and arrive at

$$V\Big|_{x=L} = 0 \text{ for } 0<t<T, \text{ and}$$

$$\frac{1}{2V_o} V\Big|_{x=l} = \sum_{n=o}^{N-l} (-1)^n e^{-\alpha[u]} \ln(2\alpha u)$$

for $(2N-1)T < t < (2n+1)t \quad N = 1,2,3 \ldots$

where $u = t - (2n+1)T$

To obtain V in terms of V_o, enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
[F] π	3.141592654	
[÷]	3.141592654	
4	4	
[=]	7.853981634 -01	T
[STO n] 1	7.853981634 -01	T in memory register 1
[+/-]	-7.853981634 -01	
[+]	-7.853981634 -01	
2	2	
[×]	1.214601837	U_0 where n=0
1.3	1.3	
[+/-]	-1.3	
[=]	-1.578982388	$-\alpha U_o$
[e^x]	2.061848077 -01	
[STO n] 2	2.061848077 -01	$e^{-\alpha U_o}$ in register 2
[Ln]	-1.578982388	
[+/-]	1.578982388	
[×]	1.578982388	
[2]	2	
[=]	3.157964775	

STO$_n$ 3	3.157964775	$2\alpha U_o$ in register 3
0	0	n = 0
ENT a	0	
RCL$_n$ 3	3.157964775	$2\alpha U_o$
F Lv(x)	1	$L_0(2\alpha U_o)$
RCL$_n$ 1	7.853981634 -01	T
×	7.853981634 -01	
3	3	2n+1 where n = 1
=	2.35619449	
+/-	-2.35619449	
+	-2.35619449	
2	2	
=	-3.561944902 -01	U_1 where n=1
×	-3.561944902 -01	
1.3	1.3	
=	-4.630528373 -01	
+/-	4.630528373 -01	$-\alpha U_1$
e^x	1.588917294	
STO$_n$ 4	1.588917294	$e^{-\alpha U_o}$ in register 4
Ln	-4.630528373 -01	
×	-4.630528373 -01	
2	2	
=	9.261056745 -01	
+/-	-9.261056745 -01	
STO$_n$ 3	-9.261056745 -01	$2\alpha U_1$ in register 3
1	1	n = 1
ENT a	1	
RCL$_n$ 3	-9.261056745 -01	

F Lv(x)	1.926105675	$L_1(2\alpha \dot{U}_1)$
x	1.926105675	
RCLn 4	1.588917294	
=	3.060422617	$e^{-x} \times L_1(2\alpha U_1)$
+/-	-3.060422617	$(-1)'(e^{-\alpha U_1})(L_1[2 \times V_1]$
SUMn 2	-3.060422607	above $+(-1)^o$ $(e^{-\alpha U_0}) L_o(2\alpha U_o)$ in register 2
RCLn 1	-7.853981634 -01	T
x	-7.853981634 -01	
5	5	2n + 1 where n=2
=	3.926990817	
+/-	-3.926990817	
+	-3.926990817	
2	2	
x	-1.926990817	U_2
1.3	1.3	
=	-2.505088062	
+/-	2.505088062	$-\alpha U_2$
e^x	12.24463721	
STOn 4	12.24463721	$e^{-\alpha U_2}$ in register 4
Ln	2.505088062	
+/-	-2.505088062	
x	-2.50508806	
2	2	
=	-5.010176124	
STOn 3	-5.010176124	$2\alpha U_2$ in register 3

Key	Display	Comment
2	2	n = 2
ENT a	2	
RCL n 3	-5.010176124	
F Lv(x)	23.57128465	$L_2(2\alpha U_2)$
×	23.57128465	
RCL n 4	12.24463721	
=	288.621829	$(-1)^2(e^{-\alpha U2})$ $(L_2[2\alpha U_2])$
SUM n 2	288.62118291	
RCL n 2	285.7675912	n=3 Σ $(-1)^n e^{-\alpha U} \ln(2\alpha U)$ 0
÷	285.7675913	0
2	2	
=	142.8837956	

Therefore, V in terms of V_o, =

$$\frac{142.8837957}{V_o}$$

13. Legendre Polynomial

Calculate the gravitational potential ψe, of a homogeneous solid oblate spheroid (potential of outside). Introducing spherical coordinates r, θ, and ϕ, find the gravitational potential of outside, ie ψe, for c = 3, where c is the distance from the origin to the focus, let r = 2.1 $\theta = 37^\circ$ $\phi = 25^\circ$ and m = 12 kg.

We solve the equation by finding the solution to the equation

$\Delta^2 \psi e = 0$, which

satisfies the boundary conditions

$\psi e_{\infty} = 0$

we arrive at the solution:

$$\psi e\Big|_{c \to o} \simeq m \left[\frac{1}{r} + \frac{c^2}{5r^3} P_2 (\cos\theta) \right]$$

Where m = Mass

We solve the equation by substituting in the values. Enter as follows to solve:

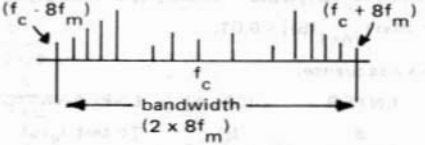
KEY ENTRY	DISPLAY
2	2
ENT a	2
37	37
Cos	7.986355101 -01
Pv(x)	4.567280169 -01
÷	4.567280169 -01
5	5
X	9.134560338-02
3	3
x^2	9
=	8.221104304 -01
STO_n 1	8.221104304 -01

KEY ENTRY	DISPLAY
2.1	2.1
[y^x]	2.1
3	3
[=]	9.261
[1/X]	1.079796998 -01
[X]	1.079796998 -01
[RCL_n] 1	8.221104304 -01
[+]	8.871123749 -02
[2.1]	2.1
[$\frac{1}{X}$]	4.761904762 -01
[=]	5.649617137 -01
[X]	5.649617137 -01
12	12
[=]	6.779540564

Therefore, the gravitational potential, e = 6.78

14. **Bessel Function on FM**

The FCC has fixed the maximum value of change of frequency (Δf) at 75 kHz for commercial FM broadcasting stations. If fm_{max} is 15 kHz (typically the maximum audio frequency in FM transmission), what is the required bandwidth for the FM station?



A single sine-wave which is frequency modulated is given by:

(Eq. 1)

$$f(t) = \cos(2\pi f_c(t) + \beta \sin 2\pi f_m(t))$$

where $f_c \triangleq$ carrier frequency

$f_m \triangleq$ frequency of the sine-wave

and $\beta \triangleq \frac{\Delta f}{f_m}$, for this case $= \frac{75}{15} = 5$

The bandwidth of this signal is obtained by counting the significant number of sidebands. The word "significant" is usually taken to mean those sidebands which have a magnitude of at least 1% of the magnitude of the unmodulated carrier. If Eq. 1 is expanded,

$f(t) = J_o(\beta) \cos w_o t - J_1 (\beta) [\cos(w_o - w_m) t - \cos$

$(w_o - w_m)t] + J_2 (\beta) [\cos(w_o - 2w_m)t + \cos$

$(w_o + 2w_m)t] - J_3 (\beta) [\cos(w_o - 3w_m)t + \cos$

$(w_o + 3w_m)t] + \ldots\ldots$

where $w_o = 2\pi f_c(t)$

$w_m = 2\pi f_m(t)$

$J_n(\beta)$ = Bessel function of nth order

Therefore, the significant sidebands will be those for which $|J_n(\beta)| > 0.01$. Since β is 5, find the smallest n that $|J_{n+1}(5)| < 0.01$.

Key sequence:

ENTER	DISPLAY	EXPLANATION
5	5	To test $J_5(5)$
ENT a	5	
5	5	
Jν(x)	2.611405461 -01	
8	8	To test $J_8(5)$
ENT a	8	
5	5	
Jν(x)	1.840521665 -02	
9	9	To test $J_9(5)$
ENT a	9	
5	5	
$J_v(x)$	5.520283139 -03	

Since $J_9(5) < 0.01$, n = 8 and the required bandwidth for the FM station is $2 \times 8f_m = 16f_m = 16 \times 15 =$ 240kHz.

(b) Bessel Function on Heat Loss

Suppose you have to find the heat loss of an infinitely long cylinder. This problem requires knowing the first order Bessel function, $J_1(x)$, for a calculated x. Assuming x equals 2.40, find the heat loss.

Solution: Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
1	1	
[ENT a]	1	
2.40	2.40	
[Jv(x)]	5.201852682 -01	

Therefore, the heat loss of the indefinitely long cylinder is 0.520 to 3 decimal places.

15. Hyperbolic Functions on Resonant Circuits

a) Find the amplitude at resonance of a magnetic field if the terminations are dissipative. The attenuation factors are given by

$A_o = 0$ $A_s = 1.77$ Also $gSp = 1.17$

Let $K = 2.4$

b) Find the efficiency of the transmission, i.e. P_s/P_o.

c) Find the decibel loss

Solution:

the amplitude is given by,

$$Kp = \frac{K}{\sinh(\alpha gSp + A_s + A_o)}$$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
1.77	1.77	A_s
[+]	1.77	
1.17	1.17	gSp
[=]	2.94	
[F] [sinh]	9.431490292	
[1/X]	1.060277824 -01	
[×]	1.060277824 -01	
2.4	2.4	
[=]	2.544666777 -01	

Therefore, the amplitude is 0.25 to 2 decimal places

b) to find the efficiency of the transmission, i.e.

$\frac{P_s}{P_o}$, we use the relationship

$$\frac{P_s}{P_o} = \frac{\sinh 2 A_s}{\sinh 2 (\alpha g S p + A_s)}$$

Enter as follows:

KEY ENTRY	DISPLAY	EXPLANATION
1.77	1.77	
+	1.77	
1.17	1.17	
×	2.94	
2	2	
=	5.88	
F sinh	178.9032235	
1/x	5.58961421 -03	
STO_n 1	5.58961421 -03	
1.77	1.77	
×	1.77	
2	2	
=	3.54	
F sinh	17.21895293	
F Prod 1	17.21895293	
RCL_n 1	9.624730398 -02	

The efficiency is 0.096.

c) We find the decibel loss by:

$$\text{Loss (db)} = 10 \log_{10} \frac{P_o}{P_s}$$

Enter as follows:

KEY ENTRY	DISPLAY
RCL$_n$ 1	9.624730398 -02
1/X	10.38990142
F log	1.016611427
×	1.016611427
10	10
=	10.16611427

Decibels loss = 10.16 to 2 decimal places.

16. **Using the Exchange key** $\underline{x \longleftrightarrow y}$

Find $3^{\ln 2 + \sin 30}$

KEY ENTRY	DISPLAY
2	2
[1n]	6.931471806 -01
[+] 30	30
[sin]	5. -01
[=]	1.193147181
[y^x]	1.193137181
3	3
[F] $x \longleftrightarrow y$	1.193147181
[=]	3.709162666

$\therefore\ 3^{1n\, 2\, +\, \sin 30} = 3.709162666$

5. APPENDIX

A. ERROR CONDITION

An error condition results when an improper operation is performed or when the result of an operation over-flows or under flows the absolute range of the calculator.

When an error condition occurs the word "ERROR" is displayed.

Press the clear key to clear the error condition.

Improper Operation:

X ÷ Y	where Y = 0
y^x	where y < 0
[F] $\sqrt[x]{y}$	where X < 0
[F] $\sqrt{x}$	where X < 0
[1/x]	where X = 0
[F] s	where number of entries is 0
[1n] X	where X < 0
[F] log X	where X < 0
[F] $\sin^{-1}$ y	where y > 1
[F] $\cos^{-1}$ y	where y > 1
X [F] X↔Y Y [→R]	where X = 0

Overflow

Occurs when a computed result is greater than $9.999999999 \times 10^{99}$

Underflow

Occurs when a computed result is less than 1.0×10^{-99}

B. OPERATING ACCURACY

The precision of your calculator depends upon the operation being performed. Basic addition, subtraction, multiplication, division and reciprocal assignments have a maximum error of ± one count in the tenth or least significant digit.

While countless computations may be performed with complete accuracy, the accuracy limits of particular operations depend upon the input argument as shown below.

Function	Input Argument	Mantissa Error (Max)
[F] $\sqrt{x}$	x positive	1 count in D_{10}
[1n] x	x positive	1 count in D_{10}
[F] log x	x positive	1 count in D_{10}
e^x		1 count in D_{10}
[y^x]	y positive	1 count in D_9
[sin] x	$0^\circ < o < 360^\circ$ or $0 < o < 2\pi$	2 counts in D_{10}
[cos] x	$0^\circ < o < 360^\circ$ or $0 < o < 2\pi$	2 counts in D_{10}
[tan] x	$0 < o < 89^\circ$	2 counts in D_9
	$89^\circ < o < 89.95^\circ$	1 count in D_6
[F] $\sin^{-1}$ y	$10^{-10} < y < 1$	$E < 5 \times 10^{-8}$
[F] $\cos^{-1}$ y	$10^{-10} < y <^{1}$	$E < 5 \times 10^{-8}$
[F] $\tan^{-1}$ y		$E < 5 \times 10^{-8}$
[F] lnΓ(x)	x positive	1 count in D_9
[F] γ(a,x)	a,x positive	1 count in D_9
[F] erf(x)	x positive	1 count in D_9
[Lv(x)]	v positive integer	1 count in D_9
[F] [Jv(x)]	v positive integer	1 count in D_9
[Pv(x)]	v positive integer	1 count in D_9

F C_k^n	n>k n,k integers	1 count in D_9
A^{-1}		1 count in D_5
F sinh x		1 count in D_{10}
F cosh x		1 count in D_{10}
F tanh x		1 count in D_{10}
$\sinh^{-1} y$	Negative or zero Positive	$E < 2 \times 10^{-10}$ 6 counts in D_{10}
$\cosh^{-1} y$		6 counts in D_{10}
$\tanh^{-1} y$		$E < 2 \times 10^{-10}$
n!		4 counts in D_{10}
F QUAD		1 count in D_9
∫		1 count in D_9
Linear Regression, Mean and Standard Deviation, Gaussian Distribution, Poisson Distribution		1 count in D_9

Dn = Nth display digit assuming a left justified 10 digit result.

C. SOME USEFUL FORMULAS AND TOPICS

Hyperbolic Functions

$\cosh x \pm \sinh x = e^{\pm x}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh(a + jb) = \sinh a \cdot \cos b + j(\cosh a \cdot \sin b)$

$\cosh(a + jb) = \cosh a \cdot \cos b + j(\sinh a \cdot \sin b)$

hyperbolic jb = j trigonometric (b)

$$\tanh^{-1}(a + jb) = 1/2 \tanh^{-1} + \frac{j}{2} \tan^{-1} \frac{2b}{1 - a^2 - b^2}$$

Factorial of Even Numbers

$(2n)!! = 2 \cdot 4 \cdot 6 \cdots 2n = 2^n n!$

Factorial of Odd Numbers

$$(2n - 1)!! = 1 \cdot 3 \cdot 5 \cdots (2n - 1) = \frac{1}{\sqrt{\pi}} 2^n \Gamma(n + 1/2)$$

Gamma and Beta Functions

$\Gamma(n + 1) = n\Gamma(n) = n!$

$$\beta(x,y) = \frac{\Gamma(x) \times \Gamma(y)}{\Gamma(x + y)}$$

Error Function and Related Functions

$$\text{Erf } z = \int_0^z e^{-t^2}\, dt = \frac{\sqrt{\pi}}{2}\, \phi(z)$$

where $\phi(z)$ = probability integral

$$\text{Erfc } z = \int_z e^{-t^2} dt = 1 - \text{erf } z$$

Cumulative distribution function of Gaussian variable related to Error function

$$p^*(x) = \frac{1}{2}\left[1 + \operatorname{erf}\frac{(x - \bar{x})}{\sqrt{2\sigma^2}}\right]$$

Incomplete gamma function

$$\gamma(a,x) = \int_0^x e^{-t}t^{a-1}dt$$

Incomplete Gamma Related to Chi-Square Distribution

$$\frac{\gamma(a,x)}{\Gamma(a)} = P(\chi^2/\nu) \quad \nu = 2a \quad \chi^2 = 2x$$

Gaussian Probability Function

$$P(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_{x}^{\infty} e^{-t^2/2}dt$$

$$P(x) = 1 - Q(x)$$

$$\operatorname{erf} x = 2P(x\sqrt{2}) - 1$$

Incomplete Gamma related to Cumulative Poisson Distribution

$$1 - \gamma(a,x) = \frac{\Sigma e^{-\lambda}\lambda^k}{k!} \qquad \lambda = x = np, \quad k = a - 1$$

Legendre polynomials defined by Rodrigues Formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n} \qquad n = 0,\ 1,\ 2, \ldots$$

$$\int_0^1 x^m \left[\ln\left(\frac{1}{x}\right)\right]^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}$$

for $m+1>0$, $n+1>0$

$$\int_0^1 (\ln x)^n \, dx = (-1)^n \cdot n!$$

Recurrence Relationships

$$J_{n-1}(x) = \frac{2n}{x} J_n(x) - J_{n+1}(x)$$ where $J_n(x)$ is the Bessel function

$$P_{n+1}(x) = \frac{(2n+1)x\,P_n(x) - nP_{n-1}(x)}{n+1}$$

$$L_{n+1}(x) = \frac{(2n+1-x)\,L_n(x) - nL_{n-1}(x)}{n+1}$$

Congruent Hypergeometric function related to Bessel function

$$\lim_{a\to\infty}\left[\frac{m(a,b,-z/a)}{\Gamma(b)}\right] = z^{\frac{1-b}{2}} J_{b-1}(2\sqrt{z})$$

Legendre Polynomial related to Gebenbauer's polynomials

$$P_n(x) = C_n^{(1/2)}(x)$$

Legendre Polynomial related to the Gaussian Hyper-geometric function

$$P_n(1-2x) = F(-n, n+1; 1; x)$$

Useful Definite Integrals

$$\int_0^\infty \frac{\cosh 2yt}{(\cosh t)^{2x}}\, dt = 2^{2x-2} \frac{\Gamma(x+y)\,\Gamma(x-y)}{\Gamma(2x)}$$

for Real x > 0 Real x > Real y.

$$\int_0^{\frac{\pi}{2}} \cos^n \theta \, d\theta = \int_0^{\frac{\pi}{2}} \sin^n \theta \, d\theta = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n+2}{2})}$$

for Real n > - 1.

$$\int_0^{\frac{\pi}{2}} \cos^m \theta \sin^n \theta \, d\theta = \frac{1}{2} \frac{\Gamma(\frac{m+1}{2})\,\Gamma(\frac{n+1}{2})}{\Gamma\,\frac{m+n+2}{2}}$$

$$\int_0^\infty \frac{a\,dx}{a^2 + x^2} = \frac{\pi}{2} \text{ if } a > o; = o \text{ if } a = 0;$$

$$= -\frac{\pi}{2} \text{ if } a < 0.$$

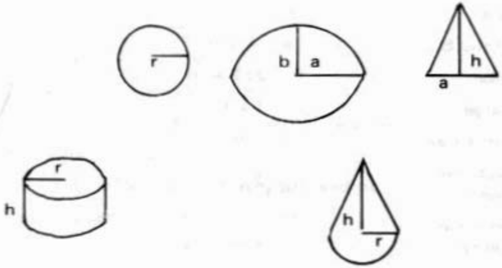
$$\int_0^\infty e^{-nx}\sqrt{x}\, dx = \frac{1}{2n}\sqrt{\frac{\pi}{n}}$$

$$\int_0^1 \frac{\ln x}{1 - x}\, dx = -\frac{\pi^2}{6}$$

$$\int_0^1 \frac{\ln x}{1 + x}\, dx = -\frac{\pi^2}{12}$$

$$\int_0^1 (\ln x)^n\, dx = (-1)^n \cdot n!$$

Geometric Formulas



1. **Circumference:**

   Circle $2\pi r$

2. **Area:**

   Circle πr^2

   Ellipse πab

   Sphere $4\pi r^2$

   Cylinder $2\pi r + hr$

   Triangle $\frac{1}{2}$ ah

3. **Volume:**

Ellipsoial of Revolution	$^4/_3\, b^2 a$
Sphere	$^4/_3\pi r^3$
Cylinder	$\pi r^2 h$
Cone	$\dfrac{\pi r^2 h}{12}$

Electrical Engineering Formulas

Force	$F = ma$
Work	$W = F \times d$
Kinetic E.	$ke = 1/2\ mv^2$
Power	$P = 2\pi T \times n$
Charge	$Q = I \times t$
Ohm's Law	$I = V/R$
Electrical Power	$= I^2R$
Electrical Energy	$= IVt$ joules

For resistances in series,

$$R = R_1 + R_2 + \dots\dots$$

For resistances in parallel,

$$1/R = 1/R_1 + 1/R_2 + \dots\dots$$

Charge $Q = C \times V$

For capacitors in parallel,

$$C = C_1 + C_2 + \dots\dots$$

For capacitors in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots\dots$$

For a circuit with R, L and C in series,

$$\text{Impedance} = Z = \sqrt{\left\{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2\right\}}$$

where $2\pi fL$ = inductive reactance in Ω

and $1/2\pi fC$ = capacitive reactance in Ω

If ϕ = phase difference between current and supply voltage,

$$\tan\phi = \frac{2\pi fL - 1/(2\pi fC)}{R}$$

$\cos \phi = R/Z$

$$\sin \phi = \frac{2\pi fL - 1/(2\pi fC)}{Z}$$

For purely resistive circuit,

$I = V/R$ and $\phi = 0$

For purely inductive circuit,

$I = V/2\pi fL \quad \phi = 90^\circ$, current lagging

For purely capacitive circuit,

$I = 2\pi\ fCV \quad \phi = 90$, current leading

For resonance in a series circuit,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

and Q-factor $= \dfrac{2\pi fL}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

For resonance in a parallel circuit,

$$C = \frac{L}{R^2 + (2\pi fL)^2}$$

and $f = \dfrac{1}{2\pi\sqrt{LC}}$ where $R << 2\pi fL$

Dynamic impedance of resonant parallel circuit

$$= \frac{L}{CR}$$

In an a·c circuit, power in watts = VI x power factor

power factor $= \dfrac{\text{power in watts}}{\text{r·m·s volts x r·m·s amperes}}$

General transmission-line equations

(1) rms voltage and current at the sending end,

$$E = E_s \cosh \gamma_x - I_s Z_c \sinh \gamma_x$$

$$I = I_s \cosh \gamma_x - \frac{E_s}{Z_c} \sinh \gamma_x$$

(2) voltage and current at the receiving end,

$$E = E_R \cosh \gamma_d + I_R Z_c \sin \gamma_d$$

$$I = I_R \cosh \gamma_d + \frac{E_R}{Z_c} \sinh \gamma_d$$

(3) input impedance Z_s is,

$$Z_s = Z_c \frac{Z_R + Z_c \tanh \gamma_s}{Z_c + Z_R \tanh \gamma_s}$$

(4) short-circuited lines:

$$E = I_R Z_c \sinh \gamma_d \qquad E_R = 0$$

$$I = I_R \cosh \gamma_d$$

and

$$Z_s = Z_c \tanh \gamma_s$$

(5) open-circuited lines:

$$E = E_R \cosh \gamma_d \qquad I_R = 0$$

$$I = E_R \sinh \gamma_d$$

$$Z_s = Z_c \tanh \gamma_s$$

Important Formulas in Diffusion

Diffused Layers

$$C(x,t) = Cs\ \mathrm{erfc}\left[\frac{x}{2\sqrt{Dt}}\right]$$

where Cs = constant concentration
x = distance
t = time
D = diffusivity

$$Q(t) = \frac{2}{\sqrt{\pi}}\ Dt,\ Cs$$ = total # of impurity atoms/cm^2

$$C(x,t) = \frac{Q}{\sqrt{\pi Dt}}\ e^{-x^2/4Dt},$$ constant Q

$$Cs(t) = \frac{Q}{\sqrt{\pi Dt}}$$

Network theory - A Summary

In a four-terminal network connecting a load to a generator, the currents in the circuit can be found by solving the set of simultaneous equations,

$$\left.\begin{aligned} i_1 &= \frac{\Delta 11}{\Delta} e_1 + \frac{\Delta 12}{\Delta} e_2 \\ i_2 &= \frac{\Delta 21}{\Delta} e_1 + \frac{\Delta 22}{\Delta} e_2 \end{aligned}\right\} \quad 1.$$

where i_1 = generator or input current

i_2 = load or output current

Δ = determinant of the system of simultaneous eqn.

and Δij = cofactor of the element in the jth row and the kth column.

The behaviour of a four-terminal network can be described by four parameters. If a parameter is given by the ratio of current to voltage, it has dimensions of an admittance; if by the ratio of voltage to current its dimensions are those of an impedance. Otherwise it is a dimensionless ratio of two voltages or two currents.

When the two quantities are measured at different parts of the network, it is better to indicate this fact by qualifying their ratio as a transfer ratio. Thus we have four forms of transfer functions.

(1) The ratio of two voltages or voltage-transfer ratio.

(2) The ratio of two currents or current-transfer ratio.

(3) The ratio of one current to another voltage, or transfer.

(4) The ratio of one voltage to another current, admittance, impedance or transfer.

Since a coefficient $\Delta jk/\Delta$ in equation 1 has dimensions of an admittance it can be replaced by a parameter y_{jk}. Using this notation,

$$\left.\begin{aligned} i_1 &= y_{11}e_1 + y_{12}e_2 \\ i_2 &= y_{21}e_1 + y_{22}e_2 \end{aligned}\right\} \quad 2.$$

The four parameters, known as the admittance parameters or y parameters, completely describe the external hehaviour of the network.

The node analysis of the same four-terminal network produces the following equations:

$$\left.\begin{aligned} e_1 &= z_{11}i_1 + z_{12}i_2 \\ e_2 &= z_{21}i_1 + z_{22}i_2 \end{aligned}\right\} \quad 3.$$

in which the z-parameters have the physical dimensions of an impedance and are readily recognized as duals of the y-parameters of the same four-terminal network.

There are several other systems of network parameters. In on system the relations connecting currents and voltages at the input and output terminals are written as

$$\left.\begin{aligned} e_2 &= h_{11}i_1 + h_{12}e_2 \\ i_2 &= h_{21}i_1 + h_{22}e_2 \end{aligned}\right\} \quad 4.$$

The external behaviour of the network is now described in terms of the h-parameters which are dimensionally different, hence the name hybrid and the symbol h.

Remark: For an example on network theory, refer to the Applications section.

Appendix D.

Rechargeable Battery

AC Operation
Connect the charger to any standard electrical outlet and plug the jack into the Calculator. After the above connections have been made, the power switch may be turned "ON". (While connected to AC, the batteries are automatically charging whether the power switch is "ON" or "OFF").

Battery Operation
Disconnect the charger cord and push the power switch, "ON". With normal use a full battery charge can be expected to supply about 2 to 3 hours of working time.

When the battery is low, figures on display will dim. Do not continue battery operation, this indicates the need for a battery charge. Use of the calculator can be obtained during the charge cycle.

Battery Charging
Simply follow the same procedure as in AC operation. The calculator may be used during the change period. However, doing so increases the time required to reach full charge. If a power cell has completely discharged, the calculator should not be operated on battery power until it has been recharged for at least 3 hours, unless otherwise instructed by a notice accompanying your machine. Batteries will reach full efficiency after 2 or 3 charge cycles.

Use proper Commodore/CBM adapter-recharger for AC operation and recharging.

Adapter 640 or 707 North America

Adapter 708 England

Adapter 709 West Germany

Low Power

If battery is low calculator will:

a. Display will appear erratic
b. Display will dim
c. Display will fail to accept numbers

If one or all of the above conditions occur, you may check for a low battery condition by entering a series of 8's. If 8's fail to appear, operations should not be continued on battery power. Unit may be operated on AC power.

CAUTION
A strong static discharge will damage your machine.

Shipping Instructions

A defective machine should be returned to the authorized service center nearest you. See listing of service centers.

Temperature Range

Mode	Temperature °C	Temperature °F
Operating	0° to 50°	32° to 122°
Storage	–40° to 55°	–40° to 131°

WARRANTY

Your new electronic calculator carries a parts and labor warranty for 12 months from date of purchase.

We reserve the right to repair a damaged component, replace it entirely, or, if necessary, exchange your machine.

This warranty is valid only when a copy of your original sales slip or similar proof of purchase accompanies your defective machine.

This warranty applies only to the original owner. It does not cover damage or malfunctions resulting from fire, accident, neglect, abuse or other causes beyond our control.

The warranty does not cover the repair or replacement of plastic housings or transformers damaged by the use of improper voltage. Nor does it cover the replacement of expendable accessories and disposable batteries.

The warranty will also be automatically voided if your machine is repaired or tampered with by an unauthorized person or agency.

This warranty supersedes, and is in lieu of, all other expressed warranties.

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